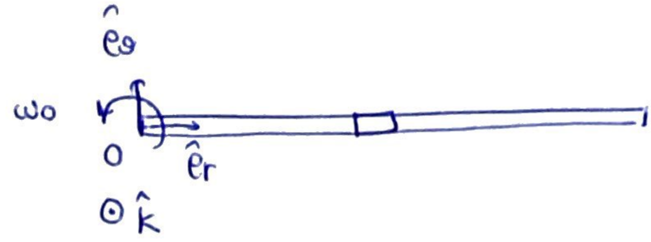
[1]



(a) $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta + \ddot{z}\hat{k}$

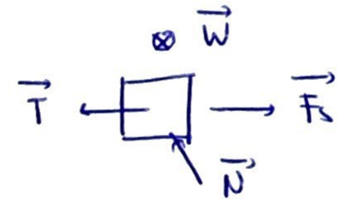
$r = \frac{L}{2}$, $\dot{r} = \ddot{r} = 0$

$\dot{\theta} = \omega_0$, $\ddot{\theta} = 0$

$\ddot{z} = 0$ (motion in fixed horizontal plane)

so, $\boxed{\vec{a} = -\frac{L}{2}\omega_0^2 \hat{e}_r .}$

(b)



$\vec{W} = -mg\hat{k}$

$\vec{N} = N_z\hat{k} + N_\theta \hat{e}_\theta$

$\vec{T} = -T\hat{e}_r$

$\vec{F_s} = k\frac{L}{2}\hat{e}_r$

$\vec{F} = m\vec{a}$

$-mg\hat{k} + N_z\hat{k} + N_\theta\hat{e}_\theta - T\hat{e}_r + \frac{kL}{2}\hat{e}_r = -m\frac{L}{2}\omega_0^2\hat{e}_r$

$(\cdot \hat{e}_r)$ $T = \frac{kL}{2} + \frac{mL}{2}\omega_0^2$

$\boxed{T = -\left(\frac{kL}{2} + \frac{mL}{2}\omega_0^2\right)\hat{e}_r}$

(c) just after the spring is cut $r = \frac{L}{2}$

$\dot{r} = 0$

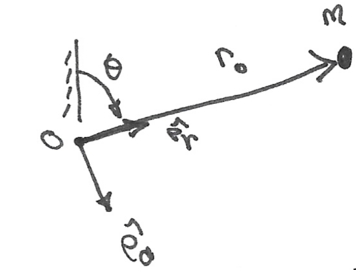
$T = 0.$

$\vec{F} = m\vec{a}$

$\underbrace{-mg\hat{k} + N_z\hat{k}}_{\text{cancel}} + \cancel{N_\theta}^{0}\hat{e}_\theta + \frac{kL}{2}\hat{e}_r = m[(\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\cancel{\ddot{\theta}}^{0} + 2\cancel{\dot{r}}^{0}\dot{\theta})\hat{e}_\theta] = m\vec{a}$

$(\cdot \hat{e}_r)$ $\frac{kL}{2} = m(\ddot{r} - r\dot{\theta}^2)$, $\boxed{\vec{a} = \frac{kL}{2m}\hat{e}_r .}$

[2] a)



$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta + \ddot{z}\hat{k}$$

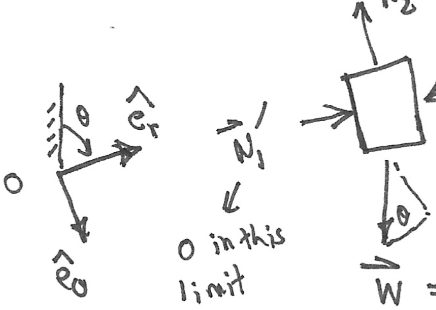
Here, $r = r_0$ const $\dot{r} = \ddot{r} = 0$

$\dot{\theta} = \omega$ const $\ddot{\theta} = 0$

$z \equiv 0 \Rightarrow \dot{z} = \ddot{z} = 0$

so $\underline{\vec{a} = -r_0\omega^2\hat{e}_r}$

b)



$\vec{N}_2 = -N_2\hat{e}_\theta$

$\vec{F}_f = -F_f\hat{e}_r$

$\vec{W} = mg[-\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta]$

$\vec{a}$ remains as in (a) for this limit

$\vec{F} = m\vec{a}$ leads to ....

$\hat{e}_r$: $-F_f - mg\cos\theta = -mr_0\omega^2$ ①

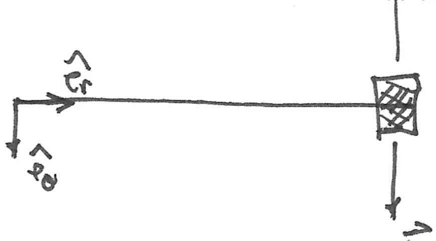
$\hat{e}_\theta$: $-N_2 + mg\sin\theta = m(0)$ ②

In this limit $F_f = \mu N_2$ from ② $N_2 = mg\sin\theta$

so ① becomes $-\mu mg\sin\theta_0 - mg\cos\theta_0 = -mr_0\omega^2$
$\theta = \theta_0$

so $\underline{\omega = \sqrt{(\mu g\sin\theta_0 + g\cos\theta_0)/r_0}}$ units are rads/sec ✓

(c) For $r = \ell$, $\theta = 90°$ ...



$-N_2\hat{e}_\theta = \vec{N}_2$

$\vec{F}_f = -F_f\hat{e}_r$

$\vec{W} = mg\,\hat{e}_\theta$

Here $\dot{r}, \ddot{r}$ are not zero anymore ....

so $\vec{a} = (\ddot{r} - \ell\omega^2)\hat{e}_r + 2\dot{r}\omega\,\hat{e}_\theta$

so $\vec{F} = m\vec{a}$ leads to ....

$\hat{e}_r$: $-F_f = m(\ddot{r} - \ell\omega^2)$ ③

$\hat{e}_\theta$: $-N_2 + mg = 2m\dot{r}\omega$ ④

However, we know $v = |\vec{v}| = |\dot{r}\hat{e}_r + \ell\omega\hat{e}_\theta| = \sqrt{\dot{r}^2 + \ell^2\omega^2}$ so $\dot{r} = \sqrt{v^2 - \ell^2\omega^2}$

we know $F_f = \mu N_2$

so ④ becomes $N_2 = mg - 2m\omega\sqrt{v^2 - \ell^2\omega^2}$

and so $F_f = \mu\left[mg - 2m\omega\sqrt{v^2 - \ell^2\omega^2}\right]$

so $\underline{\vec{F}_{reaction}} = \vec{N}_2 + \vec{F}_f = \underline{\left(mg - 2m\omega\sqrt{v^2 - \ell^2\omega^2}\right)\left(-\hat{e}_\theta - \mu\hat{e}_r\right)}$