**University of California at Berkeley**
**College of Engineering**
**Dept. of Electrical Engineering and Computer Sciences**

# EE 105 Midterm I

Spring 2005 Prof. Roger T. Howe March 2, 2005

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Your Name: SOLUTIONS

Student ID Number: ____________________

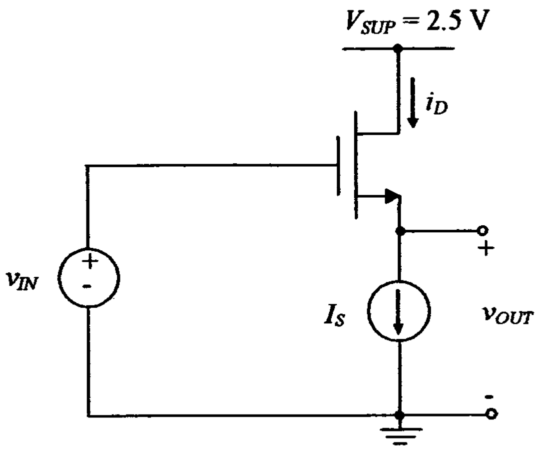
## *Guidelines*

Closed book and notes; one 8.5" x 11" page (both sides) of *your own notes* is allowed.
You may use a calculator.
Do not unstaple the exam.
Show all your work and reasoning on the exam in order to receive full or partial credit.
Time: 80 minutes = 1 hour, 20 minutes.

## *Score*

| Problem | ***Points*** Possible | Score |
|---|---|---|
| 1 | 17 | |
| 2 | 17 | |
| 3 | 16 | |
| ***Total*** | 50 | |

1. MOSFET circuit [17 points]



*Given*:
$L = 0.5\ \mu m$
$W = 5\ \mu m$
$\mu_n C_{ox} = 100\ \mu A/V^2$
$V_{Tn} = 0.5$ V
$\lambda_n = 0\ V^{-1}$
$\gamma = 0$

$I_S = 125\ \mu A$

(a) [3 pts.] Assuming that the transistor is operating in saturation, find an equation for the drain current $i_D$ in terms of the input voltage $v_{IN}$, the output voltage $v_{OUT}$, and the device parameters. It is *not* necessary to substitute numerical values.

$$i_D = \frac{1}{2}\mu_n C_{ox}(W/L)(v_{GS} - V_{Tn})^2$$

$$i_D = \frac{1}{2}\mu_n C_{ox}(W/L)(v_{IN} - v_{OUT} - V_{Tn})^2$$

(b) [4 pts.] For $v_{IN} = 1.5$ V, (i) find the numerical value of the output voltage in Volts and (ii) verify that the transistor is saturated for this case.

$i_D = I_S$ since $I_G = 0$ and $I_{OUT} = 0$.

$$\frac{1}{2}\mu_n C_{ox}(W/L)(v_{IN} - v_{OUT} - V_{Tn})^2 = I_S = 125\mu A$$

$$\frac{1}{2}(100\mu A/V^2)(5/0.5)(v_{IN} - v_{OUT} - 0.5)^2 = 125\mu A$$

$$(v_{IN} - v_{OUT} - 0.5V)^2 = \frac{125\mu A}{500\mu A/V^2} = \frac{1}{4}V^2$$

$$v_{IN} - v_{OUT} - 0.5V = 0.5V$$

$$v_{OUT} = v_{IN} - 1V \Rightarrow v_{OUT} = 1.5V - 1V = \underline{\underline{0.5V}} \quad (i)$$

TEST:

$$v_{DS} = V_{DD} - v_{OUT} = 2.5V - 0.5V = 2V$$

$$v_{DS_{SAT}} = v_{GS} - V_{Tn} = v_{IN} - v_{OUT} - V_{Tn} = 1.5V - 0.5V - 0.5V = 0.5V$$

