



PRINT your name and student ID: \_\_\_\_\_

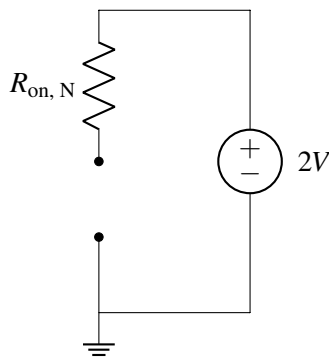
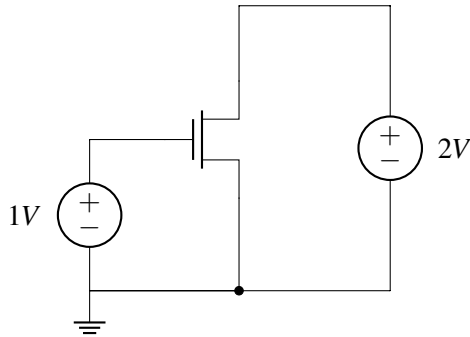
[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

PRINT your name and student ID: \_\_\_\_\_

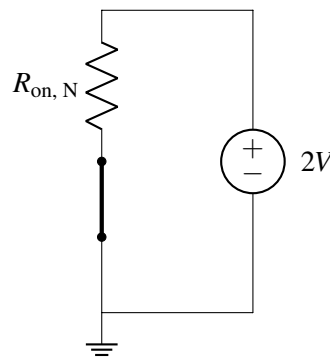
**3. Transistor Behavior (12 pts)**

For all NMOS devices in this problem,  $V_{tn} = 0.5V$ . For all PMOS devices in this problem,  $|V_{tp}| = 0.6V$ .

(a) (4 pts) Which is the equivalent circuit for the right-hand side of the circuit? **Fill in the correct bubble.**



Circuit A

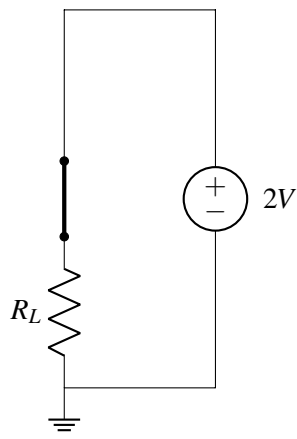
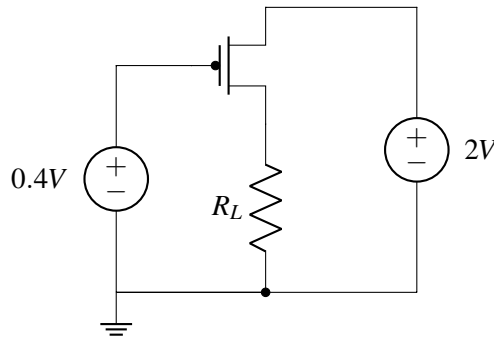


Circuit B

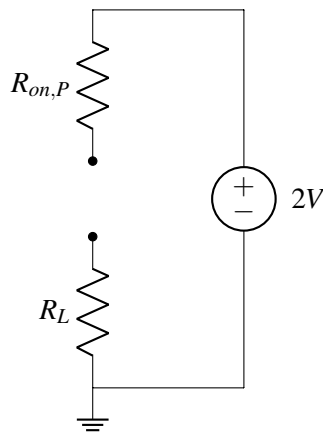
|                           | <b>A</b>              | <b>B</b>              |
|---------------------------|-----------------------|-----------------------|
| <b>Equivalent Circuit</b> | <input type="radio"/> | <input type="radio"/> |

PRINT your name and student ID: \_\_\_\_\_

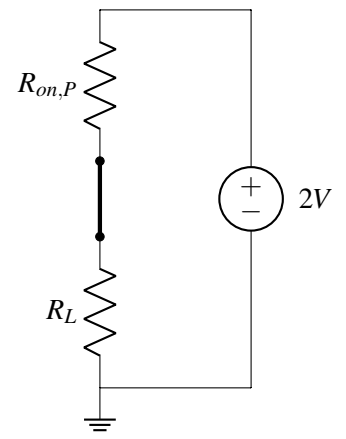
(b) (4 pts) Which is the equivalent circuit for the right-hand side of the circuit? **Fill in the correct bubble.**



Circuit A



Circuit B

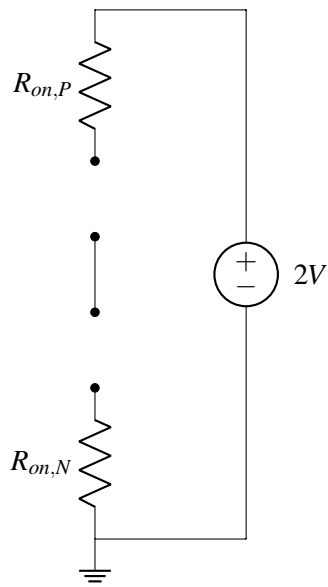
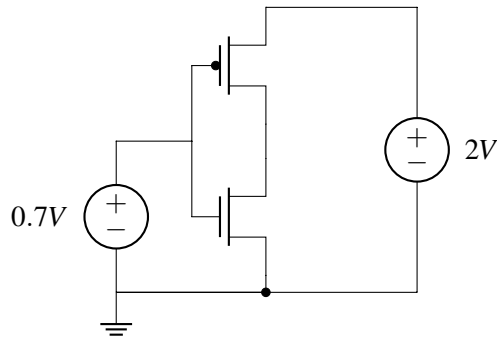


