CS 188 Spring 2014 — Introduction to Artificial Intelligence — Final

- You have approximately 2 hours and 50 minutes.
- The exam is closed book, closed notes except your two-page crib sheet.
- Mark your answers ON THE EXAM ITSELF. If you are not sure of your answer you may wish to provide a *brief* explanation. All short answer sections can be successfully answered in a few sentences AT MOST.

First name	
Last name	
SID	
edX username	
First and last name of student to your left	
First and last name of student to your right	

For staff use only:

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Q2.	Short questions	/16
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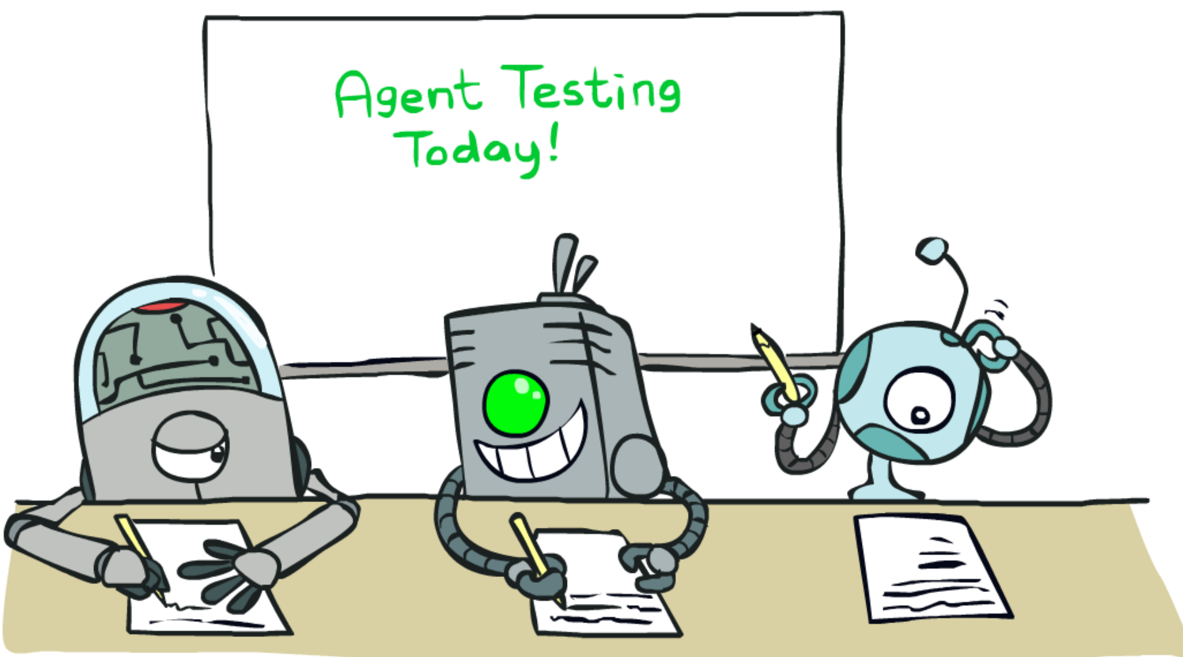
Q1. [1 pt] Agent Testing Today!

It's testing time! Not only for you, but for our CS188 robots as well! Circle your favorite robot below.



Q2. [16 pts] Short questions

(a) [4 pts] Search. If $f(s)$, $g(s)$ and $h(s)$ are all admissible heuristics then which of the following are also guaranteed to be admissible heuristics:

○ $f(s) + g(s) + h(s)$

○ $f(s)/6 + g(s)/3 + h(s)/2$

○ $\min(f(s), g(s), h(s))$

○ $\max(f(s), g(s), h(s))$

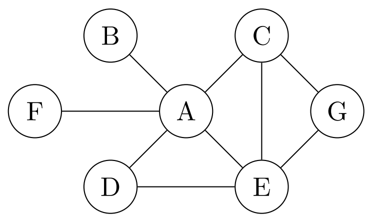
○ $f(s)/3 + g(s)/3 + h(s)/3$

○ $f(s) * g(s) * h(s)$

○ $\min(f(s), g(s) + h(s))$

○ $\max(f(s), g(s) + h(s))$

(b) CSPs. Consider solving the following CSP with backtracking search where we enforce consistency of all arcs before every value assignment. For each of the variable orderings below specify at which variables backtracking might occur. Recall that backtracking occurs at a variable X when after a value from the filtered domain of X has been assigned to the variable X the recursion returns to X without a solution and the next value from the filtered domain of X gets assigned. If enforcing arc consistency results in any empty domains then the ensuing value assignment doesn't happen and the algorithm backtracks.



(i) [1 pt] For ordering A, B, C, D, E, F, G the algorithm might backtrack at variable(s):

○ A ○ B ○ C ○ D ○ E ○ F ○ G

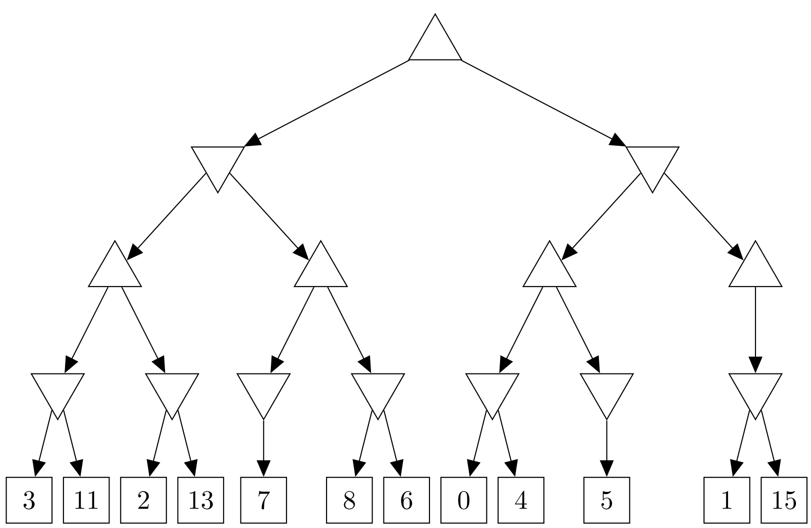
(ii) [1 pt] For ordering G, A, B, C, D, E, F the algorithm might backtrack at variable(s):

○ A ○ B ○ C ○ D ○ E ○ F ○ G

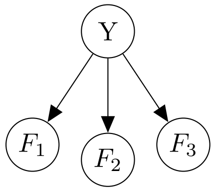
(iii) [1 pt] For ordering E, B, F, D, A, C, G the algorithm might backtrack at variable(s):

○ A ○ B ○ C ○ D ○ E ○ F ○ G

(c) [2 pts] Games. On the minimax game tree below cross out the branches removed by alpha-beta pruning assuming left to right traversal.



