University of California, Berkeley
Department of Chemical and Biomolecular Engineering

CBE150A– Midterm #1
Transport Processes

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2023/03/03

Name: ________________________________ (in uppercase)

Student ID: ________________________________

This exam contains 12 pages (including this cover page) and 2 questions. Total of points is 50.

You have 50 minutes to complete this in-person exam. Do not begin the exam until the instructors tell you to start.

Please write your answers in the box if provided. Do your calculations in the space provided for the corresponding part. Any work done outside of specified areas will **not be graded**. Use the blank white full pages behind the question pages as scratch sheets (your work here will **not be graded**). Also remember to provide your signature for the honor code below. Good luck!

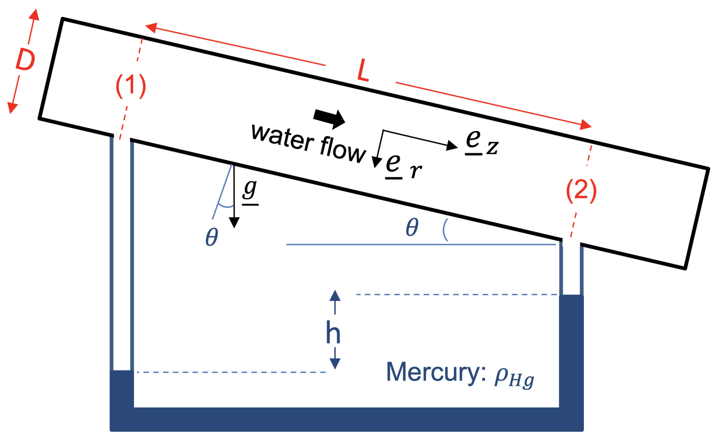
Distribution of Marks

Question	Points	Score
1	25	
2	25	
Total:	50	

Honor Code: As a member of UC Berkeley, I act with honesty, integrity, and respect for others.

Signature: ________________________________

1. **Pipe with a Manometer**. Water (density ρ_w) flows steadily down the inclined pipe at a constant mass flow rate $\dot{m}$ as indicated in the figure below. The velocity profile is uniform. Answer the questions below listing all your assumptions and clearly define your control volume. **Each part (a-c) can be done independently.**



(a) (9 points) What is the pressure difference between section (1) and (2), i.e. $(P_1 - P_2)$, in terms of known variables? Note: The pressure does not vary radially within the pipe.

Answer:

(b) (11 points) **For part (b) leave the pressure drop as** $(P_1 - P_2)$ **in your solution.** What is the net force in the direction of the flow (z-direction) exerted by the pipe wall on the flowing water between sections (1) and (2)?

Answer:

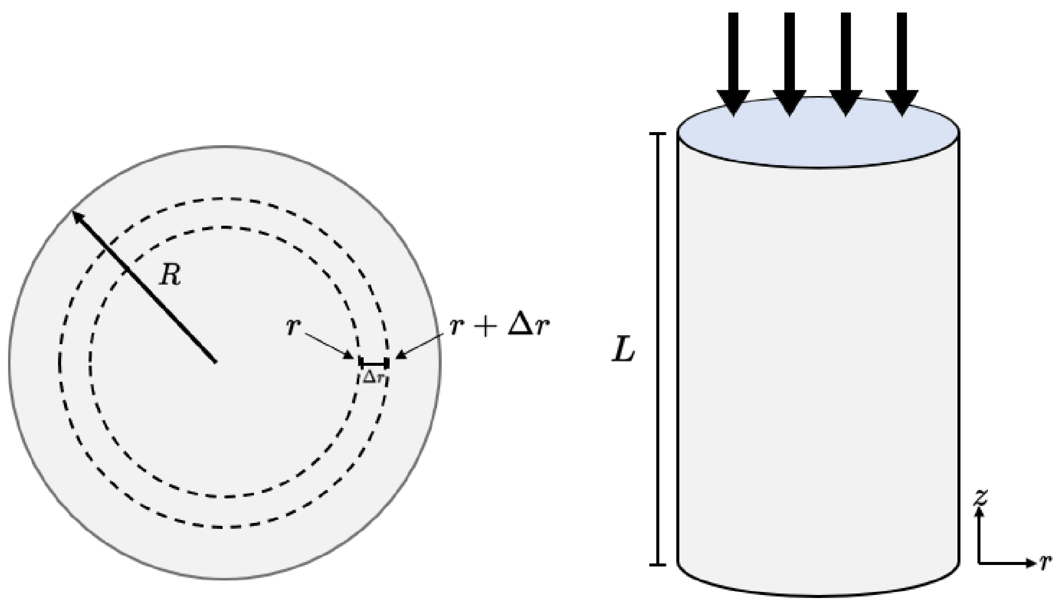
(c) (5 points) **For part (c) leave the pressure drop as** $(P_1 - P_2)$ **in your solution.** What is the power loss caused by friction between sections (1) and (2)?

Answer:

2. **Microfluidic Device**. You are looking at a cylindrical microfluidic device through which laminar fluid axially flows.

   - The fluid flow is at steady-state,
   - The fluid flow is laminar (droplet-based microfluidics approaches explicitly aim to ensure laminar flow within the system!),
   - The fluid flow is downwards (in the z-direction) because of a pressure difference and gravity, where $P = P(z)$,
   - The fluid is Newtonian,
   - The fluid is incompressible.

   Note that it is easiest to use a cylindrical coordinate system to solve this problem.



Top View Side View

(a) (4 points) Perform a kinematics analysis and use continuity to determine which variables the velocity, v_z, depends on.

Answer:

(b) (6 points) Which of the nine components of the viscous stress tensor $\boldsymbol{\tau}$, are non-zero? Explain why each component is or isn't non-zero. The nine components are as follows:

$$\tau_{rr} = 2\mu \left[\frac{\partial v_r}{\partial r} - \frac{1}{3} \left(\boldsymbol{\nabla} \cdot \boldsymbol{v} \right) \right]$$

$$\tau_{\theta\theta} = 2\mu \left[\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} - \frac{1}{3} \left(\boldsymbol{\nabla} \cdot \boldsymbol{v} \right) \right]$$

$$\tau_{zz} = 2\mu \left[\frac{\partial v_z}{\partial z} - \frac{1}{3} \left(\boldsymbol{\nabla} \cdot \boldsymbol{v} \right) \right]$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]$$

$$\tau_{\theta z} = \tau_{z\theta} = \mu \left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right]$$

$$\tau_{zr} = \tau_{rz} = \mu \left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right]$$

Answer:

(c) (6 points) Consider a thin shell, with thickness Δr, as illustrated in the figure on Page 5. What is the volume of the shell, the area at the top of the shell, and the area of the side of the shell?

Answer:

(d) (9 points) What are the different force contributions in this system? Name each one, and write down the expression for it. Express your answer in terms of each force acting on the shell fluid element.

Answer:

Scratch Paper. The work on this page will not be graded.

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