University of California at Berkeley
College of Engineering
Dept. of Electrical Engineering and Computer Sciences

EE 105 Midterm 2

Spring 2006 | Prof. Ming C. Wu | April 6, 2006

Guidelines

- Closed book and notes.
- Two pages of information sheets allowed.
- Total time = 90 minutes

(1) For the circuit shown in Fig. 1, W/L = 2 for both M_1 and M_2, $\mu_n C_{ox}$ = 100 $\mu A/V^2$, $\lambda = 0.05\ V^{-1}$, V_{Tn} = 1V, V_{DD} = 5V.

a) [5 pt] Find the DC drain current at M_2 when V_{OUT} = 3V. Use $\lambda = 0$ for this part.
b) [5 pt] Find the DC gate bias (V_G) of M_2 such that the DC output voltage V_{OUT} = 3V. Use $\lambda = 0$ for this part.
c) [5 pt] Draw the small-signal equivalent circuit. Find the values of all circuit elements in the small signal circuit (e.g., g_m, r_0, …).
d) [5 pt] Find the voltage gain, $A_V = v_{out} / v_s$.
e) [5 pt] Find the output resistance of the circuit (both expression and numeric value).
f) [5 pt] Find the input resistance, and construct the two-port model of this voltage amplifier.

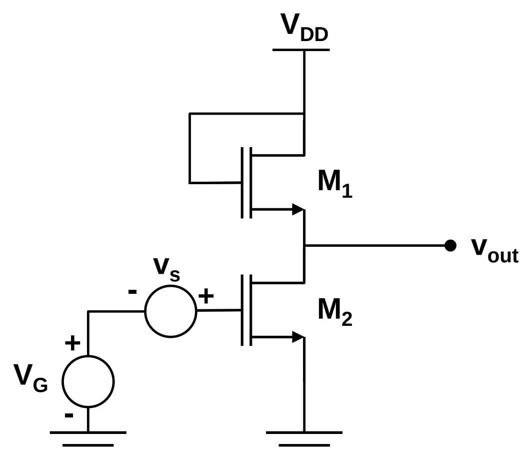


Fig. 1

(2) Consider the following circuit with $(W/L)_1 = 2$, $(W/L)_2 = 1$, $\mu_p C_{ox}$ = 50 $\mu A/V^2$, $\mu_n C_{ox}$ = 100 $\mu A/V^2$, $\lambda_n = \lambda_p = 0.05\ V^{-1}$, V_{Tn} = 1V and V_{Tp} = -1V, V_{DD} = 5V :

a) [10 pt] The gate is biased at 2.5V DC. Show that both transistors are in saturation regime. Find the expression and numeric value of small-signal voltage gain, $A_V = v_{out} / v_s$
b) [10 pt] Find the maximum and the minimum voltage at the output of this circuit when both transistors stay in saturation regime.

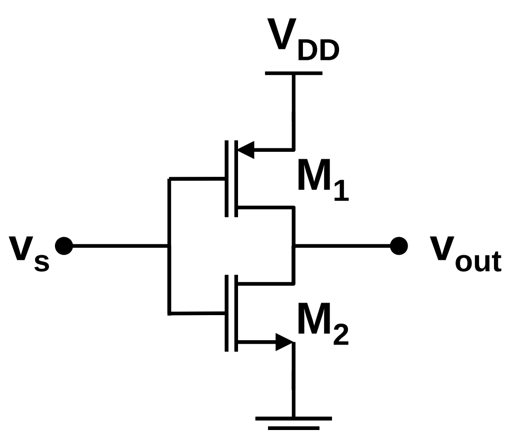


Fig. 2

(3) Consider the following circuit with 3 PMOS transistors: $(W/L)_1 = 10$, $(W/L)_2 = 20$, $(W/L)_3 = 100$, $\mu_p C_{ox} = 50\ \mu A/V^2$, $\lambda = 0.05\ V^{-1}$, $V_{Tp} = -1$ V, $I_{REF} = 10\ \mu A$, $V_+ = 3$ V, $V_- = -3V$, $R_L = 100\ K\Omega$.

a) [5 pt] Can you identify any functional block in this circuit (i.e., any portion of the circuit that performs a known function)? Replace that functional block, and draw a simplified circuit of the amplifier.

b) [10 pt] Find the expression of the voltage gain, $A_V = v_{out} / v_s$, and then find its numerical value.

c) [10 pt] Find both the expression and the numeric value of the output resistance of the amplifier.