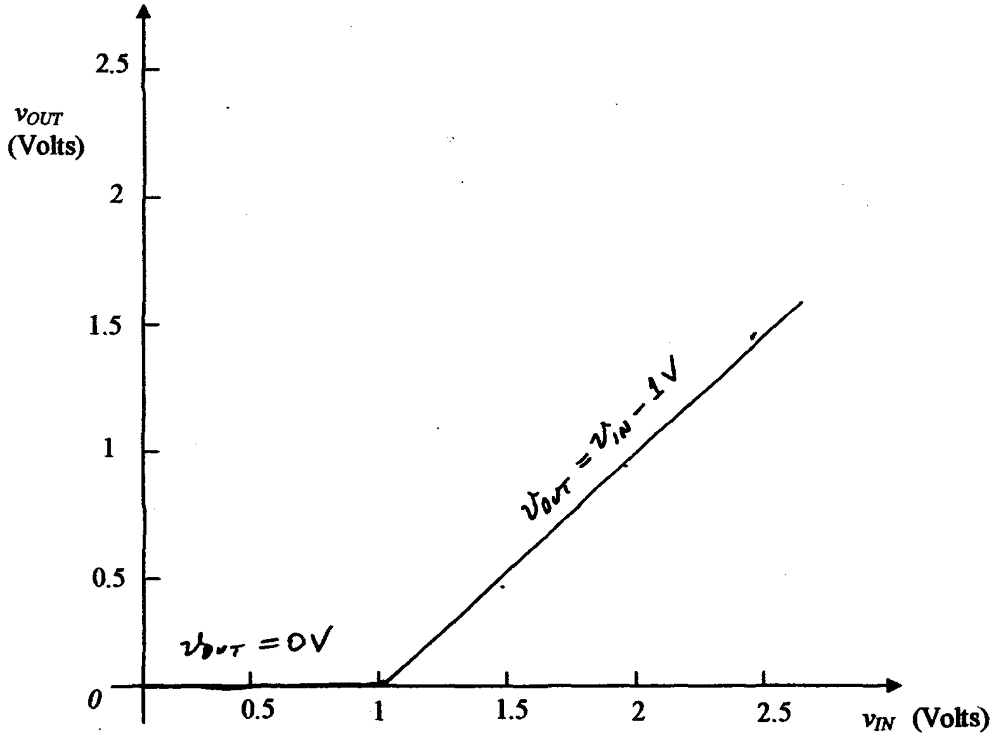
$$\therefore \quad v_{DS} > v_{DS_{SAT}} \Rightarrow \text{saturated} \checkmark \ (ii)$$

(c) [3 pts.] For $v_{IN} = 0.5$ V, (i) find the numerical value of the output voltage and (ii) identify the transistor's operating region.

$v_{IN} = V_{Tn}$ ; $v_{OUT} \geq 0$ V $\Rightarrow$ transistor must have $v_{GS} \leq V_{Tn}$ $\Rightarrow$

$v_{OUT} = 0$ V (i) and it is cutoff (ii)

(d) [4 pts.] Sketch the output voltage $v_{OUT}$ as a function of the input voltage $v_{IN}$ over the range $0 \text{ V} \leq v_{IN} \leq 2.5$ V on the graph below. *Note:* the current source $I_S$ only works for $v_{OUT} > 0$ V and is a short-circuit for $v_{OUT} = 0$ V.

