BioE102:Final Exam

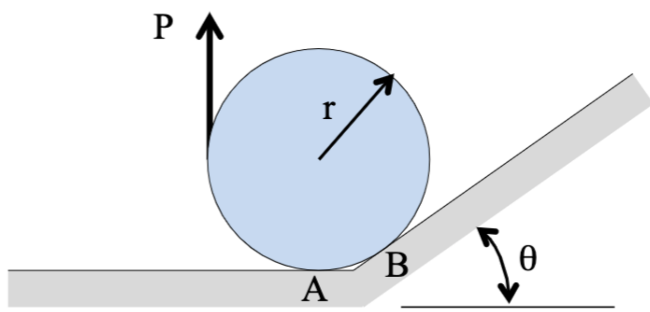
Note: Please make sure

- To turn on your CAMERA
- Not to use any virtual background
- To write your name, your last name and your student ID on top of every pages of your work.
- To draw a box around your final answer.
- To draw FBD and coordinate system for all problems.

Problem 1 (35 Points)

A cylinder of radius r and weight W is at the intersection between a horizontal plane (the "floor") and an inclined plane that makes an angle θ with the horizontal. The cylinder is in contact with the floor at point A and with the inclined surface at point B. A vertical force P is applied at the left side of the cylinder. Let μ be the coefficient of static friction at both A and B, and let μ be sufficiently large that the cylinder does not slip with respect to the contacting surfaces, but instead begins to roll up the incline when P is large enough.

1. Determine the minimum value of μ that will ensure that the impending motion is rolling about B, not slipping at A and B. Express your answer in terms of W, r, and θ.
2. Determine P when this rolling motion is impending. Express your answer in terms of W, r, and θ.
3. Determine the normal and frictional forces at both A and B when motion is impending. Again, express your answers in terms of W, r, and θ.

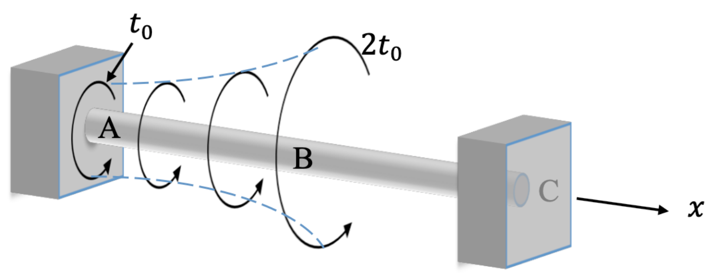


Problem 2 (35 Points)

The circular shaft shown below has polar moment of inertia J, shear modulus G and length L. It is rigidly supported at both ends. A distributed torque $t(x) = t_0 \left(1 + \left(\frac{2x}{L}\right)^2\right)$ is applied along the shaft between A at $x = 0$ and B at $x = \frac{L}{2}$. Recall that the distributed torque has dimensions of moment per unit length, so in SI, t_0 would have units of $\frac{Nm}{m}$.

1. Determine the reaction torques at A and C.
2. Derive an equation for the twist angle for $x \leq \frac{L}{2}$.

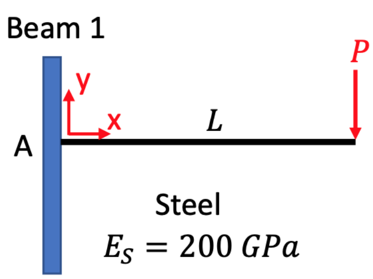
Express your answers in terms of the parameters given: J, L, G, and t_0.



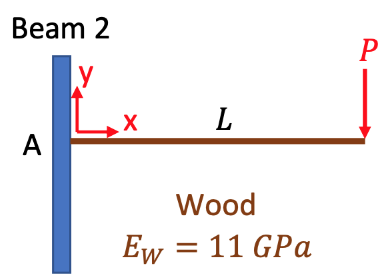
Problem 3 (30 Points)

We have two beams with the same length, same width and same height (so these beams have identical cross-section and identical geometry) but one of them is made of stainless steel (with modulus of elasticity $E_S = 200$ GPa) and the other one is made of wood (with modulus of elasticity $E_W = 11$ GPa) as shown below. These beams are rigidly attached to the wall at one end and on the other end, we applied the force P (which is equal for both beams).

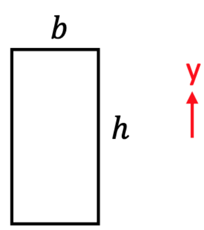
For this problem, find the beam that has the highest normal stress at each cross section and you should justify your answer mathematically.