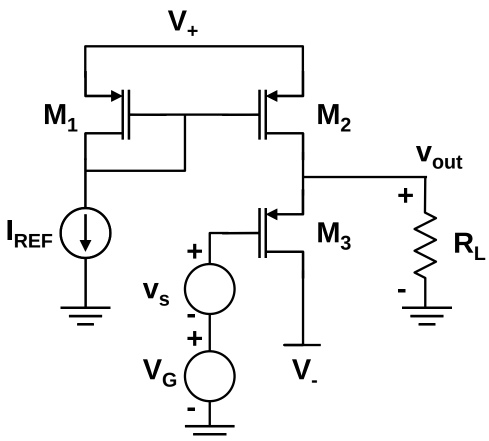


Fig. 3

(4) The frequency response of an amplifier is shown in the figure below.

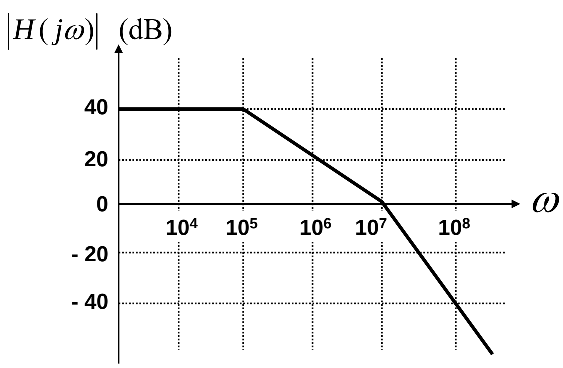


Fig. 4A

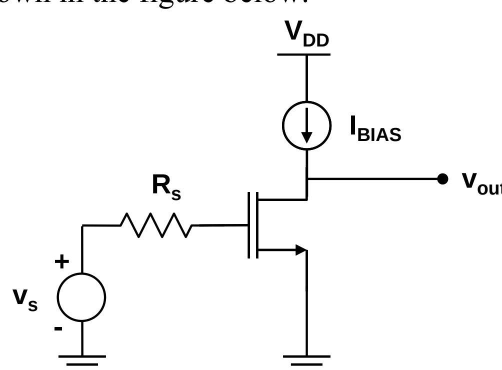


Fig. 4B

a) [8 pt] Find the transfer function of the frequency response shown in Fig. 4A.

b) [7 pt] This transfer function can be realized by the circuit in Fig. 4B. Draw the small-signal circuit that includes C_{gd}. For simplicity, we will neglect C_{gs}. Analysis of this circuit can be simplified by Miller approximation. Draw the simplified small-signal equivalent circuit. Show the Miller capacitances explicitly in terms of other circuit parameters.

c) [10 pt] The following parameters of the circuit are given:
$I_{BIAS} = 10\ \mu A$, $\lambda = 0.1\ V^{-1}$, $r_{oc} = \infty$ (ideal current source).
If the frequency response of the amplifier matches the transfer function shown in Fig. 4A, find the numeric values of the transistor parameters: C_{gd}, r_0, g_m, and the circuit parameter, R_s.

Some equations

Threshold voltage (NMOS)

$$V_{Tn} = V_{FB} - 2\phi_p + \frac{1}{C_{ox}}\sqrt{2q\varepsilon_s N_a(-2\phi_p)} \qquad \phi_p = -\frac{kT}{q}\ln\frac{N_a}{n_i}$$

$$V_{Tn} = V_{Tn0} + \gamma\left(\sqrt{V_{SB} - 2\phi_p} - \sqrt{-2\phi_p}\right)$$

NMOS equations:

$$I_D = 0, \quad V_{GS} < V_{Tn}$$

$$i_D = \frac{W}{L}\mu C_{ox}\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right)v_{DS}(1+\lambda V_{DS}), \quad V_{GS} > V_{Tn}, V_{DS} < V_{GS} - V_{Tn}$$

$$i_D = \frac{W}{L}\frac{\mu C_{ox}}{2}(v_{GS} - V_{Tn})^2(1+\lambda V_{DS}), \quad V_{GS} > V_{Tn}, V_{DS} > V_{GS} - V_{Tn}$$