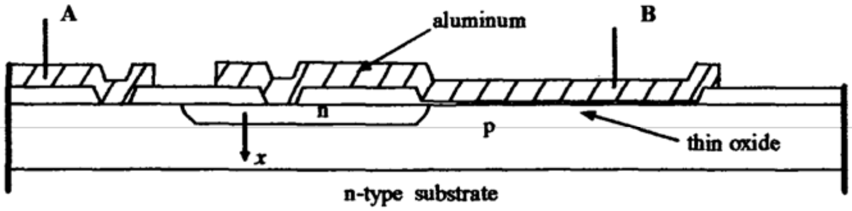


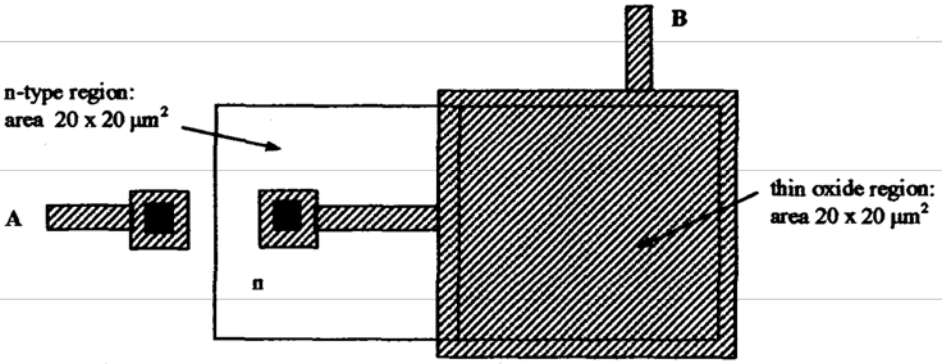
(e) [3 points] For a DC input voltage $V_{IN} = 1.5$ V, find the numerical value of the transconductance $g_m$. Note that you need not have solved either parts (a) or (d) to solve this part.

transistor is saturated $\rightarrow$ $g_m = \dfrac{2I_D}{V_{GS} - V_{Tn}} = \dfrac{2(125\mu A)}{(1.5V - 0.5V) - 0.5V}$

$\underline{g_m = 500 \mu S}$

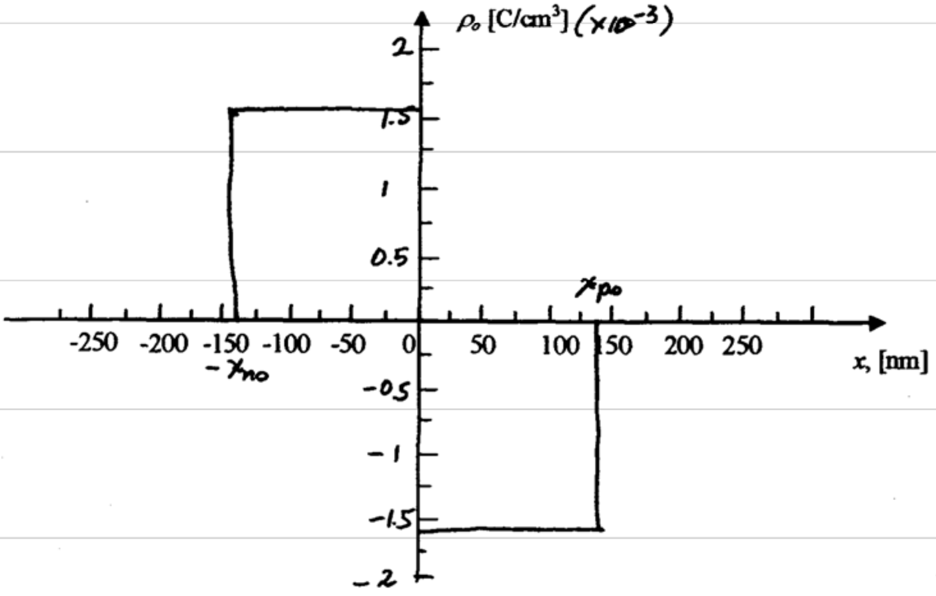
2. Integrated charge-storage element [17 points]





*Given*: The n region connected to electrode **B** is doped with phosphorus with $N_d = 2 \times 10^{16}$ cm$^{-3}$ and with boron ($N_a = 1 \times 10^{16}$ cm$^{-3}$). The p region connected to electrode **A** is doped with only boron ($N_a = 1 \times 10^{16}$ cm$^{-3}$). The permittivity of silicon is $\varepsilon_s = 1.035 \times 10^{-12}$ F/cm and the permittivity of oxide is $\varepsilon_{ox} = 3.45 \times 10^{-13}$ F/cm. The thin oxide has a thickness $t_{ox} = 100$ nm. The built-in potential of aluminum is $\phi_{Al} = -360$ mV.