Circuit C

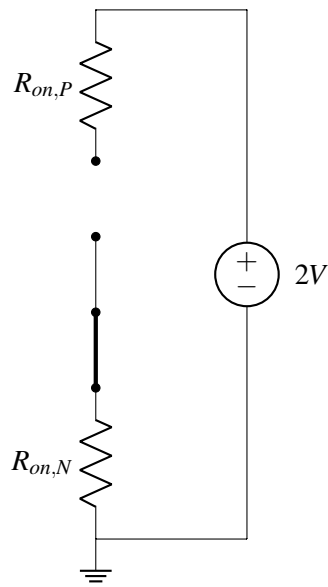
|                           | A                     | B                     | C                     |
|---------------------------|-----------------------|-----------------------|-----------------------|
| <b>Equivalent Circuit</b> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

PRINT your name and student ID: \_\_\_\_\_

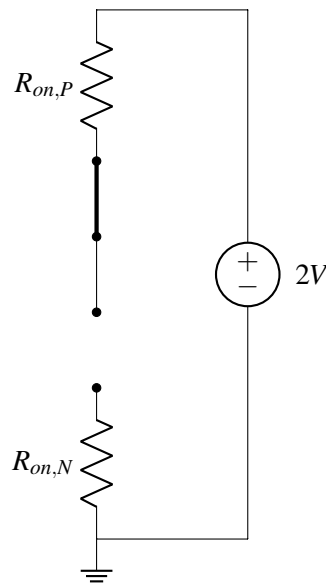
(c) (4 pts) Which is the equivalent circuit for the right-hand side of the circuit? **Fill in the correct bubble.**



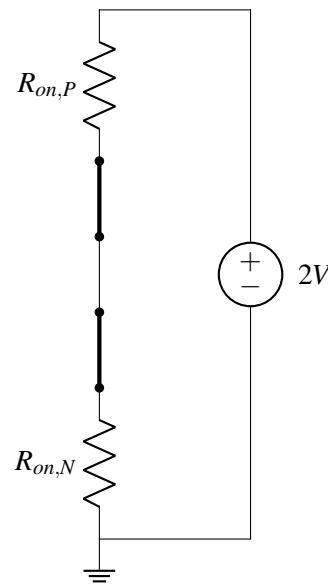
Circuit A



Circuit B



Circuit C



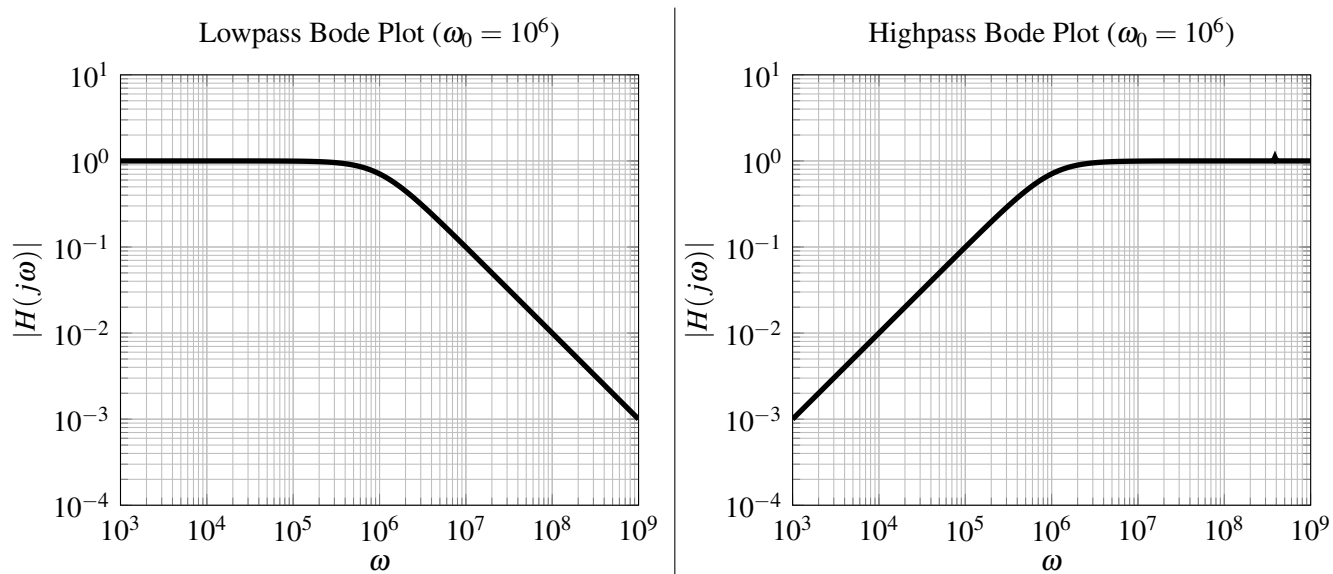
Circuit D

|                    | A                     | B                     | C                     | D                     |
|--------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Equivalent Circuit | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

PRINT your name and student ID: \_\_\_\_\_

**4. Filter Design and Bode Plots (28 pts)**

On the Bode plots below, we have plotted the magnitude responses of first-order low pass filters and high pass filters using the example of cutoff frequency  $\omega_0 = 10^6$ .



Recall that the transfer functions for such simple low pass filters and high pass filters are:

$$H_{lowpass}(j\omega) = \frac{1}{1 + \frac{j\omega}{\omega_0}}; \quad H_{highpass}(j\omega) = \frac{\frac{j\omega}{\omega_0}}{1 + \frac{j\omega}{\omega_0}}.$$

- (a) (6 pts) We want to design a bandpass filter that can pass through a 2.4GHz WiFi signal while blocking other interfering signals — FM radio at 100MHz and WiGig at 60GHz. (Recall: Mega =  $10^6$  and Giga =  $10^9$ .) We will achieve this by cascading lowpass and highpass filters, using ideal op-amp buffers in between to prevent any loading effects.

Unfortunately, when we look in the lab, we only see inductors,  $1\text{k}\Omega$  resistors, and op-amps.

We will start by cascading a single highpass filter followed by a single lowpass filter, with an op-amp buffer in between. **Using only op-amps, two inductors, and resistors (as many as needed), draw the full band-pass filter.** Label  $V_{in}$  and  $V_{out}$  and label the two inductors with  $L_1$  and  $L_2$ . Do not worry about picking the values for  $L_1$  and  $L_2$  in this part.