(d) Naive Bayes. Consider training the Naive Bayes model shown on the left with the training data provided in the table on the right.



F_1	0	0	1	0	1	1	1	1
F_2	0	1	0	1	1	0	1	1
F_3	1	1	1	0	0	1	1	0
Y	0	0	0	1	1	0	0	1

(i) [1 pt] The maximum likelihood estimate of $P(F_1 = 1 \mid Y = 0)$ is ________.

(ii) [1 pt] Assuming Laplace smoothing with $k = 1$, the estimated $P(F_2 = 1 \mid Y = 1)$ is ________.

(iii) [1 pt] Assuming Laplace smoothing with $k = 2$, the estimated $P(Y = 1)$ is ________.

(e) Perceptron. We are training a Dual Perceptron for a three-class problem. There are four training examples x_1, x_2, x_3, x_4. The dual weights are currently:

$$\alpha_A =< -1, -1, -1, -1 > \text{ for class } A$$
$$\alpha_B =< -1, +1, +1, -1 > \text{ for class } B$$
$$\alpha_C =< +1, -1, -1, +1 > \text{ for class } C$$

Consider the fourth training example x_4 with correct label A and kernel evaluations:

$$K(x_1, x_4) = 1, \quad K(x_2, x_4) = 2, \quad K(x_3, x_4) = 1, \quad K(x_4, x_4) = 3$$

(i) [1 pt] Which classification label is predicted for the fourth training example x_4 with the current dual weights?

○ A ○ B ○ C

(ii) [3 pts] What are the dual weights after the update that incorporates the fourth training example?

$\alpha_A =$ ____________________________

$\alpha_B =$ ____________________________

$\alpha_C =$ ____________________________

Q3. [12 pts] Finding the Best k Paths

The optimal search algorithms we covered in CS188 find **one** optimal path (or return failure). We will explore how to find the best k (with $k \geq 1$) paths.

The following assumptions can be made regarding all the questions in this problem :

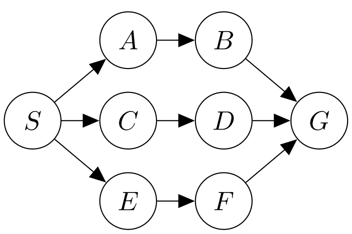
1. There are at least k paths from the start state to a goal state.
2. All edge costs are positive numbers (cost > 0).
3. No ties occur.

Consider a modified implementation of the **Uniform Cost Graph Search (UCS)** algorithm with the following **basic modifications:**

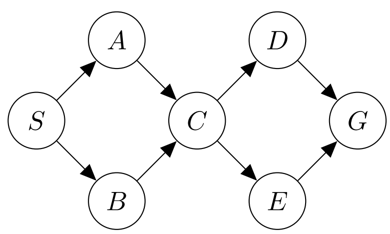
1. Maintain a list of successful paths to a goal state found so far. When a path from the start state to a goal state is found (i.e., whenever a path ending in the goal state is *popped from* the fringe), it is added to this list.
2. Exit the algorithm only if the length of the above list is k (success) or the fringe is empty (failure).

For each of the **additional modifications** on the next page, mark whether or not it would correctly give the top k unique least cost paths from the start state to a goal state. If a modification does not work, select **all** of the below graphs where there exists at least one set of edge weights and value for k (subject to the constraint that there are at least k paths through the graph) that would cause the algorithm to fail. Note that some modifications may even lead to failure for $k = 1$.

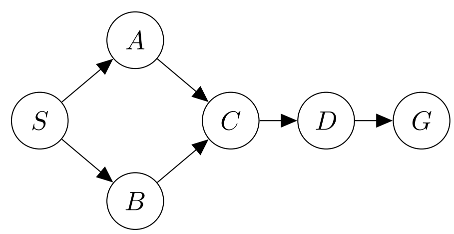
1)



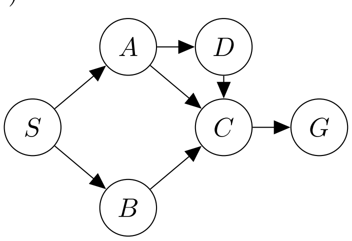
2)



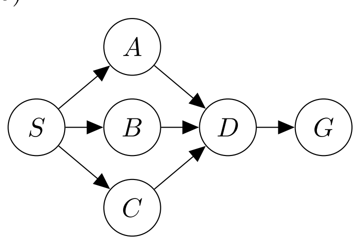
3)



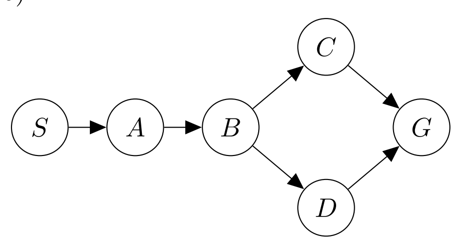
4)



5)



6)



(a) [2 pts] Everytime after a path is found, empty out the closed set.

○ Will work correctly ○ Will not work correctly

Graphs for which this modification fails:

○ 1	○ 2	○ 3
○ 4	○ 5	○ 6

(b) [2 pts] For each state s, maintain a count `count_expand(s)` of how many times a path ending in state s has been popped from the fringe. Only add a state s to the closed set if `count_expand(s)` $= k$.