The beam is rigidly attached to the wall at A



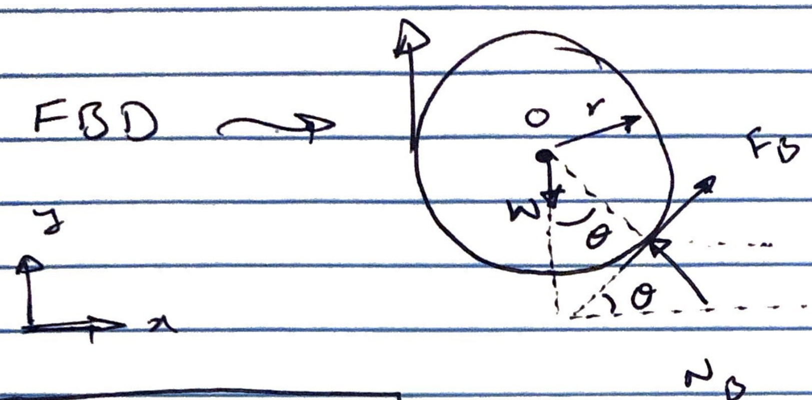
The beam is rigidly attached to the wall at A



The cross section of both beams

(P1)

FBD $\rightsquigarrow$



$$\boxed{N_A = F_A = 0}$$

because rolling motion is impending

(1) $\overset{+}{\curvearrowleft} \sum M_o = 0 \rightsquigarrow F_B r - P r = 0 \rightsquigarrow \boxed{F_B = P}$

$\overset{+}{\rightarrow} \sum F_x = 0 \rightsquigarrow F_B \cos\theta - N_B \sin\theta = 0$

$\rightsquigarrow F_B = N_B \tan\theta$

also $F_B \leq \mu N_B \rightsquigarrow N_B \tan\theta \leq \mu N_B$

$\Rightarrow \boxed{\mu \geq \tan\theta} \Rightarrow \mu_{Min} = \tan\theta$

(2) $+\uparrow \sum F_y = 0 \rightsquigarrow P - W + F_B \sin\theta + N_B \cos\theta = 0$

also $F_B = P$ and $F_B = N_B \tan\theta$

$\Rightarrow N_B = P \cot\theta$

(P1)

$$\Rightarrow P - W + P\sin\theta + P\,\mathrm{Cotg}\,\theta * \cos\theta = 0$$

$$\Rightarrow P\left[1 + \sin\theta + \frac{\cos^2\theta}{\sin\theta}\right] = W$$

$$\Rightarrow P\left[\frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\sin\theta}\right] = W$$

and $\sin^2\theta + \cos^2\theta = 1$

so $P\,\dfrac{\sin\theta + 1}{\sin\theta} = W$

$$\Rightarrow \boxed{P = W\,\frac{\sin\theta}{1+\sin\theta}}$$

(5) we found $N_A = F_A = 0$ and $N_B = P\,\mathrm{Cotg}\,\theta$

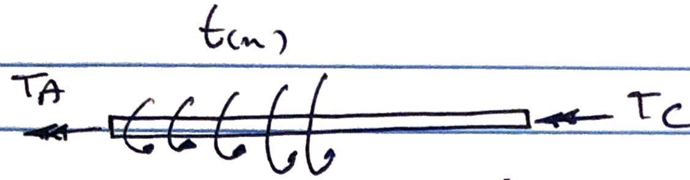
and $F_B = P$

so $F_B = P = W\,\dfrac{\sin\theta}{1+\sin\theta}$

and $N_B = P\,\dfrac{\cos\theta}{\sin\theta} = W\,\dfrac{\sin\theta}{1+\sin\theta} * \dfrac{\cos\theta}{\sin\theta}$

$$\Rightarrow \boxed{N_B = \frac{W\cos\theta}{1+\sin\theta}}$$

P2

FBD $\rightarrow$ 

$$+\rightarrow\ \Sigma T = 0 \rightarrow -T_A - T_C + \int_0^{l/2} t_0\left[1+\left(\frac{2x}{l}\right)^2\right]dx = 0$$

$$\Rightarrow T_A + T_C = t_0 \frac{l}{2} + t_0 \frac{4}{l^2}\int_0^{l/2} x^2\, dx$$

$$\rightarrow T_A + T_C = t_0 \frac{l}{2} + t_0 \frac{4}{l^2} * \frac{1}{3} \cdot \frac{l^3}{8}$$

$$\rightarrow T_A + T_C = t_0\left[\frac{l}{2} + \frac{l}{6}\right] = t_0\, l\, \frac{4}{6}$$

$$\Rightarrow T_A + T_C = \frac{4l}{6} t_0 = \frac{2}{3}\, l\, t_0$$

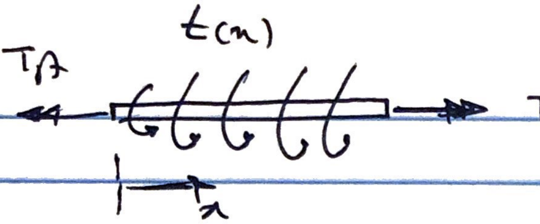
$$\Phi_A = \Phi_C = 0 \rightarrow \Phi_C - \Phi_A = \Delta\Phi_{C/A} = 0$$

$$\Phi_C - \Phi_B + \Phi_B - \Phi_A = \Delta\Phi_{C/B} + \Delta\Phi_{B/A} = 0$$

$$\Delta\Phi_{B/A} = \int_{x_A}^{x_B} \frac{T(x)\,dx}{JG} \quad \text{and} \quad \Delta\Phi_{C/B} = \int_{x_B}^{x_C} \frac{T(x)\,dx}{JG}$$

