University of California, Berkeley
Department of Chemical and Biomolecular Engineering

CBE150A– Midterm #2
Transport Processes

Instructors: Prof. Joelle Frechette & Prof. Aditi Krishnapriyan

2023/04/07

Name: ______________________________ (in uppercase)

Student ID: ______________________________

This exam contains 15 pages (including this cover page) and 2 questions. Total of points is 50.

You have 50 minutes to complete this in-person exam. Do not begin the exam until the instructors tell you to start.

Please write your answers in the box if provided. Do your calculations in the space provided for the corresponding part. Any work done outside of specified areas or on the back of pages will **not be graded**. Use the blank white full page behind the question pages as a scratch sheet (your work here will **not be graded**). Also remember to provide your signature for the honor code below. Good luck!

Distribution of Marks

Question	Points	Score
1	25	
2	25	
Total:	50	

Honor Code: As a member of UC Berkeley, I act with honesty, integrity, and respect for others.

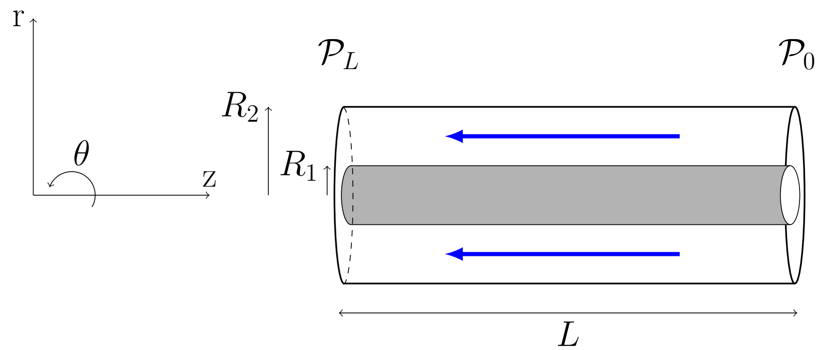
Signature: ______________________________

The two questions for the exam are written below and will be rewritten on subsequent pages for you to write down your answers.

Problem 1: Heat Exchanger (25 points). You are looking at the flow through two concentric cylinders, which may be used for example, to circulate heat through a heat exchanger (shown below).

- The heat exchanger has an inner radius of R_1 and an outer radius of R_2. Assume that $R_1 \leq r \leq R_2$.
- The inner tube is a solid core (no fluid), while in the outer tube, fluid is moving in the $-z$-direction (blue).
- The fluid is incompressible and Newtonian. The flow is laminar, fully-developed, and at steady-state. The flow is pressure-driven.
- The length of the heat exchanger is L.

Your goal is to find the velocity profile of the fluid flow in the **outer tube**, v_z.



Go through the following intermediate steps:

(a) Perform a kinematics analysis and use continuity to determine which variables the velocity, v_z, depends on. (2 pts)

(b) Start with the Navier-Stokes equations, given below, and determine which terms are zero. Write out the final expression. (6 pts)

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right] = -\frac{\partial \mathcal{P}}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] \quad (1)$$

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right] = -\frac{1}{r}\frac{\partial \mathcal{P}}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] \quad (2)$$

$$\rho\left[\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{\partial \mathcal{P}}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] \quad (3)$$

(c) Solve the expression that you get in part (b), and get an expression for v_z that includes any integration constants involved. (8 pts)

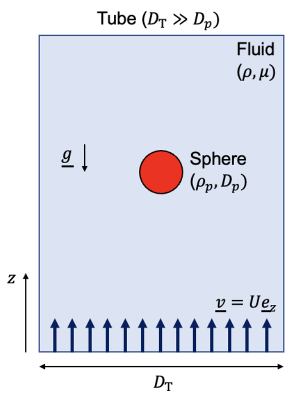
(d) Solve for the pressure drop (*Hint: Your final answer should not include any derivatives*). (2 pts)

(e) State the boundary conditions to find v_z. (2 pts)

(f) Are any of the constants zero? If so, why? If not, why not? (*Hint: Compare this to the case of flow through a regular non-annular cylinder*) (2 pts)

(g) Use the boundary conditions from part (e) to solve for the integration constants for v_z. Use these to get a final solution for v_z (Use the simplification from part (d) in this final solution). (3 pts)

Problem 2: Flow Around a Sphere (25 points). Consider a snapshot of a solid sphere in a long, wide tube filled with a Newtonian fluid ($\rho_p > \rho$). Fluid flows upward (in the positive z-direction) such that the sphere remains static (it does not move up or down). **Each part (a, b, c, d) can be done independently**. The data for the problem is below:



Sphere	Diameter: D_p	Density: $\rho_P > \rho$	
Fluid	Viscosity: μ	Density: ρ	Velocity: $\boldsymbol{v} = U\boldsymbol{e}_z$
Tube	Diameter: $D_T >> D_p$		

(a) In the limit of creeping flow (Re << 1) draw the fluid flow profile past the sphere on the schematic here (3 pts).

