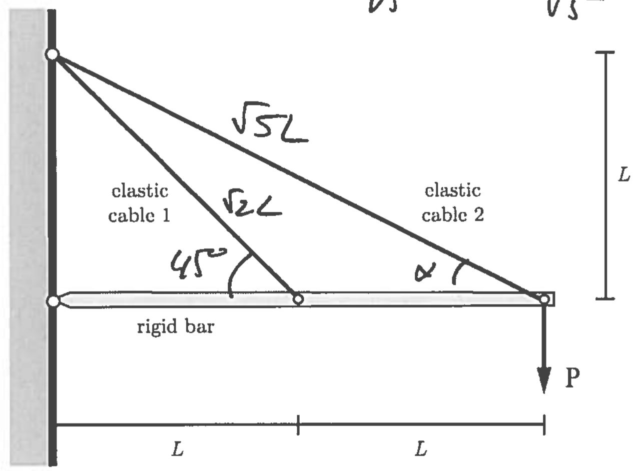
Geometry

$\sin\alpha = \frac{1}{\sqrt{5}}$ $\cos\alpha = \frac{2}{\sqrt{5}}$

Problem #1 (40%)

A rigid bar is being held horizontally by two elastic cables in the configuration depicted in the figure, while being loaded by a vertical load (P) at its right tip as shown. The cables can be considered linear elastic with an equal Young modulus E and a cross section area A. All connections are pinned, and all members can be considered weightless.

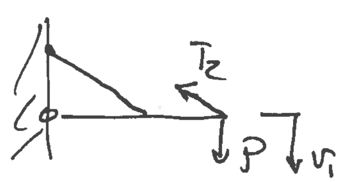
Determine: (1) the force in the cables, and (2) the deflection of the bar at its right tip.



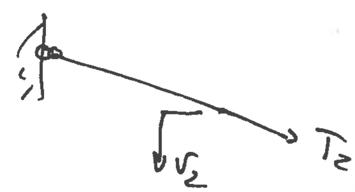
Statically indeterminate $\Rightarrow$ Force method (3 steps)
(degree of indeterminancy $= 1$)

$\boxed{\text{STEP 1}}$ Release the system, e.g., disconnect cable #2, leaving the force

Problem #1

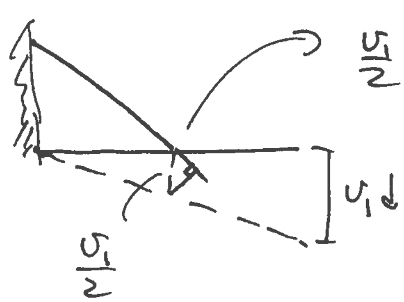


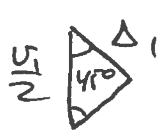
Problem #2



$\boxed{\text{STEP 2}}$ Solve for $v_1\downarrow$ and $v_2\downarrow$ in terms of P and T_2

Problem #1 Kinematics





$\frac{v_1}{2} = \frac{\Delta_1}{\sin 45}$

$\downarrow v_1 = 2\sqrt{2}\,\Delta_1$

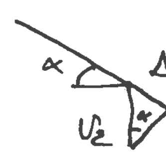
$= 2\sqrt{2}\,\frac{T_1\sqrt{2}L}{EA}$ → length of cable #1 → constitutive relation (linear elastic)

$= 4\,\frac{T_1 L}{EA}$

Problem #2

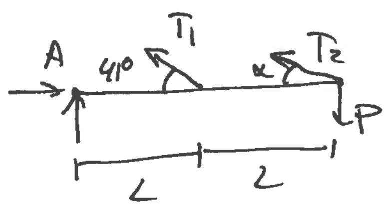
$\downarrow v_2 = \frac{\Delta_2}{\sin\alpha}$

$= 5\,\frac{T_2 L}{EA}$



$\Delta_2 = T_2\,\frac{\sqrt{5}L}{EA}$ — length of cable #2

statics



$\Sigma M_A = 0$

$\Downarrow$

$T_1 \sin 45\, L + T_2 \sin\alpha \cdot 2L = P \cdot 2L$

$\Rightarrow T_1 = 2\sqrt{2}\,[P - \sin\alpha\, T_2]$

$\rightarrow \downarrow v_1 = \frac{8\sqrt{2}L}{EA}\,[P - \underset{\frac{1}{\sqrt{5}}}{\sin\alpha}\, T_2]$

Relax ...