(P2)

$\Delta \Phi_{B/A} \rightsquigarrow$



$$\Rightarrow \overset{+}{\rightarrow} \sum T = 0$$

$$\rightsquigarrow -T_A + T(x) + \int_0^x t_0\left(1 + \frac{4x^2}{l^2}\right)dx = 0$$

$$\rightsquigarrow -T_A + T(x) + t_0 x + t_0 \frac{4}{3}\frac{x^3}{l^2} = 0$$

$$\rightsquigarrow T(x) = T_A - t_0\left(x + \frac{4}{3}\frac{x^3}{l^2}\right)$$

$$\Delta \Phi_{B/A} = \int_{x_A}^{x_B} \frac{T(x)\,dx}{JG}$$

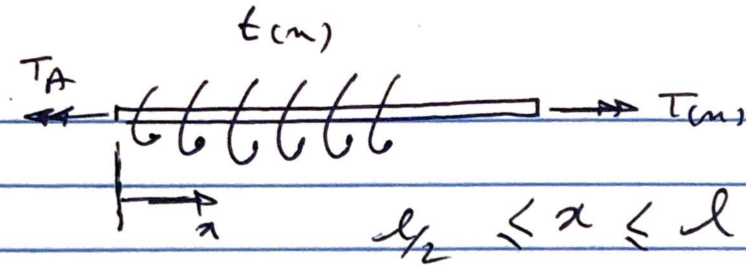
$$\rightsquigarrow \Delta \Phi_{B/A} = \frac{1}{JG}\int_0^{l/2} \left\{T_A - t_0\left(x + \frac{4}{3}\frac{x^3}{l^2}\right)\right\}dx$$

$$\Rightarrow \Delta \Phi_{B/A} = \frac{1}{JG}\left[T_A \frac{l}{2} - t_0\left(\frac{x^2}{2} + \frac{x^4}{3l^2}\right)\Big|_0^{l/2}\right]$$

$$\Rightarrow \Delta \Phi_{B/A} = \frac{1}{JG}\left[T_A \frac{l}{2} - t_0\left(\frac{l^2}{8} + \frac{1}{3l^2} * \frac{l^4}{16}\right)\right]$$

$$\Rightarrow \Delta \Phi_{B/A} = \frac{l}{JG}\left\{\frac{T_A}{2} - \frac{7}{48} t_0 l\right\}$$

(P2) $\Delta\Phi_{C/B} \rightsquigarrow$



$$\Sigma T = 0 \rightsquigarrow -T_A + T(x) + \int_0^{l/2} t(x)\,dx = 0$$

$$\Rightarrow T(x) = -T_C$$

$$\Delta\Phi_{C/B} = \int_{l/2}^{l} \frac{T(x)\,dx}{JG} = -\frac{T_C\, l/2}{JG}$$

$$\Delta\Phi_{C/B} + \Delta\Phi_{B/A} = 0$$

$$\rightsquigarrow \frac{l}{JG} * \frac{1}{2}\left[T_A - \frac{7}{24} t_0 l\right] - T_C \frac{l}{JG}\frac{1}{2} = 0$$

$$\Rightarrow T_A - T_C = \frac{7}{24} t_0 l$$

$$T_A + T_C = \frac{2}{3} l t_0$$

$$\Rightarrow 2T_A = \left(\frac{2}{3} + \frac{7}{24}\right) t_0 l$$

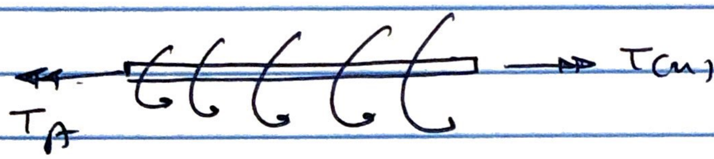
$$\rightsquigarrow 2T_A = \left(\frac{16+7}{24}\right) t_0 l$$

$$\rightsquigarrow T_A = \frac{23}{48} t_0 l \rightsquigarrow T_C = \frac{9}{48} t_0 l$$

(P2) $\Phi(x) = \Phi(x) - \Phi_A = \Phi_{x/A}$

$0 \leq x \leq l/2$

FBD



$\overset{+}{\rightarrow} \Sigma T = 0 \rightsquigarrow T(x) = T_A - t_0\left(x + \frac{4}{3}\frac{x^3}{l^2}\right)$

$$\Delta \Phi_{x/A} = \int_{x_A}^{x} \frac{T(x)\,dx}{JG} = \frac{T_A x - t_0\left(\frac{x^2}{2} + \frac{x^4}{3l^2}\right)}{JG}$$

(P3)

$$\sigma = -\frac{My}{I}$$

they have same M, y and I

so $\sigma_S^N = \sigma_W^N$