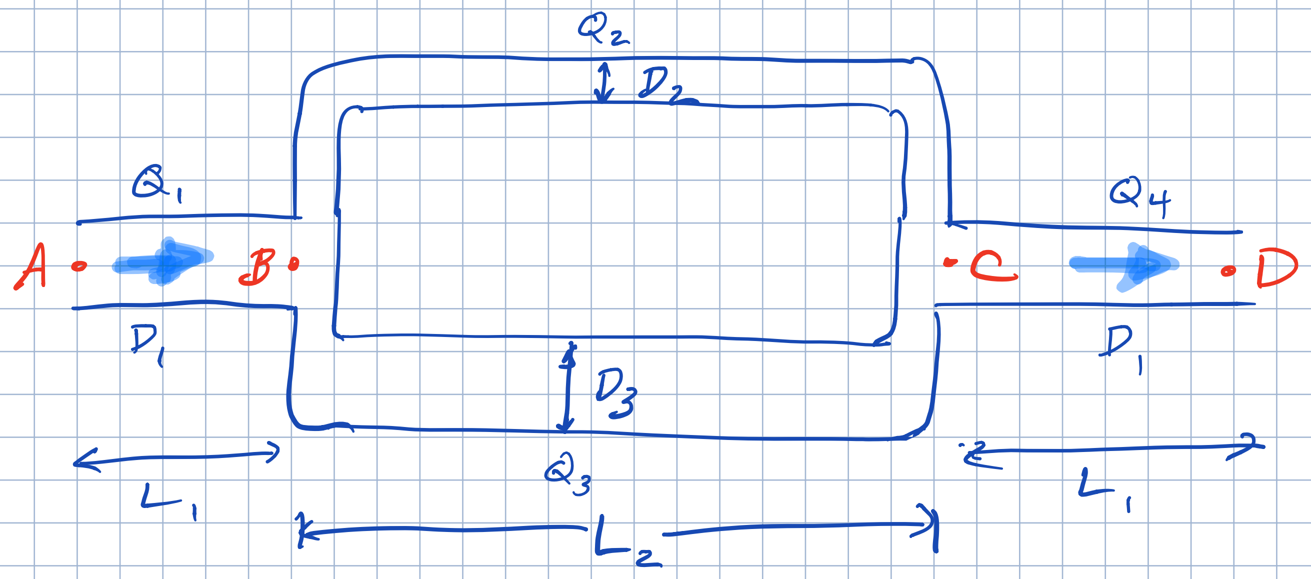
(4)



Re is low, neglect bends, splits

water: $\rho = 1000\ kg/m^3$ $\quad \eta = 0.001\ Pa \cdot s$

a) $Q_1 = Q_2 + Q_3 = Q_4$

b) $\Delta p_{AB} = \Delta p_{CD}$ $\quad$ same $\langle v \rangle$, same D, same ℓ

$$\Delta p_{AD} = \Delta p_{AB} + \Delta p_{BC} + \Delta p_{CD}$$
$$\Delta p_{AD} = 2\Delta p_{AB} + \Delta p_{BC}$$

c) Since Re is low in all pipes, the flow rates & pressure drops are related by $f = \frac{16}{Re}$ or (equivalently) the Hagen-Poiseuille eqn:

$$Q = \frac{\pi}{128} \frac{\Delta p D^4}{L \eta}$$

so $Q_2 = \frac{\pi}{128} \frac{\Delta p_{BC} D_2^4}{L_2 \eta}$, $\quad Q_3 = \frac{\pi}{128} \frac{\Delta p_{BC} D_3^4}{L_2 \eta}$

$$\Rightarrow \frac{Q_2}{Q_3} = \left(\frac{D_2}{D_3}\right)^4$$

From a)

$$Q_1 = Q_2 + Q_3 = Q_3\left(1 + \left(\frac{D_2}{D_3}\right)^4\right)$$

$$\Rightarrow \boxed{\frac{Q_3}{Q_1} = \frac{1}{\left(1 + \left(\frac{D_2}{D_3}\right)^4\right)}} \quad (*)$$

d) From Hagen-Poiseuille eqn,

$$\Delta p_{BC} = \frac{128}{\pi} \frac{\eta L_2}{D_3^4} Q_3$$

$$\Delta p_{AB} = \frac{128}{\pi} \frac{\eta L_1}{D_1^4} Q_1$$

From part b) $\quad \Delta p_{AD} = 2\Delta p_{AB} + \Delta p_{BC}$

$$\Rightarrow \Delta p_{AD} = \frac{128}{\pi} \eta \left[2\frac{L_1}{D_1^4} Q_1 + \frac{L_2}{D_3^4} Q_3 \right]$$

$$\Delta p_{AD} = \frac{128}{\pi} \eta \left[2\frac{L_1}{D_1^4} Q_1 + \frac{L_2}{D_3^4} Q_1 \frac{1}{1 + \left(\frac{D_2}{D_3}\right)^4} \right]$$

$$\frac{\Delta p_{BC}}{\Delta p_{AD}} = \frac{\frac{L_2}{D_3^4} \frac{Q_1}{\left(1 + \left(\frac{D_2}{D_3}\right)^4\right)}}{2\frac{L_1}{D_1^4} Q_1 + \frac{L_2}{D_3^4} Q_1 \frac{1}{\left(1 + \left(\frac{D_2}{D_3}\right)^4\right)}}$$

$$= \frac{\frac{L_2}{D_3^4} \frac{1}{\left(1 + \left(\frac{D_2}{D_3}\right)^4\right)}}{2\frac{L_1}{D_1^4} + \frac{L_2}{D_3^4}\left(\frac{1}{1 + \left(\frac{D_2}{D_3}\right)^4}\right)}$$