PRINT your name and student ID: \_\_\_\_\_

- (b) (8 pts) One interfering signal that we want to block is the WiGig signal at 60GHz. If we want to attenuate/reduce the magnitude of the WiGig signal by a factor of about  $\sqrt{101} \approx 10$ , **What is a candidate ‘cutoff frequency’ (in Hz) desired for this lowpass filter?**

**What inductance value should we use for the lowpass filter?** Recall that we only have resistors with  $1\text{k}\Omega$  resistance. It is fine to give your inductance as a formula — you don’t have to simplify it.

For your convenience, here are some calculations that may or may not be relevant:

|   |   |   |
|---|---|---|
| $\frac{60 \times 10^9}{2\pi} = 9.549 \times 10^9$ | $\frac{2.4 \times 10^9}{2\pi} = 382 \times 10^6$  | $\frac{100 \times 10^6}{2\pi} = 15.9 \times 10^6$ |
| $60 \times 10^9 \times 2\pi = 377 \times 10^9$    | $2.4 \times 10^9 \times 2\pi = 15.08 \times 10^9$ | $100 \times 10^6 \times 2\pi = 628 \times 10^6$   |

*(HINT: Look at the relevant Bode plot and read off how far away in frequency from  $\omega_0$  you need to be to reduce the magnitude by the desired factor of around 10.)*

PRINT your name and student ID: \_\_\_\_\_

- (c) (14 pts) Another interfering signal that we want to block is FM radio at 100MHz and we want to reduce its magnitude by a factor of around 100. We decide to use multiple highpass filters in a row (separated by ideal op-amp buffers) to attenuate the FM radio signal more strongly. We design the system with the highpass filter cutoff frequencies all at 1GHz. In this case, **what inductor value should each of the highpass filters use?** Recall that we only have resistors with  $1k\Omega$  resistance. It is fine to give your inductance as a formula — you don't have to simplify it.

For your convenience, here are some calculations that may or may not be relevant:

|   |   |   |
|---|---|---|
| $\frac{60 \times 10^9}{2\pi} = 9.549 \times 10^9$ | $\frac{2.4 \times 10^9}{2\pi} = 382 \times 10^6$  | $\frac{100 \times 10^6}{2\pi} = 15.9 \times 10^6$ |
| $60 \times 10^9 \times 2\pi = 377 \times 10^9$    | $2.4 \times 10^9 \times 2\pi = 15.08 \times 10^9$ | $100 \times 10^6 \times 2\pi = 628 \times 10^6$   |

**How many highpass filters must we cascade in order to attenuate the FM signal at 100MHz by a factor of around 100?**

**Draw the full circuit for your complete filter including op-amp buffers, the lowpass filter, and the highpass filters.**



PRINT your name and student ID: \_\_\_\_\_

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

PRINT your name and student ID: \_\_\_\_\_

**5. Controllable Canonical Form (26 pts)**

Suppose that we have a two-dimensional continuous time system governed by:

$$\frac{d}{dt}\vec{x}(t) = \begin{bmatrix} 1 & -1 \\ 0 & -4 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

We would like to put this system into Controllable Canonical Form (CCF) to use state feedback to place the eigenvalues at desired locations. For your convenience, the characteristic polynomial  $\det\left(\lambda I - \begin{bmatrix} 1 & -1 \\ 0 & -4 \end{bmatrix}\right) =$

$$(\lambda - 1)(\lambda + 4) = \lambda^2 + 3\lambda - 4.$$

(a) (4 pts) **Is the system stable? Why or why not?**

PRINT your name and student ID: \_\_\_\_\_

(b) (16 pts) Recall that our original system is:

$$\frac{d}{dt}\vec{x}(t) = \begin{bmatrix} 1 & -1 \\ 0 & -4 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) \quad \text{with } \det\left(\lambda I - \begin{bmatrix} 1 & -1 \\ 0 & -4 \end{bmatrix}\right) = \lambda^2 + 3\lambda - 4.$$

We would like to change coordinates to bring the system into CCF:

$$\frac{d}{dt}\vec{z}(t) = \begin{bmatrix} 0 & 1 \\ a_0 & a_1 \end{bmatrix} \vec{z}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

**Compute the  $T$  basis such that  $\vec{z}(t) = T^{-1}\vec{x}(t)$  or equivalently (if you want), give us  $T^{-1}$  directly.**

To help you along, here are some calculations already done for you:

$$\begin{bmatrix} 1 & 0 \\ 1 & -4 \end{bmatrix}^{-1} = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & a \end{bmatrix}^{-1} = \begin{bmatrix} -a & 1 \\ 1 & 0 \end{bmatrix}$$

**What is  $a_0$ ? What is  $a_1$ ?**

PRINT your name and student ID: \_\_\_\_\_

(c) (6 pts) Using state feedback

$$u(t) = \begin{bmatrix} \tilde{k}_0 & \tilde{k}_1 \end{bmatrix} \tilde{z}(t)$$

place the closed-loop eigenvalues at  $\lambda_1 = -1$ ,  $\lambda_2 = -2$ . **What is  $\tilde{k}_0$ ? What is  $\tilde{k}_1$ ?**(Notice that we are asking for the feedback gains in terms of  $\tilde{z}(t)$  not the original  $\tilde{x}(t)$ . If you have time, feel free to check your work by the original  $\tilde{x}(t)$ .)

PRINT your name and student ID: \_\_\_\_\_

**6. You knew this was coming (16 pts)**

In the last midterm, you began solving a minimum norm problem in which we had a wide matrix  $A$  and wanted to solve  $A\vec{x} = \vec{y}$  while minimizing the norm  $\|C\vec{x}\|$  where  $C$  was also a wide matrix. You then focused on some special cases where certain matrices were assumed to have orthonormal columns. The nonorthonormal case wasn't resolved. This question is about the heart of what remains to be resolved.