MOS capacitances in saturation $C_{gs} = (2/3)WLC_{ox} + C_{ov}$ $\qquad C_{ov} = L_D W C_{ox}$

MOS signal parameters:

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{V_{GS},V_{DS}} = \mu C_{ox}\frac{W}{L}(V_{GS} - V_{Tn})(1+\lambda V_{DS})$$

$$\approx \mu C_{ox}\frac{W}{L}(V_{GS} - V_{Tn})$$

$$= \sqrt{2i_D\left(\frac{W}{L}\right)\mu C_{ox}}$$

$$r_o = \left(\left.\frac{\partial i_D}{\partial v_{DS}}\right|_{V_{GS},V_{DS}}\right)^{-1} \approx \frac{1}{\lambda I_{DS}}$$

$$g_{mb} = \left.\frac{\partial i_D}{\partial v_{BS}}\right|_Q = \frac{\gamma g_m}{2\sqrt{-V_{BS} - 2\phi_p}}$$

(1) (a) current determined by M_1

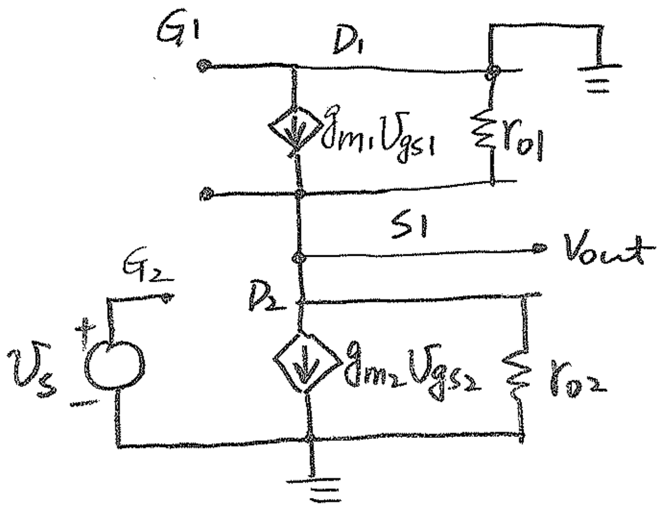
$V_{GS1} = V_{DS1} = V_{DD} - V_{out} = 5 - 3 = 2V$ when $V_{out} = 3V$

$I_{D2} = I_{D1} = \left(\frac{W}{L}\right) \cdot \frac{\mu_n C_{ox}}{2} (V_{GS1} - V_{Tn})^2 = 2 \cdot \frac{100}{2} \cdot 1^2 = 100 \mu A$.

(b) $I_{D2} = I_{D1} = \left(\frac{W}{L}\right) \cdot \frac{\mu_n C_{ox}}{2} \cdot (V_{GS2} - V_{Tn})^2 = 100$

$V_{GS2} - V_{Tn} = 1 \Rightarrow V_{GS2} = 2V$.

(c)



$g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right) \cdot (V_{GS1} - V_{Tn})$

$= 100 \cdot 2 \cdot 1 = 200 \mu S$

$g_{m2} = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS2} - V_{Tn})$

$= 200 \mu S$

$r_{o1} = \frac{1}{\lambda I_{D1}} = \frac{1}{0.05 \cdot 100 \times 10^{-6}} = 200 k\Omega$

$r_{o2} = \frac{1}{\lambda I_{D2}} = 200 k\Omega$

$v_{gs1} = 0 - v_{out} = -v_{out}$

$v_{gs2} = v_s$

(d) KCL at D_2:

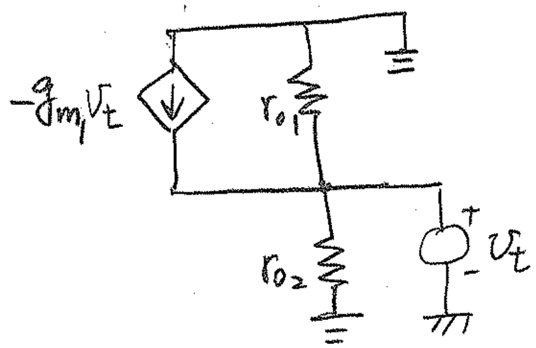
$g_{m2} v_s + \frac{v_{out}}{r_{o2}} - g_{m1}(-v_{out}) + \frac{v_{out}}{r_{o1}} = 0$

$r_{o1} = r_{o2} = r_o$

$A_v = \frac{v_{out}}{v_s} = \frac{-g_{m2}}{g_{m1} + \frac{2}{r_o}} = -\frac{2 \times 10^{-4}}{2 \times 10^{-4} + \frac{2}{2 \times 10^5}} = -\frac{2}{2.1} = -0.95$

(e) Set $v_s = 0$, $v_{out} \to v_t$

$\Rightarrow v_{gs2} = 0$



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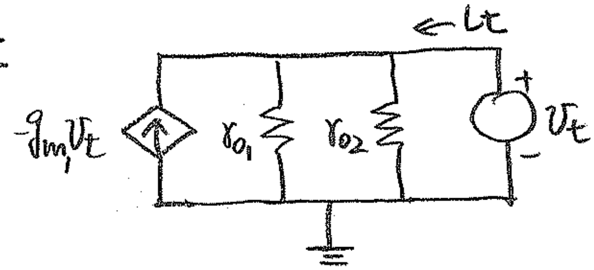
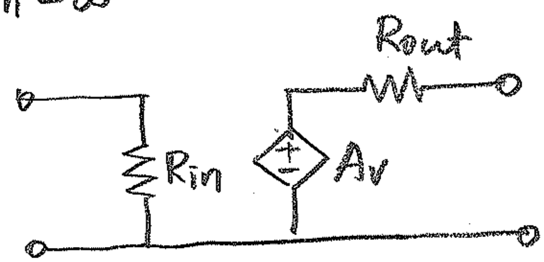


$$i_t = \frac{v_t}{r_{o1}} + \frac{v_t}{r_{o2}} - (-g_{m_1}v_t)$$

$$\Rightarrow R_{out} = \frac{v_t}{i_t} = \left[g_{m_1} + \frac{2}{r_o}\right]^{-1} = [2.1 \times 10^{-4}]^{-1} = 4.76\,k\Omega$$

(f) $R_{in} = \infty$



$R_{in} = \infty$

$R_{out} = \left[g_{m1} + \frac{2}{r_o}\right]^{-1} = 4.76\,k\Omega$

$A_v = -\frac{g_{m2}}{\left[g_{m_1} + \frac{2}{r_o}\right]} = -0.95$

(2) (a) $V_{SG1} = V_{DD} - V_S = 2.5V$, $V_{GS2} = V_S - 0 = 2.5V$

$$-I_{Dp1} = \left(\frac{W}{L}\right)_1 \cdot \frac{\mu_p C_{ox}}{2} \cdot (V_{SG_1} - |V_{Tp}|)^2 \cdot (1 + \lambda_p V_{SD1})$$

$$I_{D2} = \left(\frac{W}{L}\right)_2 \cdot \frac{\mu_n C_{ox}}{2} (V_{GS_2} - V_{Tn})^2 \cdot (1 + \lambda_n V_{DS2})$$