(a) [4 pts.] Sketch the charge density in thermal equilibrium along the $x$ axis (see location in the in the cross section above. *Given*: the width of the depletion region on the p-side of the junction is $x_{po} = 140$ nm $= 0.14$ μm.



p-side : $\rho = +qN_a = -(1.6 \times 10^{-19}\text{C})(10^{16}\text{cm}^{-3}) = -1.6\,\text{mC/cm}^3$

n-side : $\rho = +q(N_d - N_a) = (1.6 \times 10^{-19}\text{C})(2 \times 10^{16}\text{cm}^{-3} - 1 \times 10^{16}\text{cm}^{-3})$

$= +1.6\,\text{mC/cm}^3$

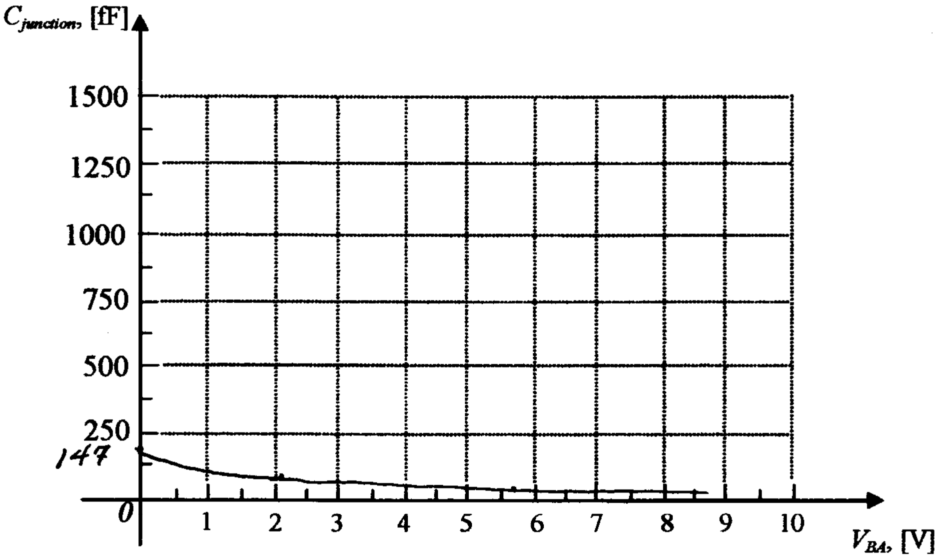
(b) [3 pts.] Find the numerical value of the junction capacitance $C_{junction}(0)$ between the $20 \times 20\ \mu m^2$ n-type region and the underlying p layer in thermal equilibrium ($v_{BA} = 0$ V) in fF. *Given*: 1 fF = $10^{-15}$ F. *Hint*: the information given in part (a) should be very useful.

$$C_{junction}(0) = \frac{\epsilon_s A}{x_{no} + x_{po}} = \frac{(1.03 \times 10^{-12} F/cm^2)(400 \times 10^{-8} cm^2)}{2.8 \times 10^{-5} cm}$$

$$= 147 fF$$

[should have made the area 10x greater]

(c) [4 pts.] Plot the junction capacitance versus $v_{BA}$ on the graph below. If you couldn't solve part (b), you can assume that the thermal equilibrium capacitance is 1000 fF in order to do this part.



$$C_{junction} = \frac{C_{junction}(0)}{\sqrt{1 - V_{AB}/\phi_B}} = \frac{C_{junction}(0)}{\sqrt{1 + V_{BA}/\phi_B}}$$

