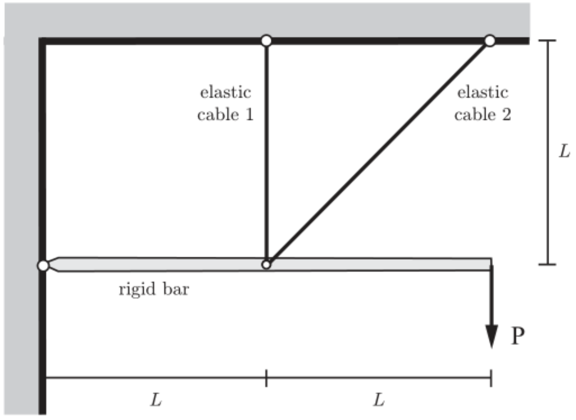
Problem #1 (40%)

A rigid bar is held horizontally by two elastic cables as shown in the figure. The cables have the same Young modulus E and cross section area A. All connections are pinned, and all members can be considered weightless.

A vertical load P is applied at the right tip of the bar, as shown. Determine:

1. The forces in the cables.
2. The displacement of the right tip of the bar.

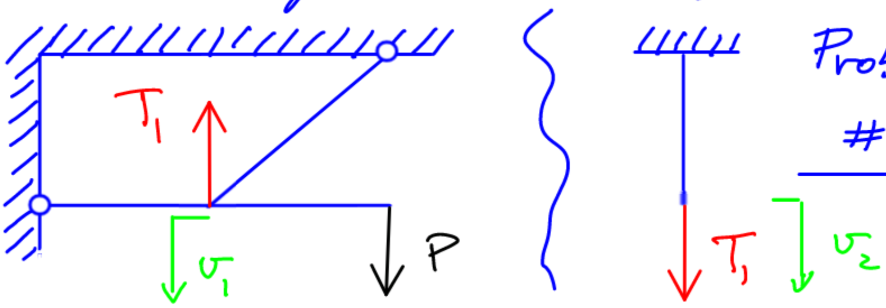


Statically indeterminate ⇒ Force method (degree of indeterminacy = 1)

$\boxed{\text{STEP 1}}$ Release the system by e.g. disconnecting cable #1 leaving the force

Problem #1

Problem #2

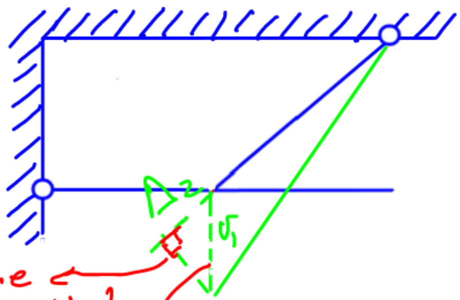


$\boxed{\text{STEP 2}}$ Solve for the deflections $v_1 \downarrow$ and $v_2 \downarrow$

Problem #2: $\downarrow v_2 = T_1 f_1 = T_1 \dfrac{L}{EA}$

Problem #1

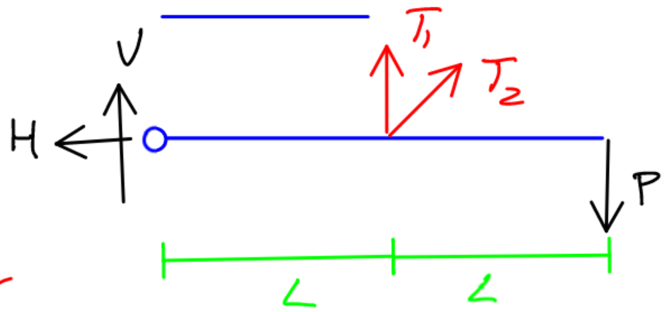
Kinematics



line perpendicular to original direction of cable #2

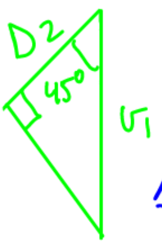
line perpendicular to the rigid bar

Statics



$\Sigma M = 0 \Rightarrow T_2 \sin 45 \cdot L + T_1 L - P \cdot 2L = 0$

$\Rightarrow T_2 = \sqrt{2}\,(2P - T_1)$



$\Rightarrow v_1 = \dfrac{\Delta_2}{\cos 45} = \sqrt{2}\,\Delta_2$

$\Delta_2 = T_2 f_2 = T_2 \dfrac{\overbrace{\sqrt{2} L}^{=\text{length of cable 2}}}{EA}$ (elastic)

$\Rightarrow \downarrow v_1 = 2\sqrt{2}\,\dfrac{L}{EA}(2P - T_1)$

Relax

STEP 3 Impose back the compatibility constraint

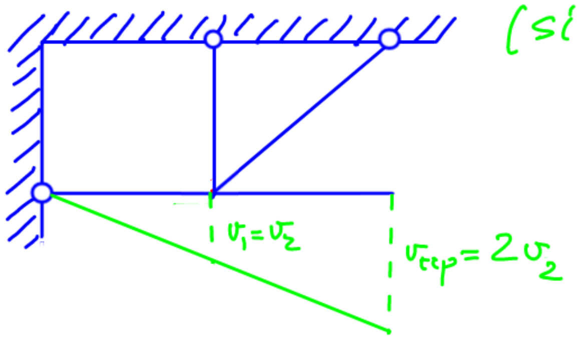
$$\downarrow v_1 = \downarrow v_2 \Rightarrow 2\sqrt{2}\frac{L}{EA}(2P - T_1) = T_1\frac{L}{EA}$$

$$\Rightarrow \boxed{T_1 = \frac{4\sqrt{2}}{1+2\sqrt{2}}P}$$

$$T_2 = \sqrt{2}(2P - T_1) \Rightarrow \boxed{T_2 = \frac{2\sqrt{2}}{1+2\sqrt{2}}P}$$