$$V_{SD1} = V_{DD} - V_{out} = 5 - V_{out}$$

$$V_{DS2} = V_{out} - 0 = V_{out}$$

$$\lambda_n = \lambda_p = \lambda$$

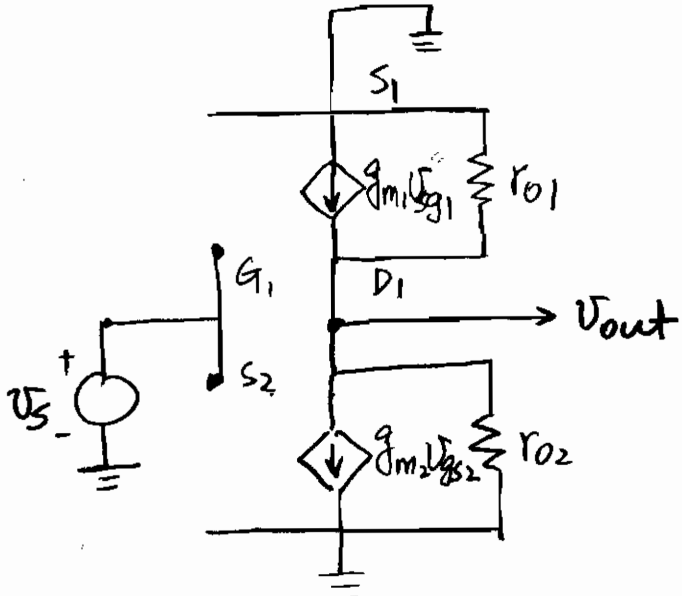
$-I_{Dp1} = I_{D2}$, since $\left(\frac{W}{L}\right)_1 \frac{\mu_p C_{ox}}{2} = \left(\frac{W}{L}\right)_2 \cdot \left(\frac{\mu_n C_{ox}}{2}\right)$

$$\Rightarrow 1 + \lambda(5 - V_{out}) = 1 + \lambda \cdot V_{out}$$

$$\Rightarrow V_{out} = 2.5V$$

For M_1. $V_{SG_1} - |V_{Tp}| = 2.5 - 1 = 1.5V < V_{SD1} = 2.5 \Rightarrow M_1$ in Saturation

For M_2. $V_{GS2} - |V_{Tn}| = 2.5 - 1 = 1.5V < V_{DS2} = 2.5V \Rightarrow M_2$ in Saturation



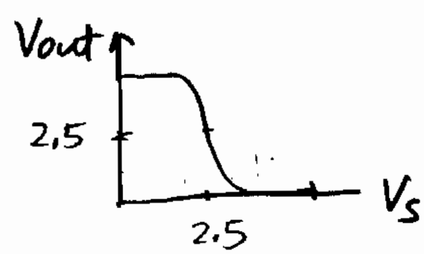
$$v_{sg1} = 0 - v_s = -v_s$$
$$v_{gs2} = v_s - 0 = v_s$$
$$g_{m1} = \left(\frac{W}{L}\right)_1 \mu_p C_{ox} (V_{SG1} - |V_{TP}|)$$
$$= 2 \cdot 50 \cdot (2.5 - 1) = 150\ \mu S$$
$$g_{m2} = \left(\frac{W}{L}\right)_2 \mu_n C_{ox} (V_{GS2} - V_{Tn})$$
$$= 1 \cdot 100\ (2.5 - 1) = 150\ \mu S$$
$$-I_{Dp1} = I_{D2} \cong \left(\frac{W}{L}\right)_2 \frac{\mu_n C_{ox}}{2} \cdot (V_{GS2} - V_{Tn})^2$$
$$= 50 \cdot 1.5^2 = 112.5\ \mu A$$
$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = 178\ k\Omega$$

KCL at $D_1 = D_2$

$$g_{m2} \cdot v_s + \frac{v_{out}}{r_{o2}} - g_{m1}(-v_s) + \frac{v_{out}}{r_{o1}} = 0$$

$$A_v = \frac{v_{out}}{v_s} = -\frac{g_{m1} + g_{m2}}{\frac{1}{r_{o1} // r_{o2}}} = -\frac{1}{2}(g_{m1} + g_{m2}) \cdot r_o = 26.7$$

(b) Maximum Voltage occurs when M_1 is on the verge of saturation:



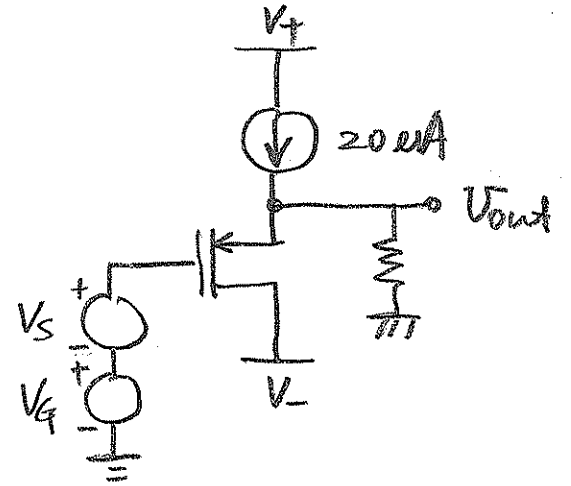
$$V_{SD1} = V_{DD} - V_{out} = V_{SG1} - |V_{Tp}|$$
$$= V_{DD} - V_s - |V_{Tp}|$$
$$\Rightarrow V_s = V_{out} - |V_{Tp}|$$
$$I_1 = \left(\frac{W}{L}\right)_1 \frac{\mu_p C_{ox}}{2} [V_{DD} - V_{out}]^2 = I_2 = \left(\frac{W}{L}\right)_2 \frac{\mu_n C_{ox}}{2} [(V_{out} - |V_{Tp}|) - V_{Tn}]^2$$
$$\Rightarrow V_{DD} - V_{out} = V_{out} - |V_{Tp}| - V_{Tn} \Rightarrow V_{out} = \frac{1}{2}(V_{DD} + |V_{Tp}| + V_{Tn}) = \underline{3.5}\ \#$$

Minimum V_{out}: M_2 at verge of sat. $\Rightarrow V_s = V_{out} + V_{Tn}$

$$I_1 = I_2 \Rightarrow V_{DD} - (V_{out} + V_{Tn}) - |V_{Tp}| = V_{out}$$
$$\Rightarrow V_{out} = \frac{1}{2}(V_{DD} - |V_{Tp}| - V_{Tn}) = \underline{1.5}\ \#$$

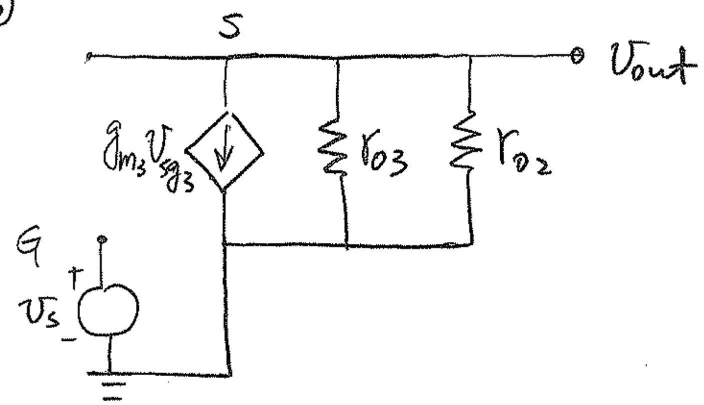
(3) (a) M_1 and M_2 form current mirror

$I_{D2} = I_{REF} \cdot \left(\frac{W}{L}\right)_2 / \left(\frac{W}{L}\right)_1 = 2 I_{REF} = 20\,\mu A$



The resistance looking into $M_2 = r_{o2}$

(b)



$v_{sg3} = (v_{out} - v_s)$

KCL at S : $g_{m3}(v_{out} - v_s) + \frac{v_{out}}{r_{o3} // r_{o2}} = 0$

$\Rightarrow A_v = \frac{v_{out}}{v_s} = \frac{g_{m3}}{g_{m3} + \frac{1}{r_{o3}//r_{o2}}}$

Use DC analysis to find g_{m3}, r_{o2}, r_{o3}

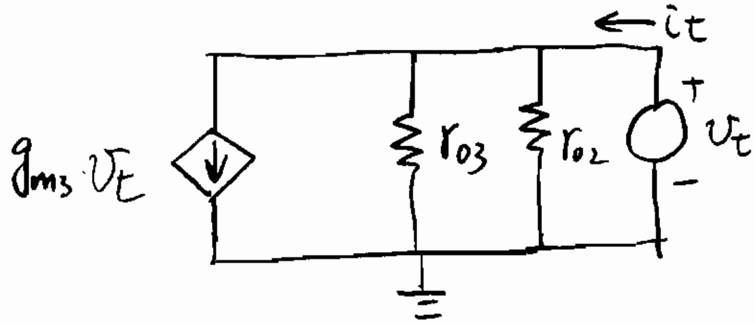
$I_{D3} = 20\,\mu A = \left(\frac{W}{L}\right)_3 \cdot \frac{\mu_p C_{ox}}{2} \cdot (V_{SG} - |V_{Tp}|)^2 = 100 \cdot \frac{50}{2} (V_{SG} - |V_{Tp}|)^2\,\mu A$