○ Will work correctly ○ Will not work correctly

Graphs for which this modification fails:

○ 1	○ 2	○ 3
○ 4	○ 5	○ 6

(c) [2 pts] Do not use a closed set.

○ Will work correctly ○ Will not work correctly

Graphs for which this modification fails:

○ 1	○ 2	○ 3
○ 4	○ 5	○ 6

(d) [2 pts] Do not use a closed set and, every time after a path is found, change the edge costs along that path by adding C, where C is a number that is at least as large as the sum of the costs of all edges in the graph. Also for each path on the fringe that contains i edges of the path that was just found, add $i \times C$ to the cost associated with this path on the fringe.

○ Will work correctly ○ Will not work correctly

Graphs for which this modification fails:

○ 1	○ 2	○ 3
○ 4	○ 5	○ 6

(e) [2 pts] Do not use a closed set and, for each state s, maintain a count `count_fringe(s)` of how many times a node ending in state s has been added to the fringe. Only add a node ending in a state s to the fringe if `count_fringe(s)` $< k$.

○ Will work correctly ○ Will not work correctly

Graphs for which this modification fails:

○ 1	○ 2	○ 3
○ 4	○ 5	○ 6

(f) [2 pts] No modification is made except for the Basic Modification described at the beginning of this question.

○ Will work correctly ○ Will not work correctly

Graphs for which this modification fails:

○ 1	○ 2	○ 3
○ 4	○ 5	○ 6

Q4. [17 pts] Probability and Bayes Nets

(a) [2 pts] Suppose $A \perp\!\!\!\perp B$. Determine the missing entries (x, y) of the joint distribution $P(A, B)$, where A and B take values in $\{0, 1\}$.

$$P(A = 0, B = 0) = 0.1$$
$$P(A = 0, B = 1) = 0.3$$
$$P(A = 1, B = 0) = x$$
$$P(A = 1, B = 1) = y$$

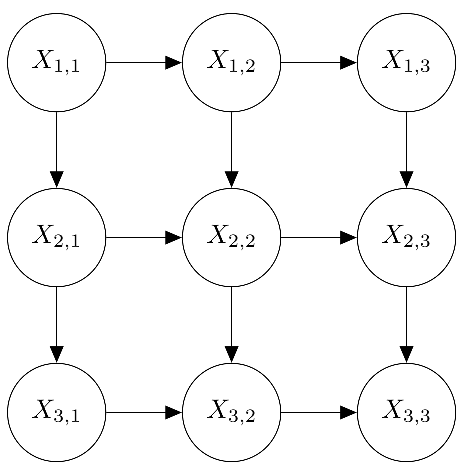
$x =$ ______________, $y =$ ______________

(b) [3 pts] Suppose $B \perp\!\!\!\perp C \mid A$. Determine the missing entries (x, y, z) of the joint distribution $P(A, B, C)$.

$$P(A = 0, B = 0, C = 0) = 0.01$$
$$P(A = 0, B = 0, C = 1) = 0.02$$
$$P(A = 0, B = 1, C = 0) = 0.03$$
$$P(A = 0, B = 1, C = 1) = x$$
$$P(A = 1, B = 0, C = 0) = 0.01$$
$$P(A = 1, B = 0, C = 1) = 0.1$$
$$P(A = 1, B = 1, C = 0) = y$$
$$P(A = 1, B = 1, C = 1) = z$$

$x =$ ______________, $y =$ ______________, $z =$ ______________

(c) [3 pts] For this question consider the Bayes' Net below with 9 variables.



Which random variables are independent of $X_{3,1}$? (Leave blank if the answer is none.)

○ $X_{1,1}$ ○ $X_{1,2}$ ○ $X_{1,3}$ ○ $X_{2,1}$ ○ $X_{2,2}$ ○ $X_{2,3}$ ○ $X_{3,2}$ ○ $X_{3,3}$

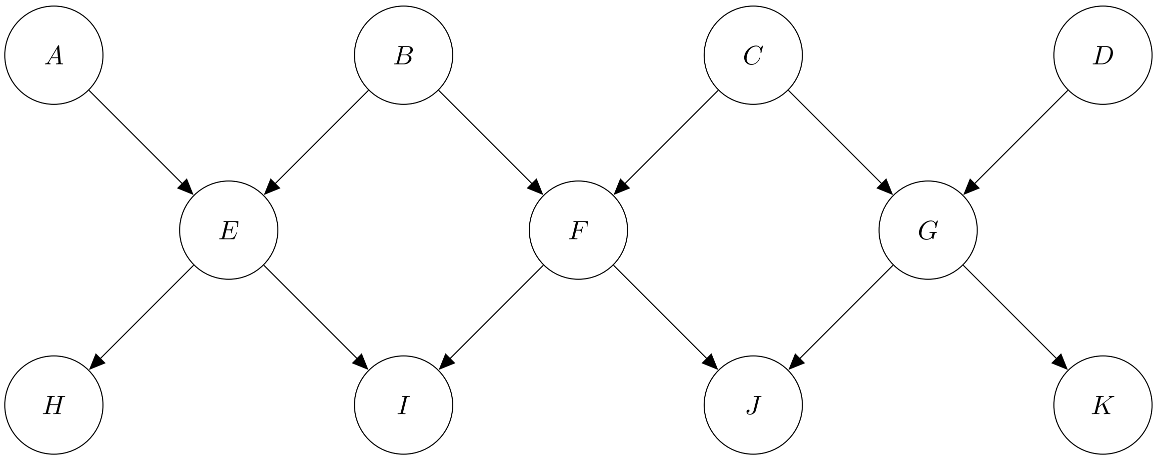
Which random variables are independent of $X_{3,1}$ *given* $X_{1,1}$? (Leave blank if the answer is none.)

○ $X_{1,2}$ ○ $X_{1,3}$ ○ $X_{2,1}$ ○ $X_{2,2}$ ○ $X_{2,3}$ ○ $X_{3,2}$ ○ $X_{3,3}$

Which random variables are independent of $X_{3,1}$ *given* $X_{1,1}$ and $X_{3,3}$? (Leave blank if the answer is none.)