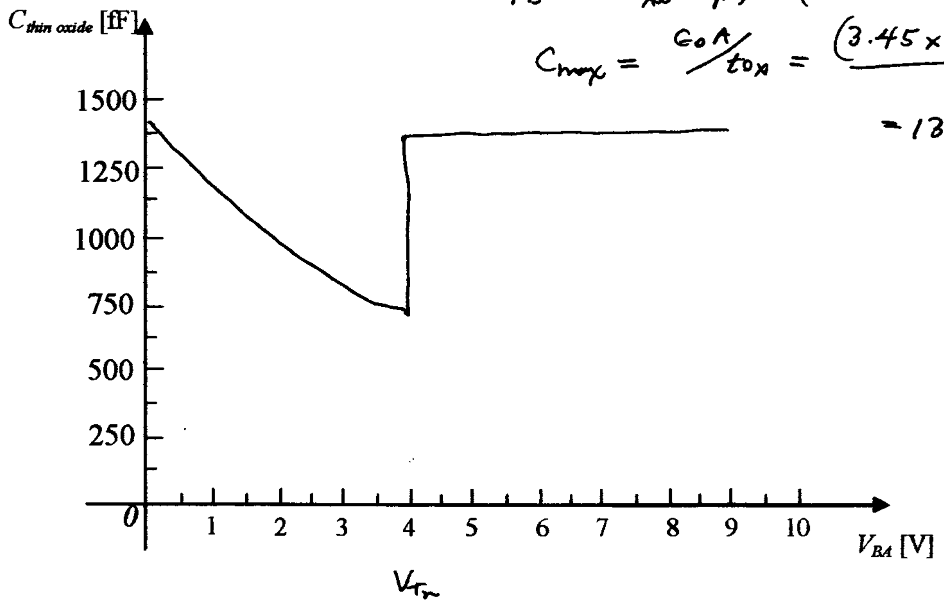
$$\phi_B = \phi_n - \phi_p = 0.72$$

(d) [3 pts.] Sketch the capacitance of the 20 x 20 μm$^2$ thin-oxide area as a function of the voltage $V_{AB}$ on the graph below. *Given*: due to oxide charges, the threshold voltage is $V_{Tn} = 4$ V, the minimum capacitance of the structure is one-half the maximum capacitance ~~and the thermal-equilibrium capacitance is three-quarters of the maximum~~.

$$V_{FB} = -(\phi_{Al} - \phi_p) = -(-360 mV - (-360 mV)) = 0$$

$$C_{max} = \epsilon_o A / t_{ox} = \frac{(3.45 \times 10^{-13} F/cm)(400 \times 10^{-8} cm^2)}{10^{-6} cm}$$

$$= 1380 fF$$



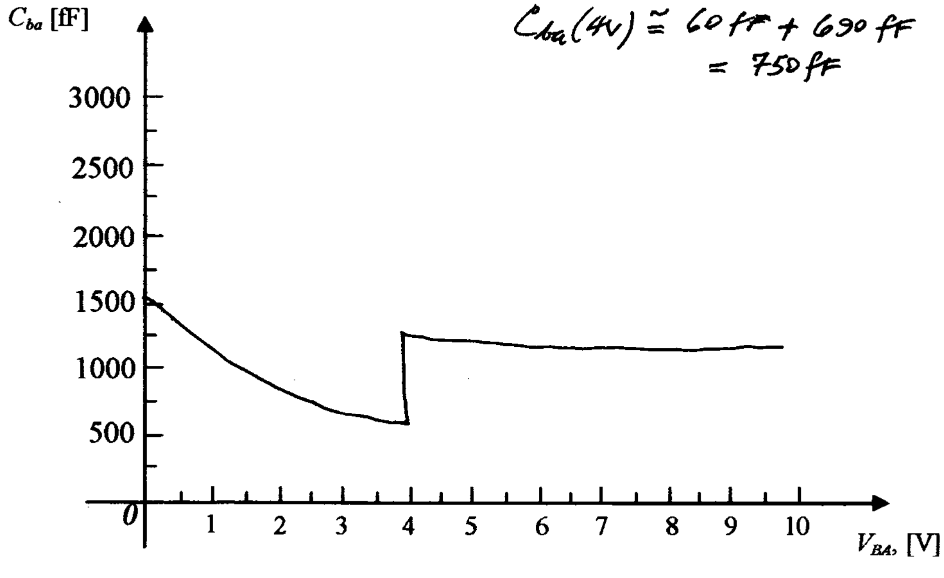
(e) [3 pts.] Sketch the capacitance $C_{ba}$ as a function of the voltage $V_{AB}$ on the graph below. Ignore the contribution of the overlap of the metal onto the thick-oxide regions.