$V_{SG} - |V_{Tp}| = \sqrt{\frac{40}{5000}} = 0.089$

$g_{m3} = \left(\frac{W}{L}\right)_3 \mu_p C_{ox} (V_{GS} - |V_{Tp}|) = \frac{2 I_{D3}}{(V_{SG} - |V_{Tp}|)} = 449\,\mu S$

$r_{o3} = \frac{1}{\lambda I_{D3}} = \frac{1}{0.05 \cdot 20 \times 10^{-6}} = 1\,M\Omega$ $\quad r_{o2} = \frac{1}{\lambda I_{D2}} = 1\,M\Omega$

(c) set $v_s = 0$, $v_{sg3} = v_{out} - 0 = v_{out} \rightarrow v_t$



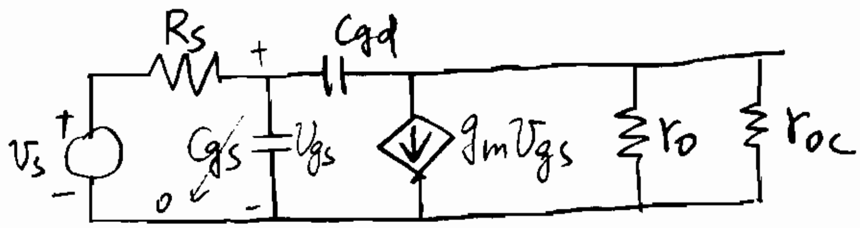
$$i_t = \frac{v_t}{r_{o2} // r_{o3}} + g_m v_t$$

$$\Rightarrow R_{out} = \frac{v_t}{i_t} = \frac{1}{g_{m3} + \frac{1}{r_{o2}//r_{o3}}} = 2.2\,k\Omega$$

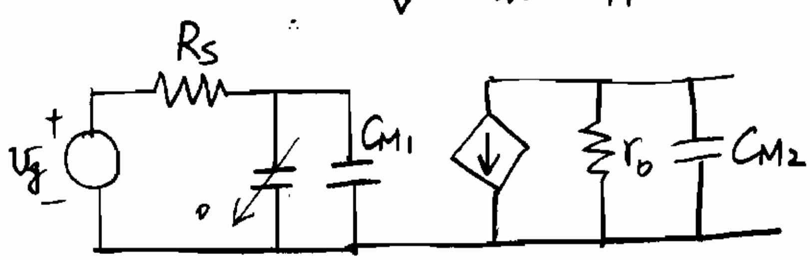
(4) (a) $\omega_{p1} = 10^5$, $\omega_{p2} = 10^7$

$$H(j\omega) = \frac{100}{(1 + j\frac{\omega}{10^5})(1 + j\frac{\omega}{10^7})}$$

(b)



⇓ Miller Approximation



$$C_{M1} = (1 + g_m r_o) \cdot C_{gd}$$

$$C_{M2} = (1 + \frac{1}{g_m r_o}) \cdot C_{gd}$$

(c) $r_o = \frac{1}{\lambda I_{BIAS}} = \frac{1}{0.1 \times 10 \times 10^{-6}} = 1\,M\Omega$

$\tau_1 = R_S C_{M1} = R_S \cdot (1 + g_m r_o) \cdot C_{gd}$

$\tau_2 \cong r_o \cdot C_{gd}$

$C_{gd} = \frac{\tau_2}{r_o} = \frac{\omega_{p2}^{-1}}{r_o} = \frac{10^{-7}}{10^6} = 10^{-13}$

$g_m r_o = 40dB = 10^2 = 100$

$g_m = \frac{100}{10^6} = 10^{-4} = 100\,\mu S$

$\tau_1 = 10^{-5} \approx R_s \cdot (101) \cdot 10^{-13} \approx R_s \cdot 10^{-11}$

$\Rightarrow R_s = 10^6 = 1\,M\Omega.$