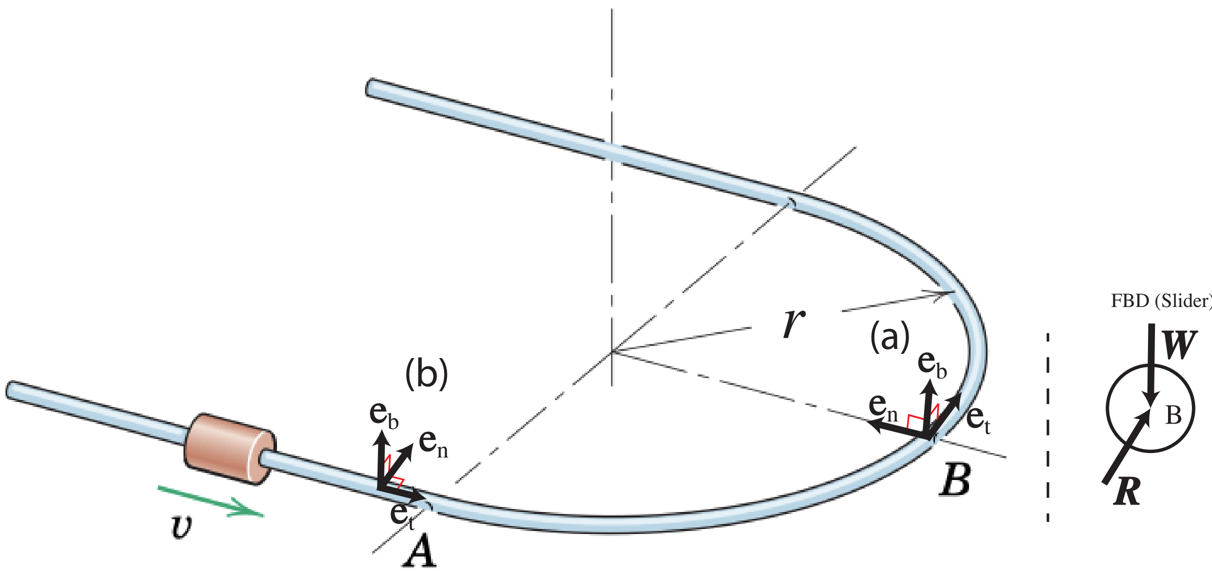
University of California, Berkeley
Department of Mechanical Engineering
ME 104, Spring 2021

Midterm Exam 1 (15 April 2021)
SOLUTIONS

Problem 1 25 points

(a) (5 pts)

- Frenet-Serret basis and free-body diagram of the slider at B



- Applying Frenet-Serret basis, reaction force ($\boldsymbol{R}$) supplied by the rod can be written as

$$\boldsymbol{R} = R_t \boldsymbol{e}_t + R_n \boldsymbol{e}_n + R_b \boldsymbol{e}_b,$$

where $\boldsymbol{W} = mg(-\boldsymbol{e_b})$ is the weight of the slider.

(b) (4 pts)
The basis for the straight-line portion is given in the figure above. Note that $\boldsymbol{e}_n$ is defined such that the basis joints continuously with the Frenet-Serret basis for the semi-circular portion.

(c) (16 pts)

- Passing point A:
  Applying Newton's second law

$$\begin{aligned} &\boldsymbol{F} = m\boldsymbol{a} \\ \Rightarrow \quad &R_t\boldsymbol{e}_t + R_n\boldsymbol{e}_n + R_b\boldsymbol{e}_b - mg\boldsymbol{e}_b = m(\dot{v}\boldsymbol{e}_t + \cancelto{0}{\kappa} v^2\boldsymbol{e}_n) = m\dot{v}\boldsymbol{e}_t. \end{aligned}$$

Taking dot products,

$$\begin{aligned} (\boldsymbol{F} = m\boldsymbol{a}) \cdot \boldsymbol{e}_t &\Rightarrow \ R_t = m\dot{v} = 0 \\ (\boldsymbol{F} = m\boldsymbol{a}) \cdot \boldsymbol{e}_n &\Rightarrow \ R_n = 0 \\ (\boldsymbol{F} = m\boldsymbol{a}) \cdot \boldsymbol{e}_b &\Rightarrow \ R_b = mg \end{aligned}$$

$$\therefore \|\boldsymbol{R}\| = mg \quad \text{N}$$

- Passing point B:
  Applying Newton's second law

$$\begin{aligned} &\boldsymbol{F} = m\boldsymbol{a} \\ \Rightarrow \quad &R_t\boldsymbol{e}_t + R_n\boldsymbol{e}_n + R_b\boldsymbol{e}_b - mg\boldsymbol{e}_b = m(\dot{v}\boldsymbol{e}_t + \kappa v^2\boldsymbol{e}_n), \end{aligned}$$

