

EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 1
Department of Electrical Engineering and Computer Sciences 5 October 2005
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name _____ FIRST Name _____
Lab Time _____

- (10 Points) Print your name and lab time in legible, block lettering in the appropriate spaces provided above AND on the back of the last page.
- This midterm should take you up to 80 minutes to complete. You will be given at least 80 minutes—up to a maximum of 90 minutes—to work on the midterm.
- Aside from a two-sided 8.5 × 11-inch formula sheet for your exclusive use, you may not access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—including, for example, commencing work prematurely or continuing beyond the announced stop time—is a serious violation of the Code of Student Conduct.
- **The midterm printout consists of pages numbered 1 through 10.** When you are prompted by the teaching staff to begin work, verify that your copy of the midterm is free of printing anomalies and contains all of the ten numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in evaluating your midterm. No exceptions.*
- **NO PROBLEM ON THIS MIDTERM REQUIRES AN EXPLANATION.** Feel free to use for scratch work the blank spaces provided herein. Note, however, that only your final answers are looked at in evaluating your work.
- We hope you wrestle and beat this midterm to the ground!

Problem 1 (42 Points) Consider the set

$$X = [\{a, b, c\} \rightarrow \{1, 2, 3\}].$$

(a) (14 Points) How many elements are in the set X ? Explicitly specify one element in X .

(b) (14 Points) Of the elements in X , how many define, respectively, a graph of an "onto" function $f : \{a, b, c\} \rightarrow \{1, 2, 3\}$? Explicitly specify one element in X that defines a graph of an onto function.

(c) (14 Points) Of the elements in X , how many define, respectively, a graph of a one-to-one function $f : \{a, b, c\} \rightarrow \{1, 2, 3\}$? Explicitly specify one element in X that defines a graph of a one-to-one function.

Problem 2 (20 Points) Consider two assertions P and Q . We know that the following compound assertion is true:

$$(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q).$$

We do not know whether Q is true; it may or may not be true.

Choose the strongest statement below that is most accurately reflective of the truth or falsehood of P . Circle your choice.

- (I) P must be true.
- (II) P could be true, but does not have to be true.
- (III) P must be false.

Feel free to use the blank space below for scratch work. There is no partial credit here, so your work below will not be considered in grading this problem. Only the choice you circle will be considered.

Problem 3 (14 Points) Consider the following system F :

$$F : [\mathbb{R} \rightarrow \mathbb{R}] \rightarrow [\mathbb{R} \rightarrow \mathbb{R}] \text{ such that } \forall x \in [\mathbb{R} \rightarrow \mathbb{R}] \text{ and } \forall t \in \mathbb{R},$$

$$(F(x))(t) = (t + 1)x(t),$$

where \mathbb{R} denotes the set of real numbers.

Choose the strongest statement below that most accurately reflects one or more properties of the system F .

- (I) The system F is memoryless and causal.
- (II) The system F is memoryless, but it is not causal.
- (III) The system F is not memoryless, but it is causal.
- (IV) The system F is neither memoryless nor causal.

There is no partial credit here. Accordingly, no explanation will be considered in grading your selection among the statements above.

(10) (14 Points) Of the elements in X , how many define, respectively, a graph of an "onto" function $f : \{a, b, c\} \rightarrow \{1, 2, 3\}$? Explicitly specify how many elements in X that defines a graph of an onto function.

6 elements / define an onto

one element = $\{(a, 1), (b, 2), (c, 2)\}$

(10) (14 Points) Of the elements in X , how many define, respectively, a graph of a one-to-one function $f : \{a, b, c\} \rightarrow \{1, 2, 3\}$? Explicitly specify how many elements in X that defines a graph of a one-to-one function.