Suppose that we have a matrix  $A_f$ , but we don't know whether or not the columns of  $A_f$  are linearly independent or orthonormal. How can you use the results from the SVD  $A_f = U_f \Sigma_f V_f^T = \sum_{i=1}^r \sigma_{f,i} \vec{u}_{f,i} \vec{v}_{f,i}^T$  to help project a vector  $\vec{s}$  onto the subspace spanned by the columns of  $A_f$ ? Here, the  $\sigma_{f,1}, \dots, \sigma_{f,r}$  are all strictly positive. **Give an expression for a matrix  $P_f$  such that  $P_f \vec{s}$  is the projection of  $\vec{s}$  in the subspace spanned by the columns of  $A_f$ .**

**Also give an expression for an  $\vec{x}_f$  so that  $A_f \vec{x}_f = P_f \vec{s}$ .**

PRINT your name and student ID: \_\_\_\_\_

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

PRINT your name and student ID: \_\_\_\_\_

### 7. Movie Ratings and PCA (36 pts)

Recall from the lecture on PCA that we can think of movie ratings as a structured set of data. For every person  $i$  and movie  $j$ , we have that person's rating  $R_{i,j}$  (thought of as a real number).

Suppose that there are  $m$  movies and  $n$  people. Let's think about arranging this data into a big  $n \times m$  matrix  $R$  with people corresponding to rows and movies corresponding to columns. So the entry in the  $i$ -th row and  $j$ -th column should be  $R_{i,j}$  above.

- (a) (6 pts) Suppose we believe that there is actually an underlying pattern to this rating data and that a separate study has revealed that every movie is characterized by a set of characteristics: say action and comedy. This means that every movie  $j$  has a pair of numbers  $a[j], c[j]$  that define it. At the same time, every person  $i$  has a pair of sensitivities  $s_a[i], s_c[i]$  that essentially define that person's preferences. A person  $i$  will rate the movie  $j$  as  $R_{i,j} = s_a[i]a[j] + s_c[i]c[j]$ .

If we arrange the sensitivities into a pair of  $n$ -dimensional vectors  $\vec{s}_a, \vec{s}_c$ , and the movie characteristics into a pair of  $m$ -dimensional vectors  $\vec{a}, \vec{c}$ , **use outer products to express the rating matrix  $R$  in terms of these vectors  $\vec{s}_a, \vec{s}_c, \vec{a}, \vec{c}$ .**

Recall that the outer product of a real vector  $\vec{u}$  with a real vector  $\vec{v}$  is  $\vec{u}\vec{v}^T$  — the reverse order as compared to the usual inner product.

PRINT your name and student ID: \_\_\_\_\_

- (b) (6 pts) Now suppose that we want to discover the underlying nature of movies from the data  $R$  itself. Suppose for this part, that you have four observed rating data vectors (corresponding to four different movies being rated by six individuals).

All of the movie data vectors just happened to be multiples of the following 6-dimensional vector

$$\vec{v} = \begin{bmatrix} 2 \\ -2 \\ 3 \\ -4 \\ 4 \\ 0 \end{bmatrix}. \text{ (For your convenience, note that } \|\vec{v}\| = 7.\text{)}$$

You arrange the movie data vectors as the columns of a matrix  $R$  given by:

$$R = \begin{bmatrix} | & | & | & | \\ -\vec{v} & -2\vec{v} & 2\vec{v} & 4\vec{v} \\ | & | & | & | \end{bmatrix} \quad (1)$$

You want to perform PCA (for movies) using the SVD of the matrix  $R$  to better understand the pattern in your data.

The first “principal component vector” is a *unit vector* that tells which direction we would want to project the columns of  $R$  onto to get the best rank-1 approximation for  $R$ .

**Find this first principal component vector of  $R$  to explain the nature of your movie data points.**

(*HINT: You don't need to actually compute any SVDs in this simple case. Also, be sure to think about what size vector you want as the answer. Don't forget that you want a unit vector!*)



PRINT your name and student ID: \_\_\_\_\_

- (c) (12 pts) Suppose that now, we have two more data points (corresponding to two more movies being rated by the same set of six people) that are multiples of a different vector  $\vec{p}$  where:

$$\vec{p} = \begin{bmatrix} 6 \\ 3 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \text{ (For your convenience, note that } \|\vec{p}\| = 7 \text{ and that } \vec{p}^\top \vec{v} = 0.\text{)}$$

We augment our ratings data matrix with these two new data points to get:

$$R = \begin{bmatrix} | & | & | & | & | & | \\ -\vec{v} & -2\vec{v} & 2\vec{v} & 4\vec{v} & -3\vec{p} & 3\vec{p} \\ | & | & | & | & | & | \end{bmatrix} \quad (2)$$

Find the first two principal components corresponding to the nonzero singular values of  $R$ . This is what we would use to best project the movie data points onto a two-dimensional subspace.

**What is the first principal component vector? What is the second principal component vector? Justify your answer.**

*(Hint: Think about the inner product of  $\vec{v}$  and  $\vec{p}$  and what that implies for being able to appropriately decompose  $R$ . Again, very little computation is required here.)*