$$C_{ba}(0) = C_{junction}(0) + C_{thin\ oxide}(0) = 147 + 1380\ fF$$

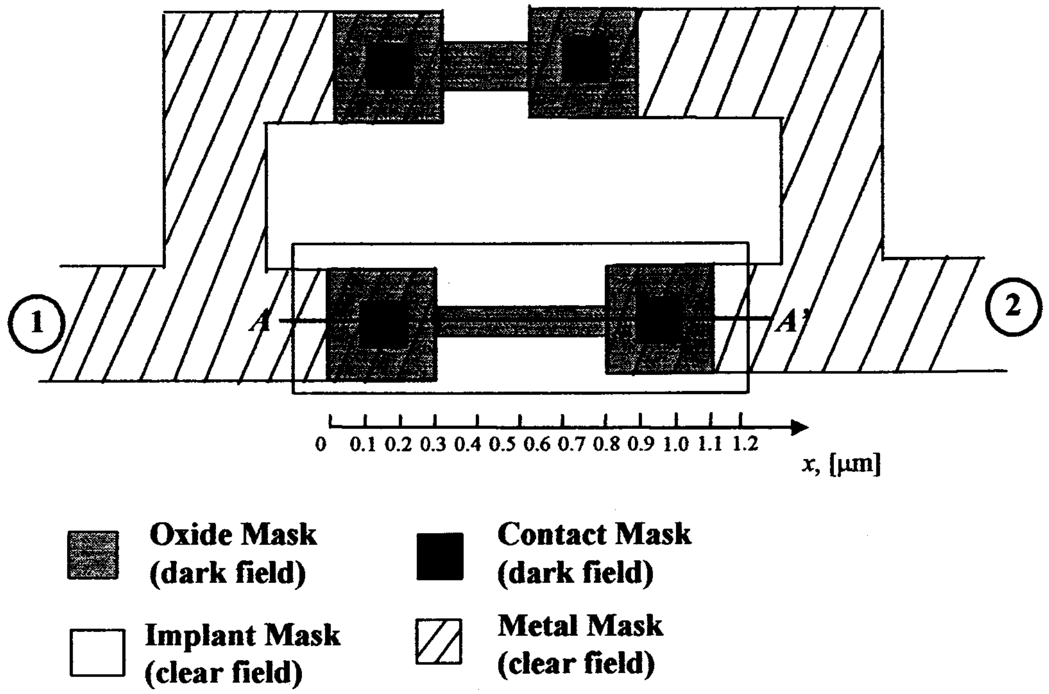
$$= \underbrace{1527 fF}_{\text{should have been larger}}$$