$$\frac{\Delta p_{BC}}{\Delta p_{AD}} = \frac{L_2/D_3^4}{2\frac{L_1}{D_1^4}\left(1 + \left(\frac{D_2}{D_3}\right)^4\right) + L_2/D_3^4} \quad (**)$$

e) $D_1 = 0.005\ m$, $D_2 = 0.0021\ m$, $D_3 = 0.003\ m$

$L_1 = 1\ m$, $L_2 = 2.5\ m$, $\Delta p_{AD} = 180\ Pa$

From (*)
$$\frac{Q_3}{Q_1} = \frac{1}{1+\left(\frac{D_2}{D_3}\right)^4} = 0.8064$$

From (**)
$$\frac{\Delta p_{BC}}{\Delta p_{AD}} = \frac{2.5/(0.003)^4}{2\frac{1}{(0.005)^4}\left(1+\left(\frac{.0021}{.003}\right)^4\right) + 2.5/(0.003)^4}$$

$$\Delta p_{BC} = 0.886\ \Delta p_{AD}$$
$$= 0.886\,(180\ Pa) = \underline{159.5\ Pa}$$

$$\Delta p_{AB} = \frac{1}{2}\left(\Delta p_{AD} - \Delta p_{BC}\right) \qquad \text{from a)}$$
$$= \underline{10.25\ Pa}$$

$$Q_3 = \frac{\pi}{128\,\eta}\frac{D_3^4}{L_2}\Delta p_{BC} = 1.268 \times 10^{-7}\ \frac{m^3}{s}$$

$$Q_1 = \frac{Q_3}{0.8064} = \underline{1.573 \times 10^{-7}\ \frac{m^3}{s}}$$

$$Q_2 = Q_1 - Q_3 = \underline{3.049 \times 10^{-8}\ \frac{m^3}{s}}$$

f) check Re

$$Re = \frac{\rho V D}{\eta}$$

$$Re_1 = \frac{\rho V_1 D_1}{\eta} = \frac{\rho}{\eta}\frac{Q_1}{\frac{\pi}{4}D_1^2}D_1 = \frac{4\rho Q_1}{\pi\,\eta\,D_1} = \underline{40.06}$$

since $Re_1 < 2100$ (or 4000), flow is laminar & eqns relating Q and Δp are appropriate, Assumption is ok.

g) No effect. Pipe roughness doesn't matter in laminar flow regime.

CBE 150A MT1 P2

(a) Buckingham Π analysis (7)

	Variable	Dimensions
0.5	Φ	ML^2T^{-3}
0.5	D	L
0.5	u	LT^{-1}
0.5	ρ	ML^{-3}
0.5	η	$ML^{-1}T^{-1}$

5 variables − 3 independent dimensions (M, L, T)

$\Rightarrow$ 2 dimensionless groups!

$D, u, \rho \rightarrow$ core variables.

+2 • $N_1 = \dfrac{\Phi}{D^a u^b \rho^c}$ $\Rightarrow$ $ML^2T^{-3} = M^c L^{a+b-3c} T^{-b}$

$\Rightarrow c = 1,\ b = 3,\ a = 2$

$\hat{=} \dfrac{\Phi}{D^2 u^3 \rho}$

+1 • $N_2 = \dfrac{Du\rho}{\eta} \equiv Re$ (as expected).

$\Rightarrow$ General functional relation : $N_1 = f(N_2)$

+1.5 $\Rightarrow$ $\boxed{\Phi = D^2 u^3 \rho\, f(Re)}$

(b) $\Phi \propto m^{3/4}$

+1

But $m = \rho_f V \sim \rho_f D^3$ assuming $V \sim D^3$ +1

density of fish

Volume of fish.

$$\Rightarrow \Phi \sim \rho_f^{3/4} D^{9/4} = \underbrace{k \rho_f^{3/4}}_{C_0} D^{9/4}$$ (Klieber's law) +1

Thus all we need to assume is that the volume V scales as D^3

(c) $$Re_m = \frac{0.5\,\text{m} \times 1\,\text{m/s} \times 1000\,\text{kg/m}^3}{10^{-3}\,\text{Pa.s}} = 5 \times 10^5 \gg 1$$ + 3

(d) $$F_{D,m} = C_1 D_m^2 \rho U_m^2$$ → Modern fish

$$F_{D,G} = C_1 D_G^2 \rho U_G^2$$ → Giant prehistoric relative.

$$\Rightarrow \phi_m \propto F_{D,m} U_m \;;\; \phi_G \propto F_{D,G} U_G$$

+3 +1

$$\Rightarrow \underbrace{\frac{\phi_G}{\phi_m} = \frac{F_{D,G} U_G}{F_{D,m} U_m} = \frac{\cancel{C_0} D_G^{9/4}}{\cancel{C_0} D_m^{9/4}}}$$ (from part c)

$$\Rightarrow \frac{D_G^2 U_G^3}{D_m^2 U_m^3} = \frac{D_G^{9/4}}{D_m^{9/4}}$$

$$\Rightarrow \frac{U_G^3}{U_m^3} = \frac{D_G^{1/4}}{D_m^{1/4}}$$

$$\Rightarrow U_G = U_m \left(\frac{D_G}{D_m}\right)^{1/12} = 1\,\text{m/s} \times (10)^{1/12}$$ + 2

$$\Rightarrow \boxed{U_G = 1.21\,\text{m/s}}$$ +1