○ $X_{1,2}$ ○ $X_{1,3}$ ○ $X_{2,1}$ ○ $X_{2,2}$ ○ $X_{2,3}$ ○ $X_{3,2}$

For the following questions we will consider the following Bayes' Net:



(d) For each of the following queries, mark which variables' conditional probability tables will affect the answer to the query. For example, by marking F you'd indicate that the values in the conditional probability table $P(F \mid B, C)$ affect the answer to the query.

(i) [1 pt] $P(A \mid +k)$

○ A ○ B ○ C ○ D ○ E ○ F ○ G ○ H ○ I ○ J ○ K

(ii) [1 pt] $P(A \mid +d)$

○ A ○ B ○ C ○ D ○ E ○ F ○ G ○ H ○ I ○ J ○ K

(iii) [1 pt] $P(A, D)$

○ A ○ B ○ C ○ D ○ E ○ F ○ G ○ H ○ I ○ J ○ K

(iv) [1 pt] $P(A, D| + i, -j)$

○ A ○ B ○ C ○ D ○ E ○ F ○ G ○ H ○ I ○ J ○ K

(v) [1 pt] $P(A| + j, +k)$

○ A ○ B ○ C ○ D ○ E ○ F ○ G ○ H ○ I ○ J ○ K

(vi) [1 pt] $P(A| + i, +k)$

○ A ○ B ○ C ○ D ○ E ○ F ○ G ○ H ○ I ○ J ○ K

(e) Consider a run of Gibbs sampling for the query $P(B, C \mid +h, +i, +j)$. The current sample value is $+a, +b, +c, +d$, $+e, +f, +g, +h, +i, +j, +k$. For each of the following scenarios, write out an expression for the distribution Gibbs sampling would sample from. *Your expression should contain only conditional probabilities available in the network, and your expression should contain a minimal number of such conditional probabilities.*

(i) [1 pt] If A were to be sampled next, the distribution over A to sample from would be:

____

(ii) [1 pt] If F were to be sampled next, the distribution over F to sample from would be:

____

(iii) [1 pt] If K were to be sampled next, the distribution over K to sample from would be:

____

Q5. [6 pts] Kernels and Feature Transforms

A kernel function $K(x, z)$ is a function that conceptually denotes the similarity between two instances x and z in a transformed space. More specifically, for a feature transform $x \to \phi(x)$, the kernel function is $K(x, z) = \phi(x) \cdot \phi(z)$. The beauty of algorithms using kernel functions is that we never actually need to explicitly specify this feature transform $\phi(x)$ but only the values $K(x, z)$ for pairs (x, z). In this problem, we will explore some kernel functions and their feature transforms. For this problem the input vectors are assumed to be 2 dimensional (i.e. $x = (x_1, x_2)$). Remember that $x \cdot z = x_1 z_1 + x_2 z_2$.

(a) For each of the kernel functions below, mark the corresponding feature transform: (mark a single option only for each question)

(i) [1 pt] $K(x, z) = 1 + x \cdot z$

- ◯ $\phi(x) = (x_1, x_2)$
- ◯ $\phi(x) = (1, x_1, x_2)$
- ◯ $\phi(x) = (1, x_1^2, x_2^2)$
- ◯ $\phi(x) = (x_1^2, x_2^2)$
- ◯ $\phi(x) = (x_1^2, x_2^2, \sqrt{2}x_1x_2)$
- ◯ $\phi(x) = (1, x_1^2, x_2^2, \sqrt{2}x_1x_2)$

(ii) [1 pt] $K(x, z) = (x \cdot z)^2$

- ◯ $\phi(x) = (x_1^2, x_2^2)$
- ◯ $\phi(x) = (1, x_1^2, x_2^2)$
- ◯ $\phi(x) = (1, x_1^2, x_2^2, \sqrt{2}x_1x_2)$
- ◯ $\phi(x) = (x_1^2, x_2^2, \sqrt{2}x_1x_2)$
- ◯ $\phi(x) = (1, x_1, x_2, x_1^2, x_2^2, \sqrt{2}x_1x_2)$
- ◯ $\phi(x) = (x_1, x_2, x_1^2, x_2^2, \sqrt{2}x_1x_2)$

(iii) [1 pt] $K(x, z) = (1 + x \cdot z)^2$

- ◯ $\phi(x) = (1, x_1^2, x_2^2)$
- ◯ $\phi(x) = (1, x_1^2, x_2^2, \sqrt{2}x_1x_2)$
- ◯ $\phi(x) = (1, x_1^2, x_2^2, x_1, x_2, \sqrt{2}x_1x_2)$
- ◯ $\phi(x) = (1, x_1^2, x_2^2, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_1x_2)$
- ◯ $\phi(x) = (1, x_1, x_2, \sqrt{2}x_1x_2)$
- ◯ $\phi(x) = (1, x_1x_2, x_1^2x_2^2)$

(b) Multiple kernels can be combined to produce new kernel functions. For example $K(x, z) = K_1(x, z) + K_2(x, z)$ is a valid kernel function. For the questions below, kernel K_1 has the associated feature transform ϕ_1 and similarly K_2 has the feature transform ϕ_2. Mark the feature transform associated with K for the expressions given below.

Note: The operator $[*, *]$ denotes concatenation of the two arguments. For example, $[x, z] = (x_1, x_2, z_1, z_2)$.

