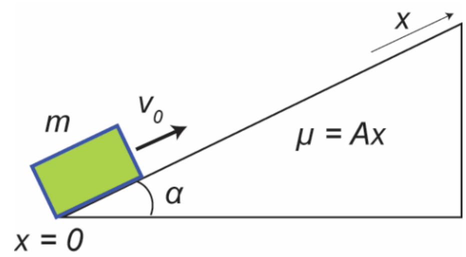
Fall 2019 Physics 7A Lec 002 (Yildiz) Midterm 2

1) A warehouse worker is shoving boxes up a rough plank inclined at an angle α above the horizontal. The plank is covered with more ice near the bottom of the plank than near the top, so that the coefficient of friction increases with the distance x along the plank: $\mu = Ax$, where A is constant and the bottom of the plank is at $x = 0$ (For this plank the coefficients of kinetic and static friction are equal). What should be the minimum velocity v_0 of the box as as it leaves the bottom of the plank in order for this box to remain at rest when it first comes to rest?

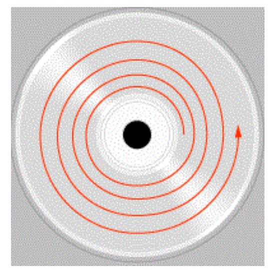


2) The engine of a rocket in outer space, far from any planet, is turned on. The rocket ejects burned fuel at a constant rate; in the first second of firing, it ejects 1/120 of its initial mass m_0 at a relative speed of 2400 m/s.

a) What is the rocket's initial acceleration?

b) Suppose that ¾ of the initial mass of the rocket is fuel, so that the fuel is completely consumed at a constant rate in 90 s. The final mass of the rocket is ¼ m_0. If the rocket starts from rest, find its speed at the end of this time.

3) On a compact disc (CD), music is coded in a pattern of tiny pits arranged in a track that spirals outward toward the rim of the disc. As the disc spins inside a CD player, the track is scanned at a constant linear speed of v. Because the radius of the track varies as it spirals outward, the angular speed of the disc must change as the CD is played. The equation of a spiral is $r(\theta) = r_0 + \beta\theta$ where r_0 is the radius of the spiral at $\theta = 0$ and β is a constant. If we take the rotation direction of the CD to be positive, β must be positive so that r increases as the disc turns.



a) When the disc rotates through a small angle $d\theta$, what is the distance ds scanned along the track?

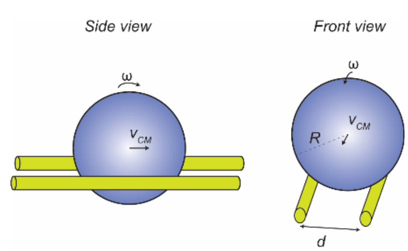
b) Integrate ds to find the total distance s scanned along the track as a function of the total angle through which the disc has rotated.

c) Since the track is scanned at a constant linear speed the distance s is equal to vt. Use this to find θ as a function of time. There will be two solutions for θ, choose the positive one, and explain why this is the solution to choose.

d) What is the angular velocity ω as a function of time?

e) What is the angular acceleration α as a function of time? Is it constant?

4) A uniform ball of mass M and radius R rolls without slipping between two rails such that the horizontal distance is d between the two contact points of the rails to the ball.

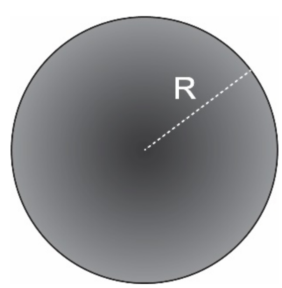


a) What is the relationship between v_{CM} and ω?

b) For a uniform ball starting from rest and descending while rolling without slipping down a ramp with angle θ, find the translational acceleration a_{CM} of the ball down the ramp.

c) Find v_{CM} of the ball after it descends vertical distance h down the incline.

5) Planets are not uniform inside. Normally, they are densest at the center and have decreasing density outward toward the surface. Model a spherically symmetric planet with radius R, as having a density that decreases *linearly* with distance from the center.



a) Let the density be ρ_0 at the center and ¼ ρ_0 at the surface. Write an equation that describes how the density changes by distance r from the radius of the planet ($r < R$).

b) What is the total mass of the planet?

c) What is the acceleration due to gravity at the surface of this planet?