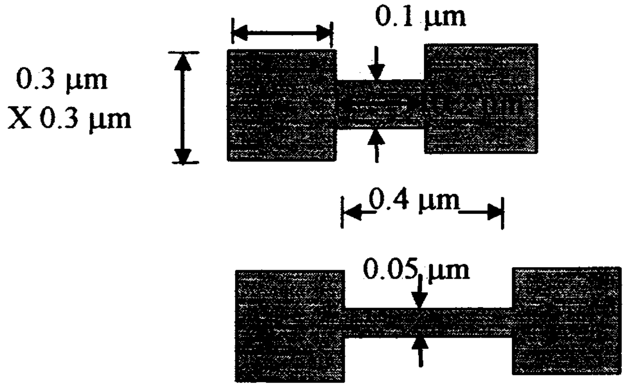
$$C_{ba}(4V) \cong 60 fF + 690 fF$$

$$= 750 fF$$



3. IC resistors [16 points]



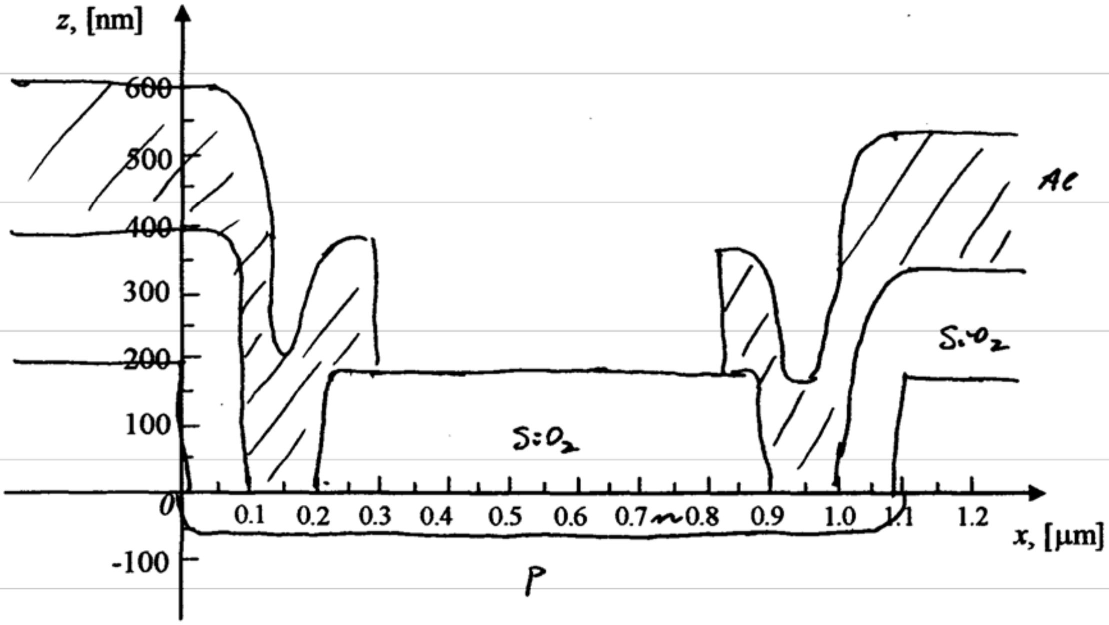


***Process Sequence***:

1. *Starting material*: boron-doped silicon wafer with a concentration of $2 \times 10^{17}$ cm$^{-3}$
2. Deposit a 0.2 µm (= 200 nm) thick $SiO_2$ layer
3. Pattern the oxide using the **Oxide Mask** (dark field) by etching it down to the silicon.
4. Implant phosphorus with dose $Q_d = 2 \times 10^{12}$ cm$^{-2}$ and anneal to form a 50 nm-thick phosphorus-doped regions where the silicon is exposed.
5. Spin on photoresist and pattern with the **Implant Mask** (clear field).
6. Implant phosphorus with dose $Q_d = 2 \times 10^{12}$ cm$^{-2}$ and then etch off the photoresist.
7. Anneal to activate the second implant; the phosphorus regions remain 50 nm thick.
8. Deposit a 200 nm-thick $SiO_2$ layer and pattern using the **Contact Mask** (dark field).
9. Deposit 200 nm of aluminum and pattern using the **Metal Mask** (clear field).

***Given***: mobilities for this problem are $\mu_n = 800$ cm$^2$/(Vs) and $\mu_p = 200$ cm$^2$/(Vs). The saturation electric field for electrons is $E_{sat} = \overset{1.25}{\cancel{2}} \times 10^4$ V/cm and their saturation velocity is $v_{sat} = 10^7$ cm/s. Count the "dogbone" contact areas as 0.65 square each for both resistors.

(a) [4 pts.] Sketch the cross section *A-A'* on the graph below **after step 9**. *Identify all layers clearly*.



(b) [4 pts.] What is the sheet resistance $R_\square$ of the 0.2 μm long, 0.1 μm wide resistor?

$$R_\square = \frac{\rho}{t} = (q n \mu_n t)^{-1} = (1.6\times10^{-19} \cdot 2\times10^{17} \cdot 800 \cdot 5\times10^{-6})^{-1}$$

$$n = N_d - N_a = Q_d/t - N_a$$

$$= \left[\frac{2\times10^{12}\,\text{cm}^{-2}}{5\times10^{-6}\,\text{cm}} = 4\times10^{17}\,\text{cm}^{-3}\right] - 2\times10^{17}\,\text{cm}^{-3}$$

$$= 2\times10^{17}\,\text{cm}^{-3}$$

$\boxed{R_\square = 7800\ \Omega/\square}$

(c) [4 pts.] What is the maximum current $I_{max}$ in $\overset{\mu}{\cancel{\mu}}$A through the 0.4 μm long, 0.05 μm wide resistor?