(i) [1 pt] $K(x, z) = aK_1(x, z)$, for some scalar $a > 0$

- ◯ $\phi(x) = \phi_1(x)$
- ◯ $\phi(x) = [a, \phi_1(x)]$
- ◯ $\phi(x) = a\phi_1(x)$
- ◯ $\phi(x) = \sqrt{a}\phi_1(x)$
- ◯ $\phi(x) = \phi_1(x) + a$
- ◯ $\phi(x) = a^2\phi_1(x)$

(ii) [1 pt] $K(x, z) = aK_1(x, z) + bK_2(x, z)$, for scalars $a, b > 0$

- ◯ $\phi(x) = a\phi_1(x) + b\phi_2(x)$
- ◯ $\phi(x) = \sqrt{a}\phi_1(x) + \sqrt{b}\phi_2(x)$
- ◯ $\phi(x) = a^2\phi_1(x) + b^2\phi_2(x)$
- ◯ $\phi(x) = [a\phi_1(x), b\phi_2(x)]$
- ◯ $\phi(x) = [\sqrt{a}\phi_1(x), \sqrt{b}\phi_2(x)]$
- ◯ $\phi(x) = [a^2\phi_1(x), b^2\phi_2(x)]$

(c) [1 pt] Suppose you are given the choice between using the normal perceptron algorithm, which directly works with $\phi(x)$, and the dual (kernelized) perceptron algorithm, which does not explictly compute $\phi(x)$ but instead works with the kernel function K. Keeping space and time complexities in consideration, when would you prefer using the kernelized perceptron algorithm over the normal perceptron algorithm.

Note: Here N denotes the total number of training samples and d is the dimensionality of $\phi(x)$.

◯ $d >> N$ ◯ $d << N$ ◯ Always ◯ Never

Q6. [9 pts] Stopping Strategy

A fair six sided dice is rolled repeatedly and you observe outcomes sequentially. Formally, dice roll outcomes are independently and uniformly sampled from the set $\{1, 2, 3, 4, 5, 6\}$. At every time step before the h^{th} roll you can choose between two actions:

Stop: stop and receive a reward equal to the number shown on the dice or,

Roll: roll again and receive no immediate reward.

If not having stopped before then, at time step h (which would be reached after $h - 1$ rolls) you are forced to take the action Stop, you receive the corresponding reward and the game ends.

We will model the game as a finite horizon MDP with six states and two actions. The state at time step k corresponds to the number shown on the dice at the k^{th} roll. Assume that the discount factor, γ, is 1.

(a) [2 pts] The value function at time step h, when it is no longer possible to roll the dice again, is $V^h(1) = 1, V^h(2) = 2, \ldots, V^h(6) = 6$. Compute the value function at time step $h - 1$:

$V^{h-1}(1) =$ ________ $V^{h-1}(4) =$ ________

$V^{h-1}(2) =$ ________ $V^{h-1}(5) =$ ________

$V^{h-1}(3) =$ ________ $V^{h-1}(6) =$ ________

(b) [2 pts] Express the value function at time step $k - 1$, with $2 < k \leq h$ recursively in terms of the value function at roll k, so in terms of $V^k(1), V^k(2), \ldots V^k(6)$:

$V^{k-1}(i) =$ ____

The Q function at time step k for action "Roll" does not depend on the state since the number shown by the dice is irrelevant once you decided to roll. We use the shorthand notation $q(k) = Q^k(\text{state}, \text{"Roll"})$ since the only dependence is on k.

(c) [1 pt] Compute $q(h-1)$:_________

(d) [2 pts] Express $q(k-1)$ recursively as a function of $q(k)$, with $2 < k \leq h$.

$q(k-1) =$ __________________________

(e) [2 pts] What is the optimal policy $\pi^k(s)$ at roll k as a decision rule based on the current state s and $q(k)$?

$\pi^k(s) =$ Roll if ____ , stop otherwise

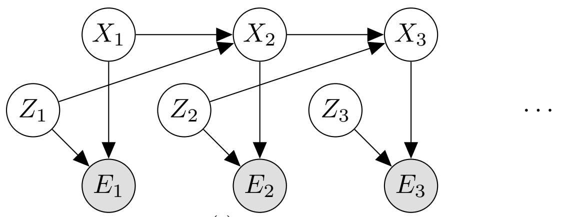
Q7. [13 pts] Inference

(a) Recall that for a standard HMM the Elapse Time update and the Observation update are of the respective forms:

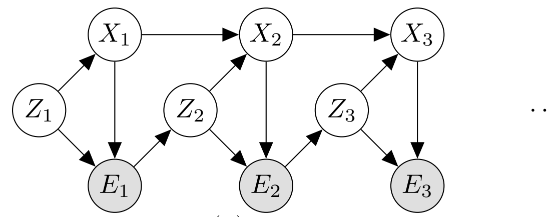
$P(X_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(X_t \mid x_{t-1})P(x_{t-1} \mid e_{1:t-1})$

$P(X_t \mid e_{1:t}) \propto P(X_t \mid e_{1:t-1})P(e_t \mid x_t)$

We now consider the following two HMM-like models:



(i)



(ii)

Mark the modified Elapse Time update and the modified Observation update that correctly compute the beliefs from the quantities that are available in the Bayes' Net. (Mark one of the first set of six options, and mark one of the second set of six options for (i), and same for (ii).)

(i) [2 pts]

- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1},z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(X_t \mid x_{t-1}, z_{t-1})P(Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1},z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(X_t \mid x_{t-1}, z_{t-1})$
- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1},z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(X_t, Z_t \mid x_{t-1}, z_{t-1})$
- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(X_t \mid x_{t-1}, z_{t-1})P(Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(X_t \mid x_{t-1}, z_{t-1})$
- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(X_t, Z_t \mid x_{t-1}, z_{t-1})$

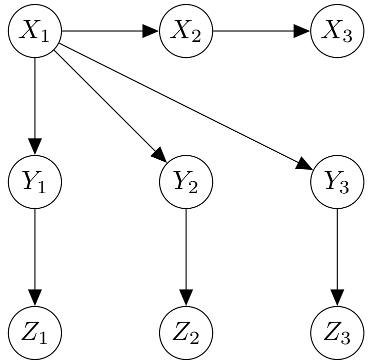
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1})P(e_t \mid X_t, Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto \sum_{X_t} P(X_t, Z_t \mid e_{1:t-1})P(e_t \mid X_t, Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto \sum_{Z_t} P(X_t, Z_t \mid e_{1:t-1})P(e_t \mid X_t, Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1})P(e_t \mid X_t)P(e_t \mid Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1})P(e_t \mid X_t)$
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1}) \sum_{X_t} P(e_t \mid X_t)$