$\boxed{\text{STEP3}}$ Impose back the compatibility constraint

$$\downarrow v_1 = \downarrow v_2$$

$$\Rightarrow \quad \frac{8\sqrt{2}L}{EA}\left[P - \frac{1}{\sqrt{5}}T_2\right] = 5\frac{T_2 L}{EA} \quad \Rightarrow \quad \underline{T_2 = \frac{8\sqrt{10}}{8\sqrt{2}+5\sqrt{5}}P}$$

From Problem #1 above

$$T_1 = 2\sqrt{2}\left[P - \frac{1}{\sqrt{5}}T_2\right] \quad \Rightarrow \quad \underline{T_1 = \frac{10\sqrt{10}}{8\sqrt{2}+5\sqrt{5}}P}$$

$$\downarrow v_1 = \downarrow v_2 = 5\frac{T_2 L}{EA} = \underline{\frac{40\sqrt{10}}{8\sqrt{2}+5\sqrt{5}}\frac{PL}{EA}}$$

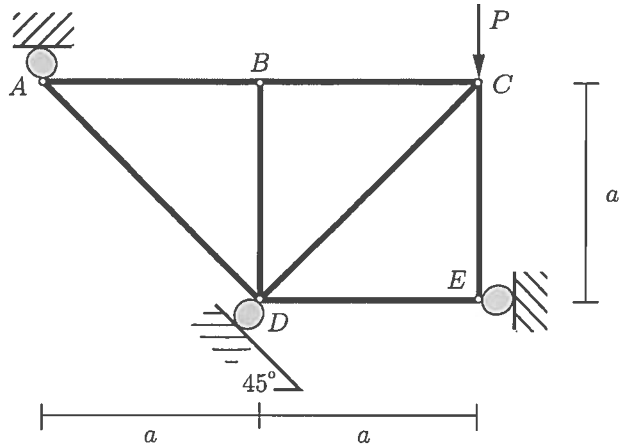
SUMMARY:

$$T_1 = \frac{10\sqrt{10}}{8\sqrt{2}+5\sqrt{5}}P = 1.4058\,P$$

$$T_2 = \frac{8\sqrt{10}}{8\sqrt{2}+5\sqrt{5}}P = 1.1247\,P$$

$$\downarrow v = \frac{40\sqrt{10}}{8\sqrt{2}+5\sqrt{5}}\frac{PL}{EA} = 5.6233\frac{PL}{EA}$$

Problem #2 (25%)

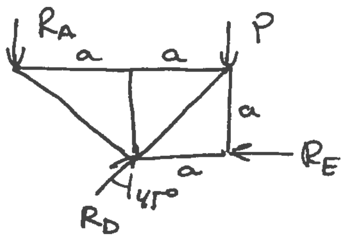


1. Determine the forces in all the members in the truss of the figure when the vertical load of value P shown in the figure is applied. Indicate clearly if the member is in tension or compression.

2. If all the members have the same $0.1 \times 0.1\ m^2$ square cross section, determine the maximum load value P that can be applied with a factor of safety of 1.5 if the material can only take $10MPa$ in tension or compression.

Remark: Express your results in terms of the length a if needed.

Reactions



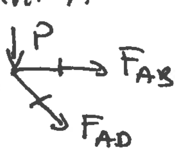
$(\Sigma M_D = 0) \Rightarrow R_A \cdot a = P \cdot a \Rightarrow R_A = P$

$(\Sigma F_y = 0) \Rightarrow R_D \cos 45 = P + R_A \Rightarrow R_D = 2\sqrt{2}P$

$(\Sigma F_x = 0) \Rightarrow R_E = R_D \sin 45 \Rightarrow R_E = 2P$

Part 1 Zero-force members $F_{CE} = F_{BD} = 0$

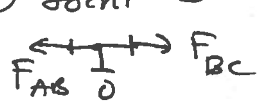
① Joint A



$F_{AD} \cos 45 + P = 0 \Rightarrow F_{AD} = -\sqrt{2}P$

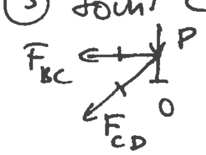
$F_{AD} \sin 45 + F_{AB} = 0 \Rightarrow F_{AB} = P$

② Joint B



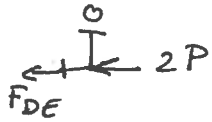
$\Rightarrow F_{BC} = F_{AB} = P$

③ Joint C



$\Rightarrow F_{CD} \cos 45 + F_{BC} = 0 \Rightarrow F_{CD} = -\sqrt{2}P$

④ Joint E



$\Rightarrow F_{DE} = -2P$

SUMMARY

$F_{CE} = F_{BD} = 0$

$F_{AB} = F_{BC} = P$ (tension)