(b) Use an arrow to draw the drag force experienced by the sphere (3 pts).

(c) You are trying to determine the drag coefficient C_{D1} for this problem. Follow the steps below.

i. Write down a force balance on the sphere and obtain an expression for the drag force, F_{D1} (4 pts).

ii. Using your answer in (i), obtain an expression for the drag coefficient, C_{D1} (4 pts).

(d) Now you increase the flow velocity to go from $\text{Re}_1 = 10^{-2}$ to $\text{Re}_2 \approx 5 \times 10^3$ (Newton's Regime, $C_{D2} \approx 0.44$).

i. Upon this change, find the ratio of the new drag force to the old drag force F_{D2}/F_{D1} (5 pts).

ii. Based on your result in (i), will the sphere move up, down, or stay in place when the Reynolds number increases (explain in one sentence or one equation) (3 pts)?

iii. What would the flow around the sphere look like (3 pts)?

1. **Heat Exchanger (25 points)**. You are looking at the flow through two concentric cylinders, which may be used for example, to circulate heat through a heat exchanger (shown below).

- The heat exchanger has an inner radius of R_1 and an outer radius of R_2. Assume that $R_1 \leq r \leq R_2$.
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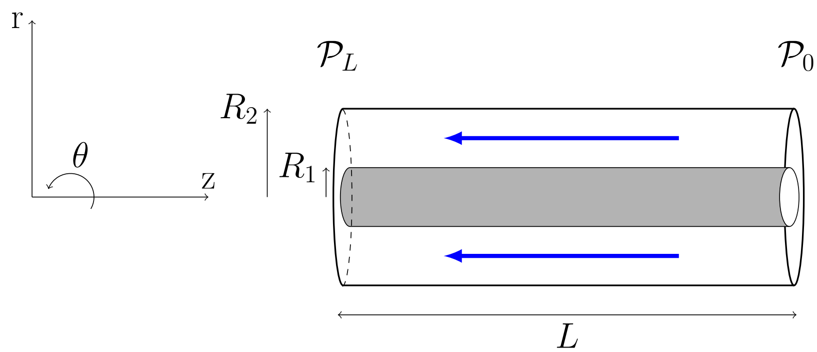
- The length of the heat exchanger is L.

Your goal is to find the velocity profile of the fluid flow in the **outer tube**, v_z.



Go through the following intermediate steps:

(a) (2 points) Perform a kinematics analysis and use continuity to determine which variables the velocity, v_z, depends on.

Answer:

(b) (6 points) Start with the Navier-Stokes equations, given below, and determine which terms are zero. Write out the final expression. (6 pts)

$$\rho\left[\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right] = -\frac{\partial \mathcal{P}}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right] \quad (4)$$

$$\rho\left[\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right] = -\frac{1}{r}\frac{\partial \mathcal{P}}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right] \quad (5)$$

$$\rho\left[\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right] = -\frac{\partial \mathcal{P}}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right] \quad (6)$$

Answer:

(c) (8 points) Solve the expression that you get in part (b), and get an expression for v_z that includes any integration constants involved.

Answer:

(d) (2 points) Solve for the pressure drop (*Hint: Your final answer should not include any derivatives*).

Answer:

(e) (2 points) State the boundary conditions to find v_z.

Answer:

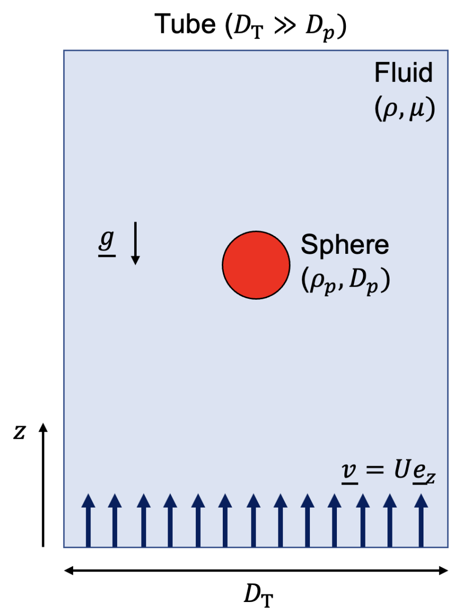
(f) (2 points) Are any of the constants zero? If so, why? If not, why not? (*Hint: Compare this to the case of flow through a regular non-annular cylinder*)

Answer:

(g) (3 points) Use the boundary conditions from part (e) to solve for the integration constants for v_z. Use these to get a final solution for v_z (Use the simplification from part (d) in this final solution).