(ii) [2 pts]

- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1},z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(X_t \mid x_{t-1}, z_{t-1})P(Z_t \mid e_{t-1})$
- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1},z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(Z_t \mid e_{t-1})P(X_t \mid x_{t-1}, Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1},z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(X_t, Z_t \mid x_{t-1}, e_{t-1})$
- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(X_t \mid x_{t-1}, z_{t-1})P(Z_t \mid e_{t-1})$
- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(Z_t \mid e_{t-1})P(X_t \mid x_{t-1}, Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1})P(X_t, Z_t \mid x_{t-1}, e_{t-1})$

- ○ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1})P(e_t \mid X_t, Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto \sum_{X_t} P(X_t, Z_t \mid e_{1:t-1})P(e_t \mid X_t, Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto \sum_{Z_t} P(X_t, Z_t \mid e_{1:t-1})P(e_t \mid X_t, Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1})P(e_t \mid X_t)P(e_t \mid Z_t)$
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1})P(e_t \mid X_t)$
- ○ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1}) \sum_{X_t} P(e_t \mid X_t)$

(b) In this question we will consider a Bayes' Net with the following structure:



(i) [3 pts] Mark *all* of the following expressions that hold true for distributions represented by the Bayes' Net above.

- ○ $P(X_1, X_2, X_3 \mid +y_1) = P(X_1, X_2, X_3 \mid -y_1)$
- ○ $P(Z_1, +x_3) = \sum_{x_1,x_2,y_1} P(x_1)P(x_2 \mid x_1)P(+x_3 \mid x_2)P(y_1 \mid x_1)P(Z_1 \mid y_1)$
- ○ $P(Z_1, +x_3) = \sum_{x_1} P(x_1) \sum_{x_2} P(x_2 \mid x_1)P(+x_3 \mid x_2) \sum_{y_1} P(y_1 \mid x_1)P(Z_1 \mid y_1)$
- ○ $P(Z_3 \mid +x_1, -y_3) = P(Z_3 \mid -x_1, -y_3)$
- ○ $P(Z_3 \mid +x_1, -y_3) = P(Z_3 \mid +x_1, +y_3)$
- ○ $P(Y_1, Y_2, Y_3) = P(Y_1)P(Y_2)P(Y_3)$

(ii) [2 pts] For the query $P(Z_1 \mid +x_3, +z_2, +z_3)$:

List a *most efficient* variable elimination ordering: ____

List a *least efficient* variable elimination ordering: ____

Note: efficiency is measured by the size of the single largest factor generated during the variable elimination process.

(iii) [4 pts] Consider sampling through likelihood weighting. For each of the following fill in the weight of the sample and fill in the probability of that sample being the one generated when using likelihood weighting with the provided evidence. (Make sure to use only conditional probabilities available from the Bayes' Net.)

Evidence: $+x_1, +x_2, +x_3$. Sample: $+x_1, +x_2, +x_3, +y_1, +y_2, +y_3, +z_1, +z_2, +z_3$.

Sample weight = ____

Probability of generating this sample = ____

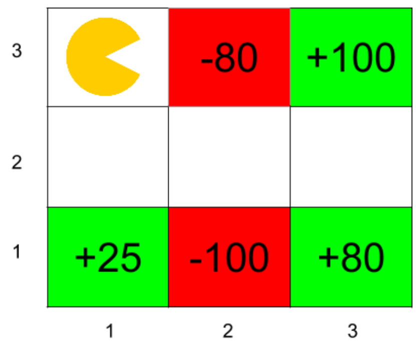
Evidence: $+z_1, +z_2, +z_3$. Sample: $+x_1, +x_2, +x_3, +y_1, +y_2, +y_3, +z_1, +z_2, +z_3$.

Sample weight = ____

Probability of generating this sample = ____

Q8. [8 pts] Q-Learning Strikes Back

Consider the grid-world given below and Pacman who is trying to learn the optimal policy. If an action results in landing into one of the shaded states the corresponding reward is awarded during that transition. All shaded states are terminal states, i.e., the MDP terminates once arrived in a shaded state. The other states have the *North, East, South, West* actions available, which deterministically move Pacman to the corresponding neighboring state (or have Pacman stay in place if the action tries to move out of the grad). Assume the discount factor $\gamma = 0.5$ and the Q-learning rate $\alpha = 0.5$ for all calculations. Pacman starts in state (1, 3).



(a) [2 pts] What is the value of the optimal value function V^* at the following states:

$V^*(3,2) =$ ________ $V^*(2,2) =$ ________ $V^*(1,3) =$ ________

(b) [3 pts] The agent starts from the top left corner and you are given the following episodes from runs of the agent through this grid-world. Each line in an Episode is a tuple containing (s, a, s', r).

Episode 1	Episode 2	Episode 3
(1,3), S, (1,2), 0	(1,3), S, (1,2), 0	(1,3), S, (1,2), 0
(1,2), E, (2,2), 0	(1,2), E, (2,2), 0	(1,2), E, (2,2), 0
(2,2), S, (2,1), -100	(2,2), E, (3,2), 0	(2,2), E, (3,2), 0
	(3,2), N, (3,3), +100	(3,2), S, (3,1), +80

Using Q-Learning updates, what are the following Q-values after the above three episodes:

Q((3,2),N) = ________ Q((1,2),S) = ________ $Q((2,2), E) =$ ________

(c) Consider a feature based representation of the Q-value function:

$$Q_f(s,a) = w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(a)$$

$f_1(s)$: The x coordinate of the state $f_2(s)$: The y coordinate of the state

$$f_3(N) = 1,\ f_3(S) = 2,\ f_3(E) = 3,\ f_3(W) = 4$$

(i) [2 pts] Given that all w_i are initially 0, what are their values after the first episode:

$w_1 =$ ________ $w_2 =$ ________ $w_3 =$ ________

(ii) [1 pt] Assume the weight vector w is equal to $(1, 1, 1)$. What is the action prescribed by the Q-function in state $(2, 2)$?