where $\kappa = \frac{1}{r}$. Taking dot products,

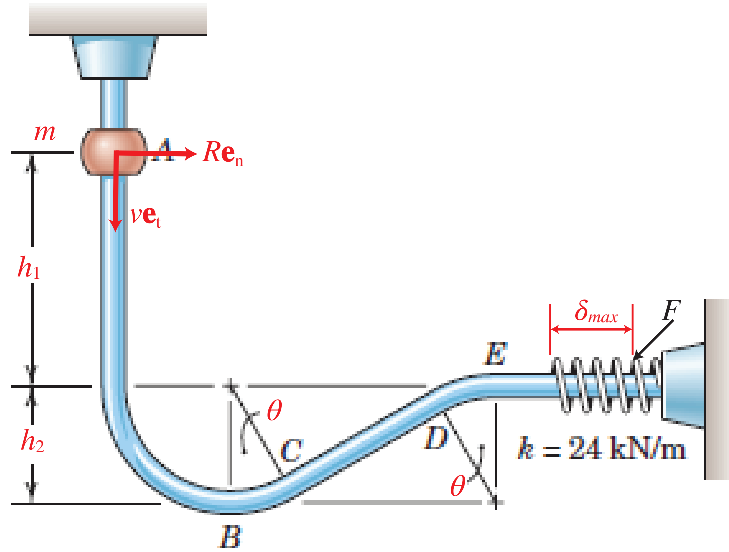
$$\begin{aligned} (\boldsymbol{F} = m\boldsymbol{a}) \cdot \boldsymbol{e}_t &\Rightarrow \ R_t = m\dot{v} = 0 \\ (\boldsymbol{F} = m\boldsymbol{a}) \cdot \boldsymbol{e}_n &\Rightarrow \ R_n = \frac{v^2}{r} \\ (\boldsymbol{F} = m\boldsymbol{a}) \cdot \boldsymbol{e}_b &\Rightarrow \ R_b = mg \end{aligned}$$

$$\therefore \|\boldsymbol{R}\| = \sqrt{\left(\frac{v^2}{r}\right)^2 + (mg)^2} \quad \text{N}$$

(d)

The existence of jump discontinuity in the normal force at point A can be problematic, which causes unnecessary impact to both track and train.

Problem 2 25 points



(a) (4 pts)
The work done by the reaction force is,

$$W_{12} = \int_{t_1}^{t_2} \boldsymbol{R} \cdot \boldsymbol{v} dt = \int_{t_1}^{t_2} Rv\boldsymbol{e}_n \cdot \boldsymbol{e}_t dt = \int_{t_1}^{t_2} 0 dt = 0.$$

Therefore, the reaction force is workless and the energy is conserved.

(b) (7 pts)
From the conservation of total mechanical energy,

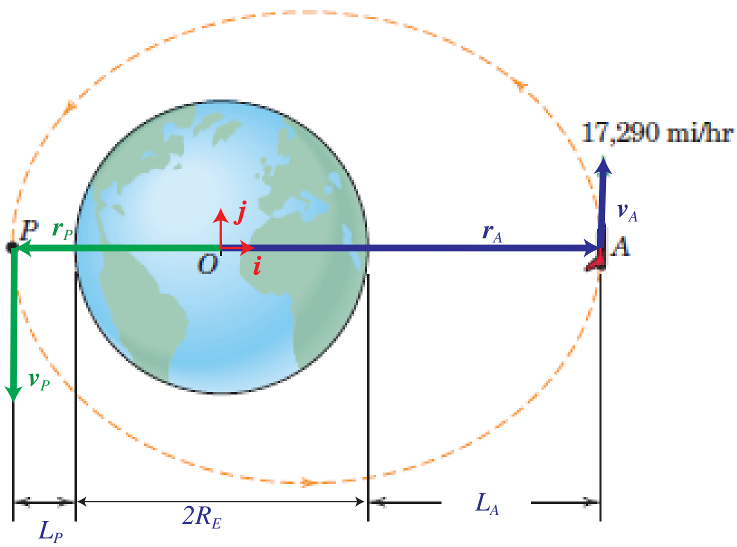
$$T_A + V_A = T_B + V_B$$
$$\Rightarrow \quad 0 + mg(h_1 + h_2) = \frac{1}{2}mv_B^2 + 0$$
$$\Rightarrow \quad v_B = \sqrt{2g(h_1 + h_2)} \quad \text{m/s}$$

(c) (8 pts)
From the similar procedure of (b),

$$T_A + V_A = T_F + V_F$$
$$\Rightarrow \quad 0 + mg(h_1 + h_2) = 0 + mgh_2 + \frac{1}{2}k\delta_{max}^2$$
$$\Rightarrow \quad \delta_{max} = \sqrt{\frac{2mgh_1}{k}} \quad \text{m}$$

(d) (6 pts)
Since the spring deforms without hitting the wall, the motions between the point A and the point F would continue periodically.

Problem 3 20 points



(a) (10pts)

From the conservation of angular momentum,

$$\frac{\boldsymbol{H}^O}{m} = \boldsymbol{r}_P \times \boldsymbol{v}_P = \boldsymbol{r}_A \times \boldsymbol{v}_A$$
$$\Rightarrow (R_E + L_A)\boldsymbol{i} \times v_A\boldsymbol{j} = (R_E + L_B)(-\boldsymbol{i}) \times v_P(-\boldsymbol{j})$$
$$\Rightarrow (R_E + L_A)v_A\boldsymbol{k} = (R_E + L_B)v_P\boldsymbol{k}$$

$$\therefore \frac{\boldsymbol{H}^O}{m} \cdot \boldsymbol{k} \;\Rightarrow v_P = \frac{(R_E + L_A)v_A}{R_E + L_B} \quad \text{ft/s} \;\; \text{or} \;\; \text{mi/hr}$$

(b) (10pts)

Since the mechanical energy per unit mass is expected to be conserved, we expect to observe

$$\frac{E_P}{m} = \frac{1}{2}v_P^2 - \frac{Gm_e}{r_P} = \frac{E_A}{m} = \frac{1}{2}v_A^2 - \frac{Gm_e}{r_A} \quad \frac{\text{lbf} \cdot \text{ft}}{\text{slug}} \;\; \text{or} \;\; \text{ft}^2/\text{s}^2$$