5 elements / define one-to-one

One element = $\{(a, 1), (b, 2), (c, 3)\}$

Problem 4 (14 Points) Determine whether the following statement is true or false:

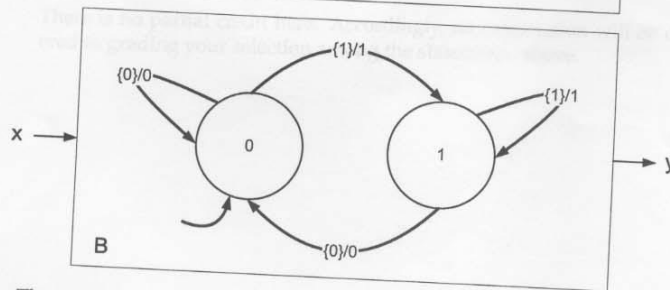
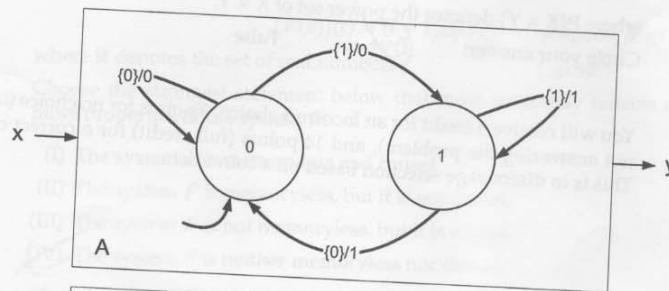
$$\{g \mid g = \text{graph}(f) \wedge f : X \rightarrow Y\} \subset P(X \times Y),$$

where $P(X \times Y)$ denotes the power set of $X \times Y$.

Circle your answer: True False

You will receive 0 credit for an incorrect choice, 8 points for no choice (i.e., for not answering the problem), and 14 points (full credit) for a correct choice. This is to discourage selection based on a coin toss!

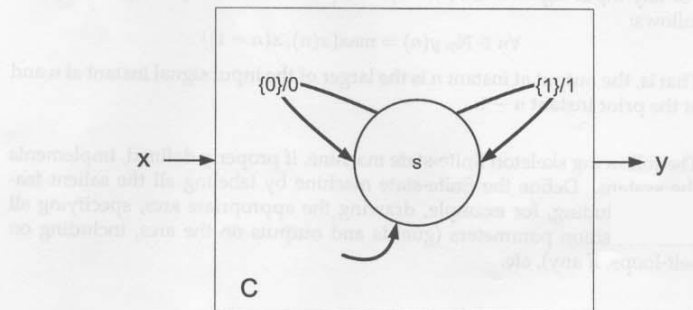
Problem 5 (30 Points) Consider two finite-state machines A and B having input and output alphabets $I_A = O_A = I_B = O_B = \{0, 1, \text{absent}\}$ and further characterized, respectively, by the state-transition diagrams shown below.



The input and output signal spaces for each machine are identical, i.e., $x, y \in [\mathbb{N}_0 \rightarrow \{0, 1, \text{absent}\}]$, where $\mathbb{N}_0 = \{0, 1, 2, \dots\}$.

- (a) (15 Points) Which one of the following four statements is true? Circle your choice.
- (I) A is not memoryless, and B is not memoryless.
 - (II) A is not memoryless, but B is memoryless.
 - (III) A is memoryless, but B is not memoryless.
 - (IV) A is memoryless, and B is memoryless.

(b) (15 Points) Consider a finite-state machine C having input and output alphabets given by $I_C = O_C = \{0, 1, \text{absent}\}$. The state-transition diagram for C is shown below:



Which one of the following four statements is true? Circle your choice.

- (I) C is bisimilar to A and is bisimilar to B .
- (II) C is bisimilar to A but is not bisimilar to B .
- (III) C is not bisimilar to A but is bisimilar to B .
- (IV) C is bisimilar neither to A nor to B .

Problem 6 (20 Points) Consider a system with an input signal space $X = [\mathbb{N}_0 \rightarrow \{0, 1, 2, 3\}]$ and an output signal space Y identical to the input signal space.

For any input signal $x \in X$, the system produces an output signal $y \in Y$ as follows:

$$\forall n \in \mathbb{N}_0, y(n) = \max(x(n), x(n-1)).$$

That is, the output at instant n is the larger of the input signal instant at n and at the prior instant $n-1$.

The following skeleton finite-state machine, if properly defined, implements the system. Define the finite-state machine by labeling all the salient features, including, for example, drawing the appropriate arcs, specifying all state transition parameters (guards and outputs on the arcs, including on self-loops, if any), etc.

For this problem, ignore stuttering.