____

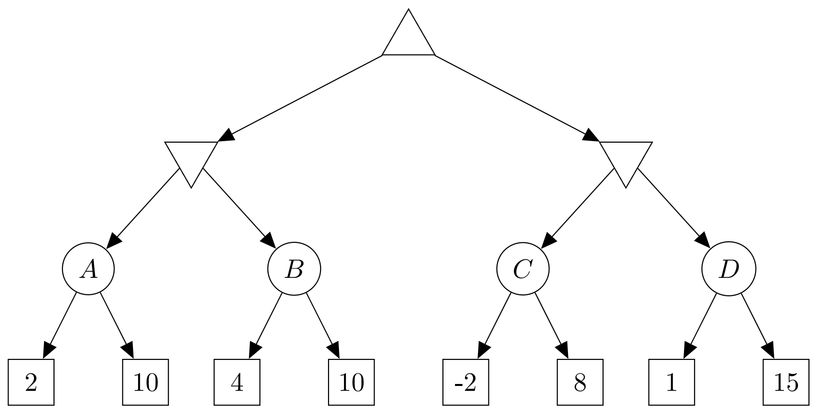
Q9. [9 pts] Adversarial VPI

In this problem you'll be considering VPI of unknown variables in an adversarial game. For this problem, assume that all observations of the random variables encoded as chance nodes are seen by both agents and that all chance nodes have equal probability for all children.

Hint: The properties of VPI presented in class were specifically for VPI when applied to a situation involving a single agent. These properties may or may not hold for situations with multiple agents. For example, the VPI of a node may be negative from the perspective of one of the agents.

When referring to VPI in the questions below, we always refer to the VPI for the maximizer.

(a) In this question we will consider the following game tree:



(i) [1 pt] What is the value of the game, for the maximizer, represented by the search tree above?

Answer: ________________

(ii) [1 pt] What is the VPI, for the maximizer, of the outcome of node A being revealed before the game is played?

Answer: ________________

(iii) [1 pt] What is the VPI, for the maximizer, of the outcome of node B being revealed before the game is played?

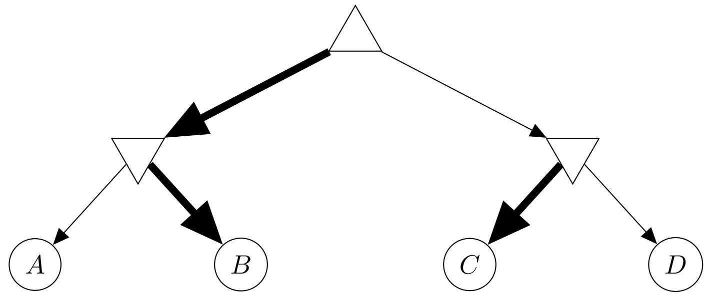
Answer: ________________

(iv) [1 pt] What is the VPI, for the maximizer, of the outcome of node C being revealed before the game is played?

Answer: ________________

(v) [1 pt] What is the VPI, for the maximizer, of the outcome of node D being revealed before the game is played?

Answer: ________________



(b) The game tree above represents a different game in which the leaf utilities are omitted, but the edges corresponding to the action that would be selected at each node are bolded. Specifically, the maximizer would select left, the left minimizer would select node B, and the right minimizer would select C. For each of the following parts, select the most accurate expression for the VPI of the specified node.

When referring to VPI in the questions below, we always refer to the VPI for the maximizer.

(i) [1 pt] VPI(A):

○ VPI(A) $= 0$ ○ VPI(A) > 0 ○ VPI(A) < 0
○ VPI(A) $\in \mathbb{R}$ ○ VPI(A) ≥ 0 ○ VPI(A) ≤ 0

(ii) [1 pt] VPI(B):

○ VPI(B) $= 0$ ○ VPI(B) > 0 ○ VPI(B) < 0
○ VPI(B) $\in \mathbb{R}$ ○ VPI(B) ≥ 0 ○ VPI(B) ≤ 0

(iii) [1 pt] VPI(C):

○ VPI(C) $= 0$ ○ VPI(C) > 0 ○ VPI(C) < 0
○ VPI(C) $\in \mathbb{R}$ ○ VPI(C) ≥ 0 ○ VPI(C) ≤ 0

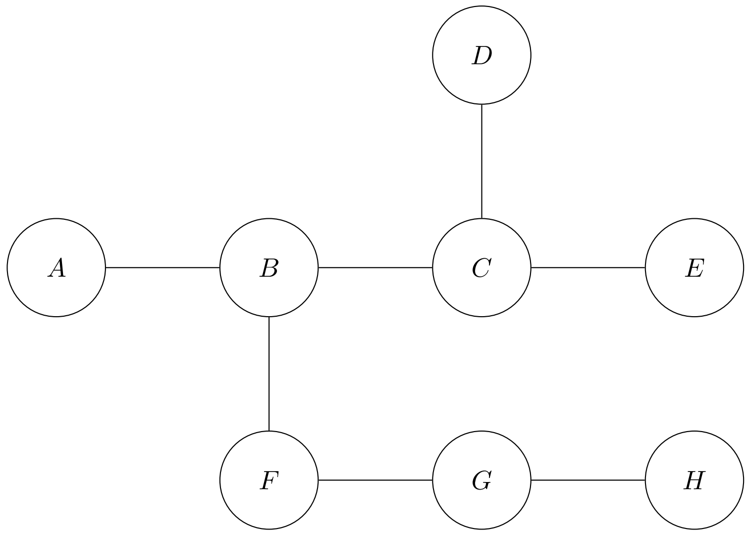
(iv) [1 pt] VPI(D):

○ VPI(D) $= 0$ ○ VPI(D) > 0 ○ VPI(D) < 0
○ VPI(D) $\in \mathbb{R}$ ○ VPI(D) ≥ 0 ○ VPI(D) ≤ 0

Q10. [9 pts] Bayes Net CSPs

(a) For the following Bayes' Net structures that are missing a direction on their edges, assign a direction to each edge such that the Bayes' Net structure implies the requested conditional independences and such that the Bayes' Net structure does not imply the conditional independences requested not to be true. Keep in mind that Bayes' Nets cannot have directed cycles.