$$J_{max} = q n v_{sat} \Rightarrow I_{max} = q W t n v_{sat}.$$

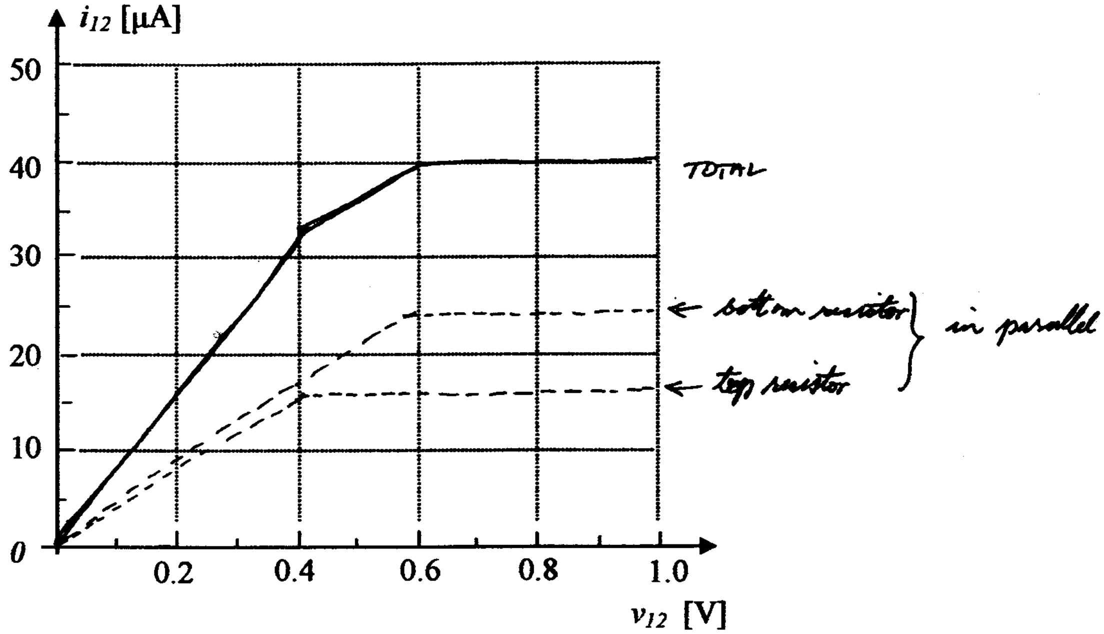
$$n = \underbrace{\left(\frac{2Q_d}{t}\right)}_{\text{double dose from step 6}} - N_a = 8\times10^{17}\,\text{cm}^{-3} - 2\times10^{17}\,\text{cm}^{-3} = 6\times10^{17}\,\text{cm}^{-3}$$

$$I_{max} = (1.6\times10^{-19})(5\times10^{-6})(5\times10^{-6})(6\times10^{17})(10^7)\ \text{A}.$$

$$= 24\,\mu\text{A}.$$

$\boxed{I_{max} = 24\ \overset{\mu}{\cancel{\text{A}}}\ \mu\text{A}.}$

(d) [4 pts.] Plot the current-voltage curve between terminals 1 and 2 over the range indicated on the graph below.



- TOP RESISTOR

$$R_T = R_\square (2 + 1.3)$$
$$= 25.7\,k\Omega.$$

$$I_{max_T} = q\,W t\,n\,v_{sat} = (1.6\times10^{-19})(10^{-5})(5\times10^{-6})(10^7) = 16\,\mu A.$$
$$V_{max_T} = R_T \cdot I_{max_T} = (25.7\,k\Omega)(16\,\mu A) = 0.41V$$

- BOTTOM RESISTOR

$$R_\square = (1.6\times10^{-19}\cdot 6\times10^{17}\cdot 800\cdot 5\times10^{-6})^{-1} = 2600\,\Omega/\square$$
$$R_B = R_\square\,(8 + 1.3) = 24.2\,k\Omega$$
$$V_{max_B} = R_B \cdot I_{max_B} = (24.2\,k\Omega)(24\,\mu A) = 0.58\,V.$$