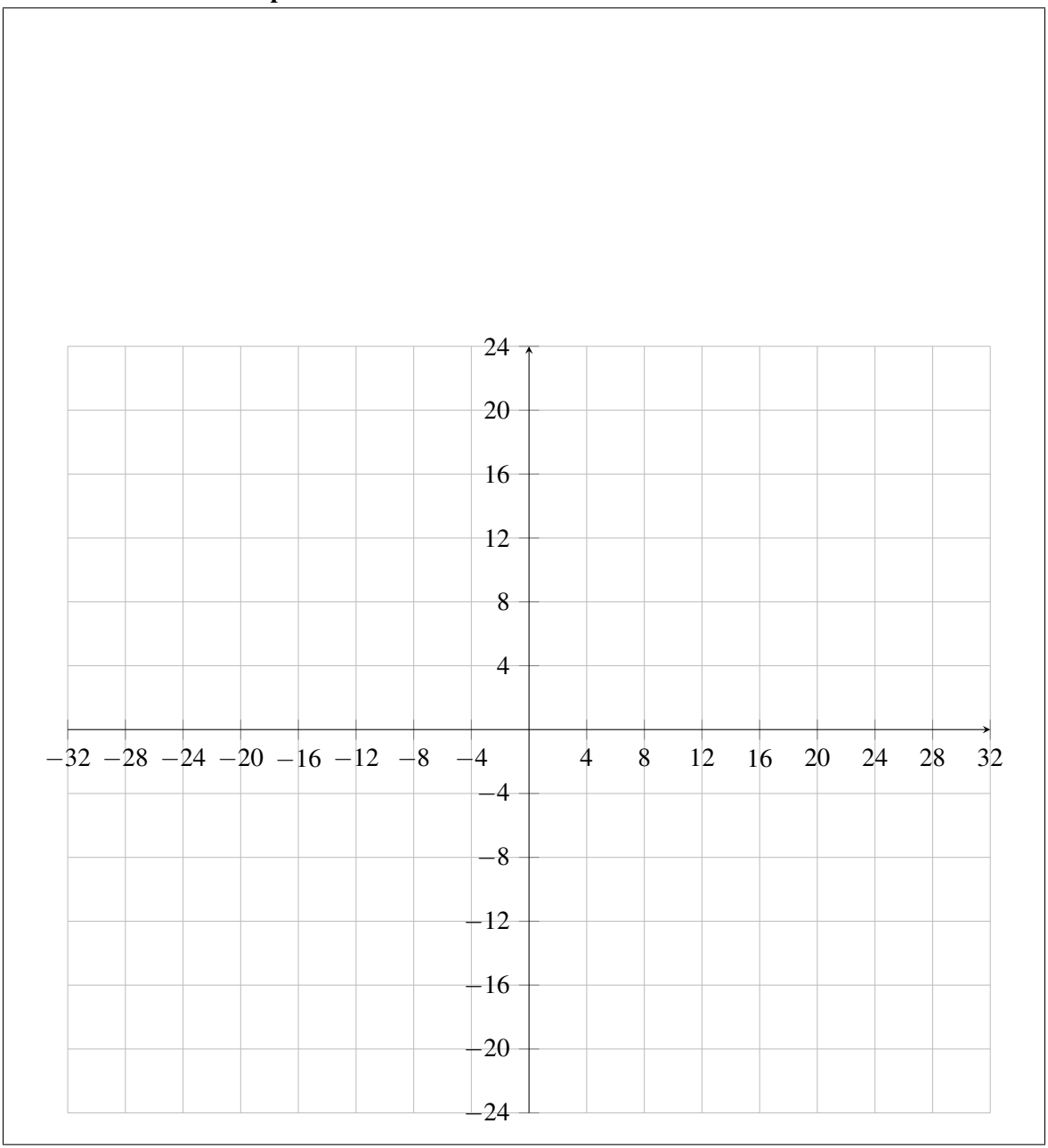
PRINT your name and student ID: \_\_\_\_\_

(d) (12 pts) In the previous part, you had

$$R = \begin{bmatrix} | & | & | & | & | & | \\ -\vec{v} & -2\vec{v} & 2\vec{v} & 4\vec{v} & -3\vec{p} & 3\vec{p} \\ | & | & | & | & | & | \end{bmatrix}$$

with  $\|\vec{v}\| = 7$  and  $\|\vec{p}\| = 7$ , satisfying  $\vec{p}^\top \vec{v} = 0$ .

If we use  $\vec{r}_i$  to denote the  $i$ -th column of  $R$ , **plot the movie data points  $\vec{r}_i$  projected onto the first and second principal component vectors.** The coordinate along the first principal component should be represented by horizontal axis and the coordinate along the second principal component should be the vertical axis. **Label each point.**



PRINT your name and student ID: \_\_\_\_\_

**8. Latch (46 pts)**

The circuit below is a type of latch, which is one of the fundamental components of memory in many digital systems. The latch is a bistable circuit, which means that there are two possible stable states: one representing a stored '1' bit and the other a stored '0' bit.

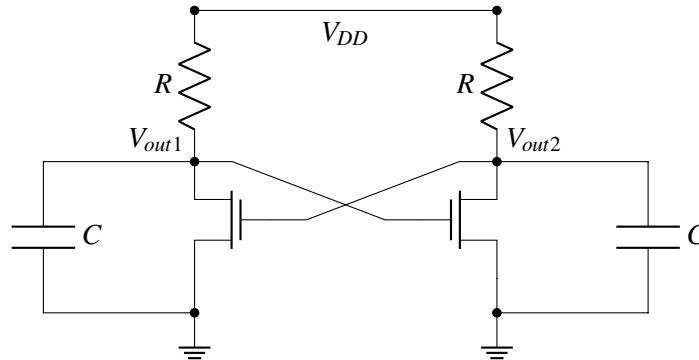


Figure 7: Simplified latch: the gate capacitances have been pulled out explicitly.

- (a) (6 pts) To get a basic understanding of the stable operating points for the latch, consider the following simplified circuit using the pure switch model for MOSFETs (and a threshold voltage of  $\frac{V_{DD}}{2}$ ).