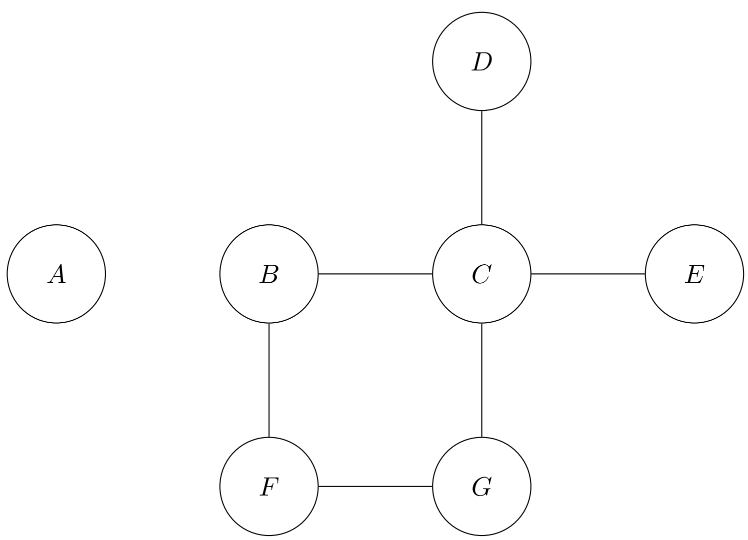
(i) [2 pts]



Constraints:

- $D \perp\!\!\!\perp G$
- not $D \perp\!\!\!\perp A$
- $D \perp\!\!\!\perp E$
- $H \perp\!\!\!\perp F$

(ii) [2 pts]

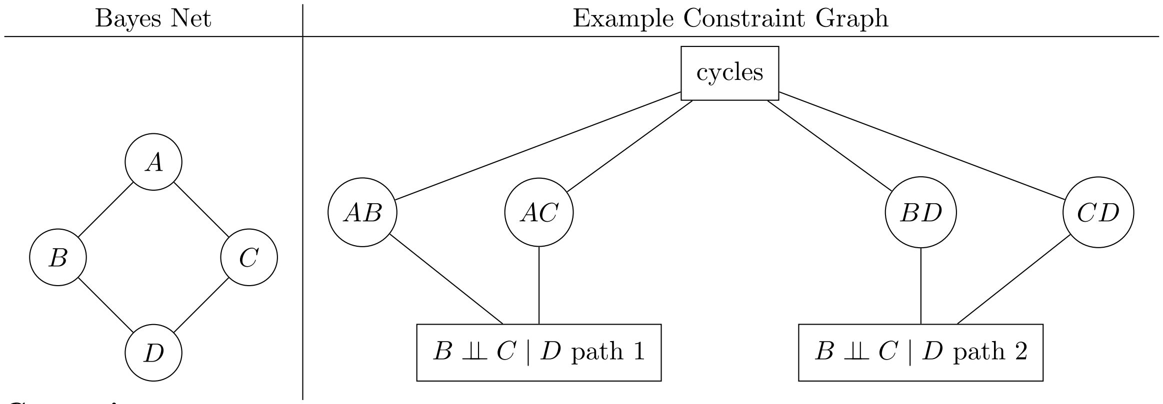


Constraints:

- $D \perp\!\!\!\perp F$
- not $D \perp\!\!\!\perp G$
- $D \perp\!\!\!\perp E$
- Bayes Net has no directed cycles

(b) For each of the following Bayes Nets and sets of constraints draw a constraint graph for the CSP. Remember that the constraint graph for a CSP with non-binary constraints, i.e., constraints that involve more than two variables, is drawn as a rectangle with the constraint connected to a node for each variable that participates in that constraint. A simple example is given below.

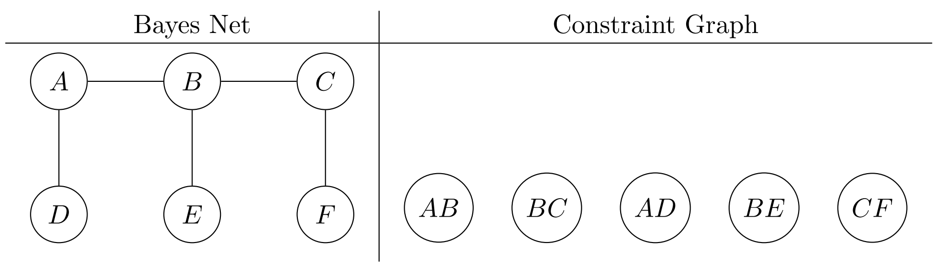
Note: As shown in the example below, if a constraint can be broken up into multiple constraints, do so.



Constraints:

- $B \perp\!\!\!\perp C \mid D$
- No directed cycles

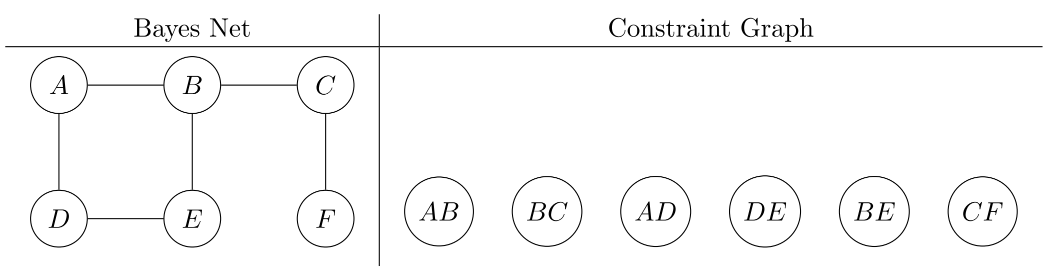
(i) [2 pts]



Constraints:

- $A \perp\!\!\!\perp F \mid E$
- not $D \perp\!\!\!\perp C$

(ii) [3 pts]



Constraints:

- $A \perp\!\!\!\perp E \mid F$
- $C \perp\!\!\!\perp E$
- No directed cycles