IEOR 165 – Midterm Exam
Spring 2021

Instructions:

- Open notes/homeworks/solutions only
- Calculators are allowed (graphing calculators are okay)
- Excel/R/Python/similar softwares are **not** allowed
- If steps/works are not shown, then points will be deducted
- Communicating with anyone other than GSI/instructor is **not** allowed
- Exam period **starts** Wednesday, March 17 at 2PM (Pacific Time) and **ends** Thursday, March 18 at 2PM (Pacific Time)
- Once you start the exam, you will have **1.5 hours** to complete the exam
- There are extra office hours during which you can get clarification about the exam questions

Name: ____________________________________

Student ID: ______________________________

1	/10
2	/10
3	/10
4	/10
5	/10

1. We are growing beans in science class and would like to study the effect of fertilizer amount (F) on the height of the bean stalks (H) after a month of time. The experiment was performed on six bean stalks with different values of F. The data collected is shown below.

H	F
1.4	10
2.2	15
5.0	30
6.5	45
8.0	50
8.2	60

Consider the linear model $H_i = \beta F_i + \epsilon_i$ where i denotes the i^{th} data. Assume that the noises $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$ are iid with known σ^2, and that the prior distribution of the coefficient is Gaussian: $\beta \sim \mathcal{N}(0, 1/(2\lambda))$. Suppose $\sigma^2 = 0.3$ and $\lambda = 2$. Find the estimate for β using the Maximum A Posteriori (MAP) method. (10 points)

2. Suppose that we are flipping a 'tilted' coin that shows a head with probability p and a tail with probability $1 - p$. Let X be a random variable that counts the number of heads after flipping the coin n times. Then, $X \sim Binomial(n, p)$ with mean np and variance $np(1-p)$. After repeating the experiment five times (each experiment consists of n coin flips), we observe $X_1, \ldots, X_5$ to be equal to $15, 20, 18, 22, 20$. Use the method of moments to estimate the parameters n and p. (10 points)

3. Let $a > 0$, $\theta > 0$ and $X_1, \ldots, X_n$ be iid random variables from the probability distribution function

$$f_{a;\theta}(x) = \frac{a-1}{\theta^{a-1}} \cdot x^{a-2}, \quad 0 < x < \theta$$

The first and second moments of the above distribution is

$$\mathbb{E}[X] = \frac{a-1}{a}\theta, \quad \mathbb{E}[X^2] = \frac{(a-1)\cdot\theta^2}{a+1}$$

Use the method of moments to derive an estimator of a and θ. (10 points)

4. (a) Suppose $x_1, \ldots, x_n$ are iid samples from a distribution with density

$$f_\theta(x) = \frac{\theta}{x^2}, \quad 0 < \theta \le x$$

Find the maximum likelihood estimator (MLE) of θ. (5 points)

(b) Suppose $y_1, \ldots, y_n$ are iid samples from a distribution with density

$$f_\theta(y) = \frac{2}{\theta} \cdot y \cdot \exp\{-\frac{y^2}{\theta}\}, \quad y > 0, \quad \theta > 0$$

Find the maximum likelihood estimator (MLE) of θ. (5 points)

5. We propose the following model for the time it takes to perform a simple task as a function of the number of times the task has been practiced

$$T \approx ts^{-n}$$

where T is the time, n is the number of times the task has been practiced, and t and s are parameters depending on the task and individual. Given the following data set

T	22.4	21.3	19.7	15.6	15.2	13.9
n	0	1	2	3	4	5

(a) Estimate t and s. (7 points)

(b) Estimate the time it takes to perform the task after 6 practices. (3 points)