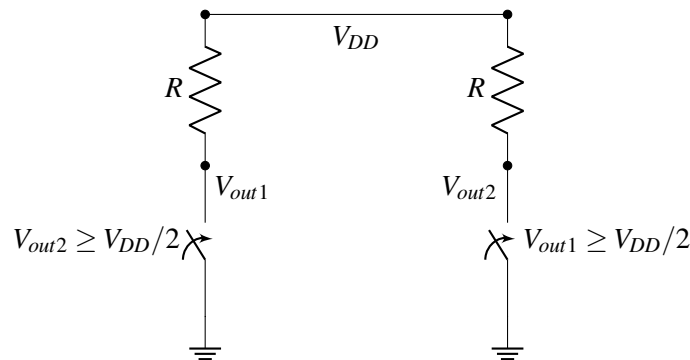


Figure 8: Pure switch model for a latch

**First assume that  $V_{out1} = 0$ . What is  $V_{out2}$ ? Are the left and right switches open or closed?**

|                     | Open or $V_{DD}$      | Closed or 0           |
|---------------------|-----------------------|-----------------------|
| <b>Left Switch</b>  | <input type="radio"/> | <input type="radio"/> |
| <b>Right Switch</b> | <input type="radio"/> | <input type="radio"/> |
| $V_{out2}$          | <input type="radio"/> | <input type="radio"/> |

**Suppose that  $V_{out1} = V_{DD}$ . What is  $V_{out2}$ ? Are the left and right switches open or closed?**

|                     | Open or $V_{DD}$      | Closed or 0           |
|---------------------|-----------------------|-----------------------|
| <b>Left Switch</b>  | <input type="radio"/> | <input type="radio"/> |
| <b>Right Switch</b> | <input type="radio"/> | <input type="radio"/> |
| $V_{out2}$          | <input type="radio"/> | <input type="radio"/> |

PRINT your name and student ID: \_\_\_\_\_

- (b) (10 pts) To get an understanding of latch dynamics, we will now break it down into smaller pieces. Below is one half of the latch circuit.

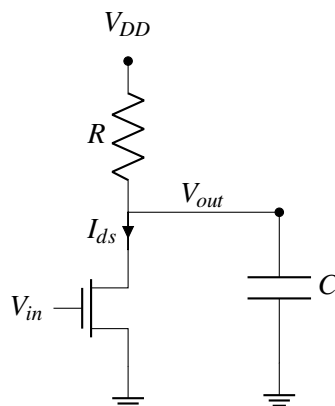
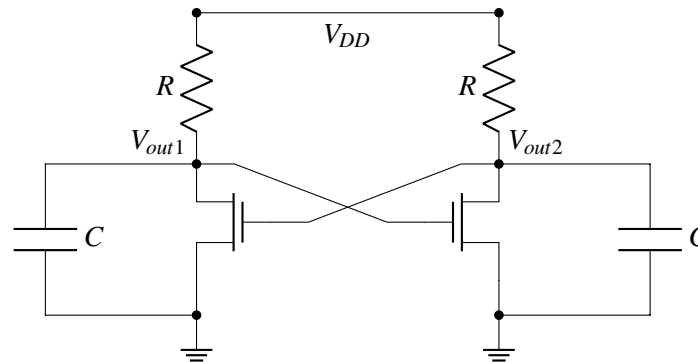


Figure 9: Latch half-circuit

**Write a differential equation for the voltage  $V_{out}$  in terms of the drain to source current  $I_{ds}$ .** Treat  $I_{ds}$  as some specified input signal and treat the transistor as a current source connected to ground. (i.e. There is no dependence on  $V_{in}$  in this part. In this part, treat the  $I_{ds}$  as a constant that you are given.)

PRINT your name and student ID: \_\_\_\_\_

(c) (10 pts)



For this circuit, we care about more detailed analog characteristics of the MOSFETs, so we will model their behavior more accurately as current sources that are controlled by their gate voltages  $V_{in}$  with the following equation:

$$I_{ds} = g(V_{in})$$

Where  $g(V_{in})$  is a some nonlinear function. **Using this  $I_{ds}$  expression together with the result from the previous part, write down a system of differential equations for  $V_{out1}$  and  $V_{out2}$  in vector form:**

$$\frac{d}{dt} \begin{bmatrix} V_{out1}(t) \\ V_{out2}(t) \end{bmatrix} = \vec{f} \left( \begin{bmatrix} V_{out1}(t) \\ V_{out2}(t) \end{bmatrix} \right).$$

(Hint: Notice that the latch above can be constructed by taking two of the circuit in Figure ??, and connecting the  $V_{out}$  of one to the gate  $V_{in}$  of the other and vice-versa.)

PRINT your name and student ID: \_\_\_\_\_

- (d) (20 pts) For the rest of this problem, assume that your analysis yields the following system of nonlinear differential equations:

$$\begin{bmatrix} \frac{dV_{out1}}{dt} \\ \frac{dV_{out2}}{dt} \end{bmatrix} = \begin{bmatrix} 1 - V_{out1} - g(V_{out2}) \\ 1 - V_{out2} - g(V_{out1}) \end{bmatrix}$$