$F_{AD} = F_{CD} = -\sqrt{2}P$ (compression)

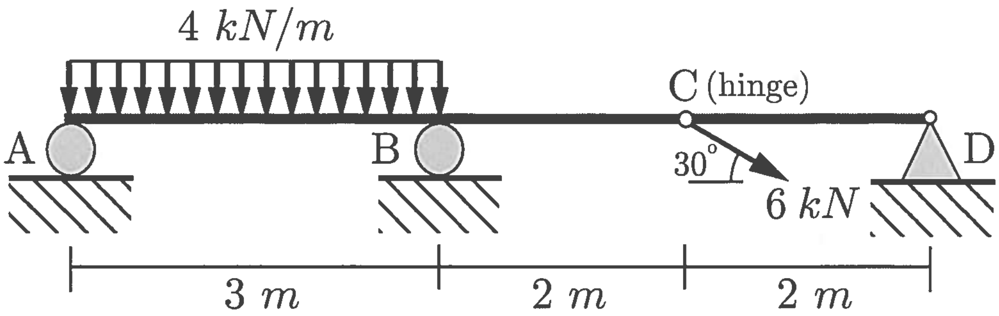
$F_{DE} = -2P$ (compression)

Part 2 Maximum force among all members $= -2P$

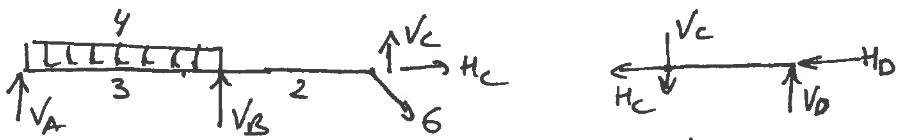
$$\Rightarrow \underset{0.1 \cdot 0.1\,m^2 = A}{\frac{+2P}{}} \le \frac{\sigma_{max} = 10MPa}{FS = 1.5} \Longrightarrow \boxed{P \le 33.\bar{3}\ kN}$$

Problem #3 (35%)

Draw the axial force, transversal shear force and bending moment diagrams for the beam shown in the figure. Indicate the characteristic values (min/max values, values at the ends and supports, slopes, linear/parabolic/cubic distributions,...).



Reactions (cut at the hinge, no moment)



(2)

$H_C + 6\cos 30 = 0 \Rightarrow H_C = -3\sqrt{3}$

$(\Sigma M_A = 0) \Rightarrow V_B \cdot 3 = 4 \cdot 3 \cdot \frac{3}{2} + 6 \sin 30 \cdot 5$

V_C already 0

$\Rightarrow V_B = 11$ kN

$(\Sigma F_y = 0) \to V_A + V_B = 4 \cdot 3 + 6 \sin 30 \Rightarrow V_A = 4$ kN

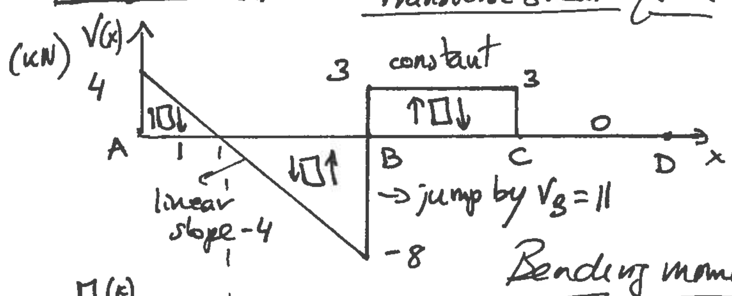
(1)

$V_C = V_D = 0$ (zero moment)

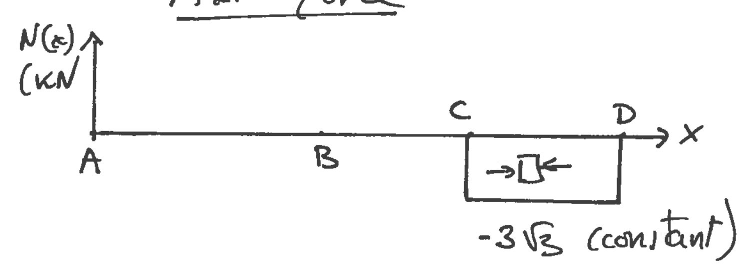
$H_D = -H_C = +3\sqrt{3}$ kN

Diagrams

Transverse shear force



Axial force



Bending moment