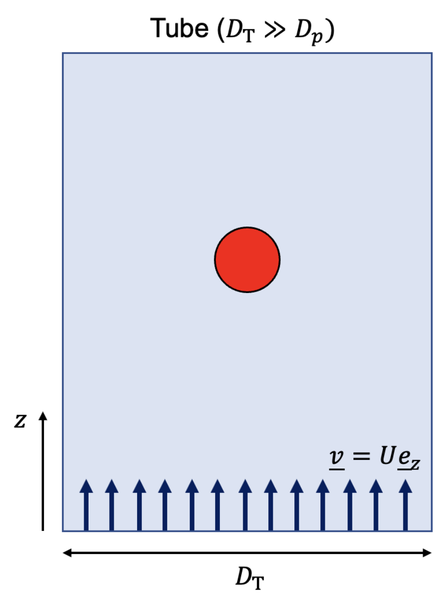
Answer:

2. **Flow Around a Sphere (25 points)**. Consider a snapshot of a solid sphere in a long, wide tube filled with a Newtonian fluid ($\rho_p > \rho$). Fluid flows upward (in the positive z-direction) such that the sphere remains static (it does not move up or down). **Each part (a, b, c, d) can be done independently**. The data for the problem is below:

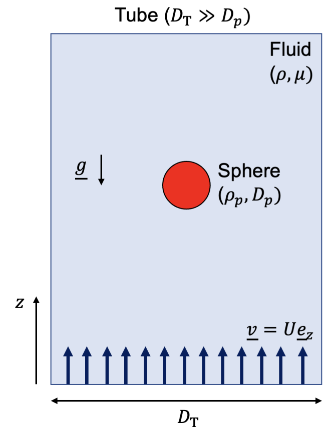


Sphere	Diameter: D_p	Density: $\rho_P > \rho$	
Fluid	Viscosity: μ	Density: ρ	Velocity: $\boldsymbol{v} = U\boldsymbol{e}_z$
Tube	Diameter: $D_T >> D_p$		

(a) (3 points) In the limit of creeping flow (Re << 1) draw the fluid flow profile past the sphere on the schematic here.



(b) (3 points) Use an arrow to draw the drag force experienced by the sphere.



(c) (8 points) You are trying to determine the drag coefficient C_{D1} for this problem. Follow the steps below.

(i) (4 points) Write down a force balance on the sphere and obtain an expression for the drag force, F_{D1}.

Answer:

(ii) (4 points) Using your answer in (i), obtain an expression for the drag coefficient, C_{D1}.

Answer:

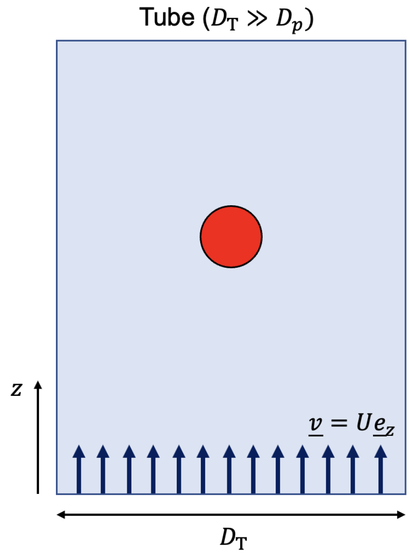
(d) (11 points) Now you increase the flow velocity to go from $\mathrm{Re}_1 = 10^{-2}$ to $\mathrm{Re}_2 \approx 5 \times 10^3$ (Newton's Regime, $C_{D2} \approx 0.44$).
(i) (5 points) Upon this change, find the ratio of the new drag force to the old drag force F_{D2}/F_{D1}.

Answer:

(ii) (3 points) Based on your result in (i), will the sphere move up, down, or stay in place when the Reynolds number increases (explain in one sentence or one equation)?

Answer:

(iii) (3 points) What would the flow around the sphere look like?



Scratch Paper. The work on this page will not be graded.