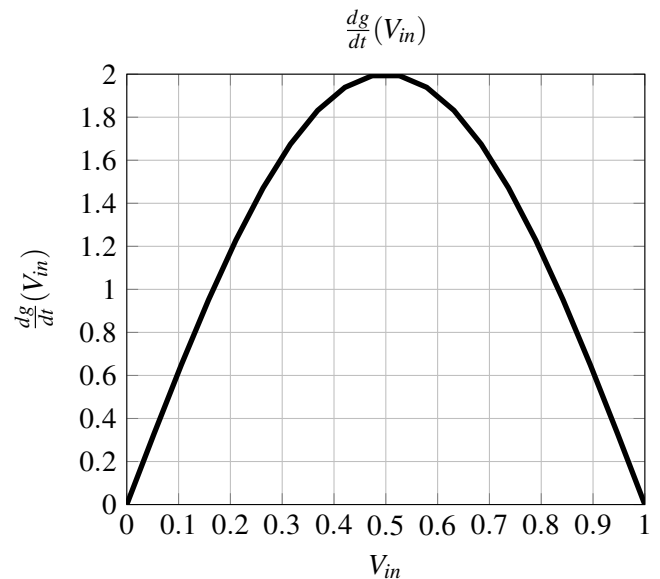
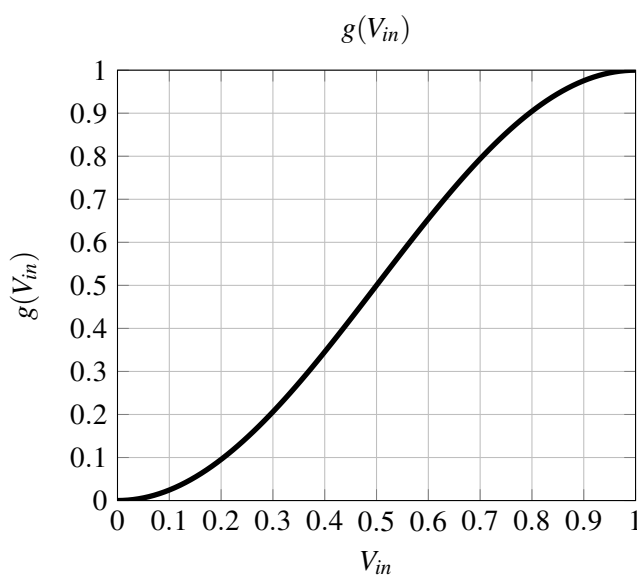
Suppose that you put this latch into an ideal circuit simulator, and measure  $g(V_{in})$  and  $\frac{dg}{dt}(V_{in})$ . The results from these measurements are shown in the graphs below. From your simulations, you also can see that for the following initial conditions, the latch voltages do not change over time:

$$\begin{bmatrix} V_{out1}^* \\ V_{out2}^* \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

Use the graphs below to linearize the differential equations around the three operating points.

Write a linearized system of differential equations around each of those operating points  $\begin{bmatrix} V_{out1}^* \\ V_{out2}^* \end{bmatrix}$ .

For which of the provided  $\begin{bmatrix} V_{out1}^* \\ V_{out2}^* \end{bmatrix}$  points is the latch locally stable? For which of the provided points is the latch locally unstable? Why?



PRINT your name and student ID: \_\_\_\_\_

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

PRINT your name and student ID: \_\_\_\_\_

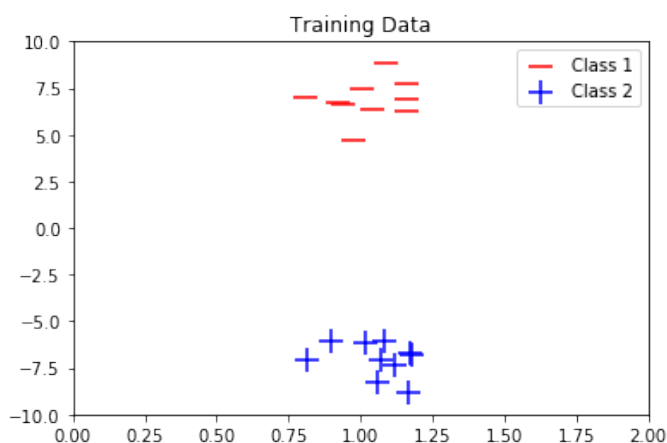
**9. Choosing cost functions for learning classification (12 pts)**

In this problem, we want to design classifiers for data that comes from two classes “+” and “-”. We have labeled data  $\{(\vec{x}_i, \ell_i)\}$  from which we want to learn a vector  $\vec{w}$  so that we can use the sign of  $\vec{w}^\top \vec{x}$  to classify  $\vec{x}$ . To do this, we are going to be trying to minimize a sum  $c_{total}(\vec{w}) = \sum_i c^{\ell_i}(\vec{x}_i^\top \vec{w})$ .

The cost functions we are considering include:

- Squared loss:  $c^+(p) = (p - 1)^2$  and  $c^-(p) = (p - (-1))^2$
- Exponential loss:  $c^+(p) = e^{-p}$  and  $c^-(p) = e^{+p}$
- Logistic loss:  $c^+(p) = \ln(1 + e^{-p})$  and  $c^-(p) = \ln(1 + e^{+p})$

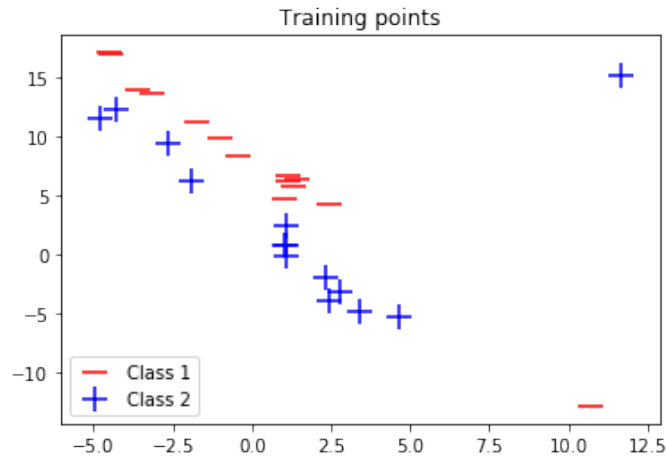
**For the plotted data, which cost functions will give reasonable answers? Select all that apply.**



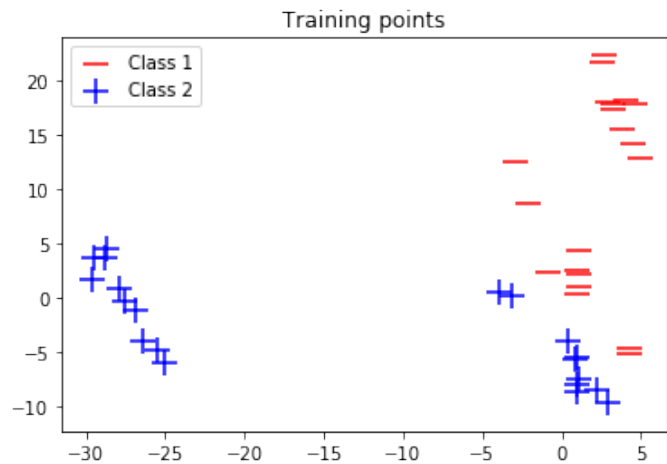
|                          | <b>Squared</b>        | <b>Exponential</b>    | <b>Logistic</b>       |
|--------------------------|-----------------------|-----------------------|-----------------------|
| <b>Reasonable Choice</b> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |



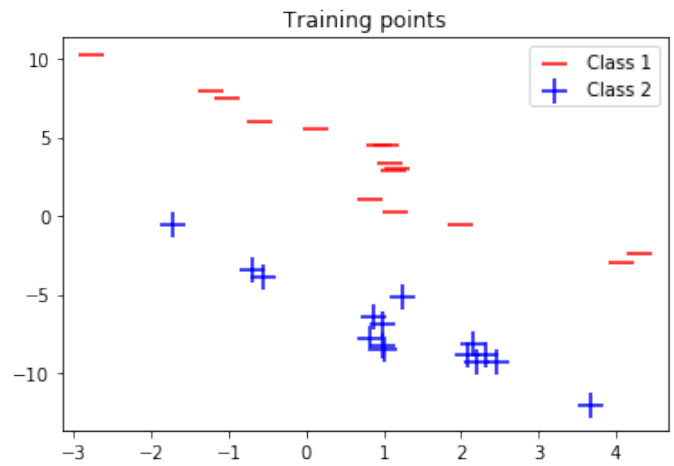
PRINT your name and student ID: \_\_\_\_\_



|                   | Squared               | Exponential           | Logistic              |
|-------------------|-----------------------|-----------------------|-----------------------|
| Reasonable Choice | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |



|                   | Squared               | Exponential           | Logistic              |
|-------------------|-----------------------|-----------------------|-----------------------|
| Reasonable Choice | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |



|                          | Squared               | Exponential           | Logistic              |
|--------------------------|-----------------------|-----------------------|-----------------------|
| <b>Reasonable Choice</b> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

PRINT your name and student ID: \_\_\_\_\_

**10. DFT and Polynomials (34 pts)**

Consider the polynomial-style DFT basis that comes from evaluating the monomials  $x^0, x^1, x^2, x^3$  on the 4-th roots of unity  $x_0 = e^{j\frac{2\pi}{4}0} = 1$ ,  $x_1 = e^{j\frac{2\pi}{4}1} = j$ ,  $x_2 = e^{j\frac{2\pi}{4}2} = -1$ ,  $x_3 = e^{j\frac{2\pi}{4}3} = -j$ .

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{j\frac{2\pi}{4}} & e^{j2\frac{2\pi}{4}} & e^{j3\frac{2\pi}{4}} \\ 1 & e^{j2\frac{2\pi}{4}} & e^{j4\frac{2\pi}{4}} & e^{j6\frac{2\pi}{4}} \\ 1 & e^{j3\frac{2\pi}{4}} & e^{j6\frac{2\pi}{4}} & e^{j9\frac{2\pi}{4}} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}.$$

- (a) (4 pts) The DFT coefficients  $\vec{F}$  are related to a vector of samples  $\vec{f}$  by the relationship  $\vec{f} = B\vec{F}$ . In other words,  $\vec{F}$  represents  $\vec{f}$  in the basis given by the columns of  $B$ . Similarly for the DFT coefficients  $\vec{G}$  and a vector of samples  $\vec{g}$  — they too satisfy the relationship  $\vec{g} = B\vec{G}$ .

**What are the DFT coefficients  $\vec{H}$  for  $\vec{h} = \alpha\vec{f} + \beta\vec{g}$  in terms of  $\vec{F}$  and  $\vec{G}$ . Here,  $\alpha$  and  $\beta$  are constant real numbers.**

- (b) (8 pts) **Explicitly find the DFT coefficients  $\vec{F}$  of the vector  $\vec{f} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$  i.e. we want  $\vec{F}$  so that  $\vec{f} = B\vec{F}$ .**

PRINT your name and student ID: \_\_\_\_\_

(c) (10 pts) **Show that if  $\vec{f}$  is a real vector, then:**

- $F[0]$  is always real and so is  $F[2]$ .
- $F[1] = \overline{F[3]}$ .

(HINT: What do you know about  $B^{-1}$ ?)

(d) (12 pts) We notice that when we use the standard polynomial interpolation approach  $\hat{f}(e^{j\theta}) = F[0] + F[1]e^{j\theta} + F[2]e^{j2\theta} + F[3]e^{j3\theta}$ , we often get nonzero imaginary parts even though our samples  $\vec{f}$  were purely real. Here  $\vec{F}$  are the DFT coefficients corresponding to  $\vec{f}$  so that  $\vec{f} = B\vec{F}$ .

**Give new constants  $k_1, k_3$  so that**

$$\hat{f}(e^{j\theta}) = F[0] + F[1]e^{jk_1\theta} + F[2]\frac{e^{j2\theta} + e^{-j2\theta}}{2} + F[3]e^{jk_3\theta}$$

**will always return a real number for each  $\theta \in [0, 2\pi]$ , and actually interpolates the data points — i.e.  $f[i] = \hat{f}(e^{j\frac{2\pi i}{4}})$  for  $i = 0, 1, 2, 3$ .**

**Prove that this  $\hat{f}(e^{j\theta})$  is always real for  $\theta \in [0, 2\pi]$  no matter what real values the vector  $\vec{f}$  has.**

(HINT: For proving realness, use what is already established by the previous part (even if you didn't get the previous part!) and write out  $\hat{f}(e^{j\theta})$  in terms of constants, sines, and/or cosines. Using a polar form for  $F[1]$  might be helpful.)

PRINT your name and student ID: \_\_\_\_\_

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

PRINT your name and student ID: \_\_\_\_\_

[Doodle page! Draw us something if you want or give us suggestions or complaints. You can also use this page to report anything suspicious that you might have noticed.]