PART 2

The rigid bar moves vertically at the right tip



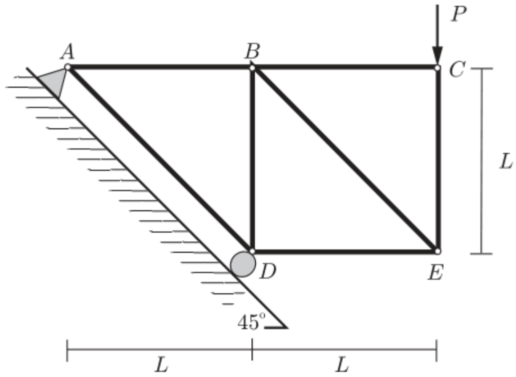
(similar triangles)

$$\Rightarrow v_{tip} = 2T_1\frac{L}{EA}$$

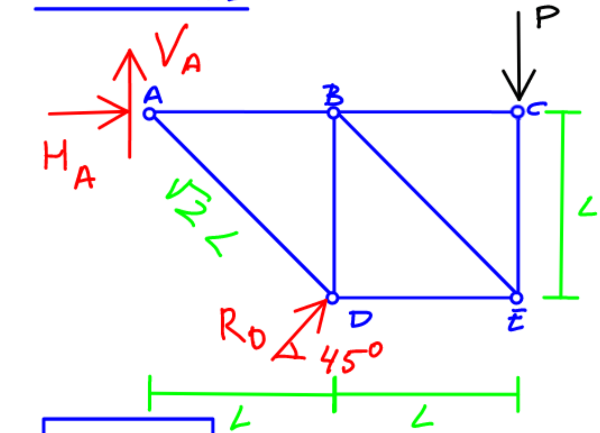
$$\Rightarrow \boxed{v_{tip} = \frac{8\sqrt{2}}{1+2\sqrt{2}}\frac{PL}{EA}}$$

Problem #2 (25%)

1. Determine the forces in all the members in the truss of the figure when the vertical load of value P shown in the figure is applied. Indicate clearly if the member is in tension or compression, and identify all zero-force members, if any.
2. If all the members have the same $0.1 \times 0.1\ m^2$ square cross section, determine the maximum load value P that can be applied with a factor of safety of 1.5 if the material can only take $10 MPa$ in tension or compression.



Reactions



$\Sigma M_A = 0 \Rightarrow R_D \sqrt{2} L = P 2L$

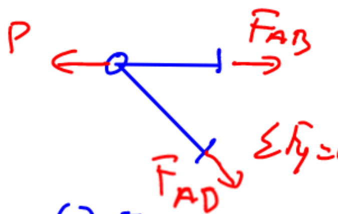
$\Rightarrow \underline{R_D = \sqrt{2} P}$

$\Sigma F_x = 0 \Rightarrow H_A + R_D \cos 45 = 0 \Rightarrow \underline{H_A = -P}$

$\Sigma F_y = 0 \Rightarrow V_A = P - R_D \sin 45 \Rightarrow \underline{V_A = 0}$

$\boxed{\text{Part 1}}$ Zero-force members $\underline{F_{BC} = F_{AD} = 0}$ (see Joint A) ← because $V_A = 0$

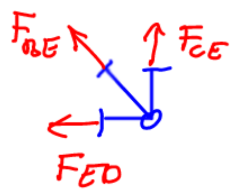
① Joint A



$F_{AB} = P$

$\Sigma F_y = 0 \Rightarrow F_{AD} = 0$

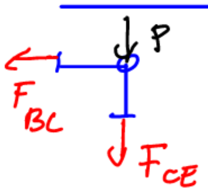
③ Joint E



$F_{BE} \cos 45 + F_{CE} = 0 \Rightarrow F_{BE} = \sqrt{2} P$

$F_{ED} + F_{BE} \sin 45 = 0 \Rightarrow F_{ED} = -P$

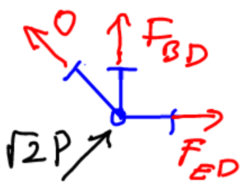
② Joint C



$F_{CE} = -P$

$F_{BC} = 0$

④ Joint D



$F_{BD} + \sqrt{2} P \cos 45 = 0 \Rightarrow F_{BD} = -P$

$F_{ED} + \sqrt{2} P \cos 45 = 0$ ✓

$\boxed{\text{Part 2}}$ Maximum force among all members $= \sqrt{2} P$

$$\Rightarrow \frac{\sqrt{2} P}{\underset{0.1 \times 0.1\, m^2}{A}} \leq \frac{\sigma_{max} = 10 MPa}{FS = 1.5}$$

$$\Rightarrow \boxed{P \leq \frac{\sqrt{2}}{3} 10^2 kN = P_{max}}$$

SUMMARY:

$F_{BC} = F_{AD} = 0$

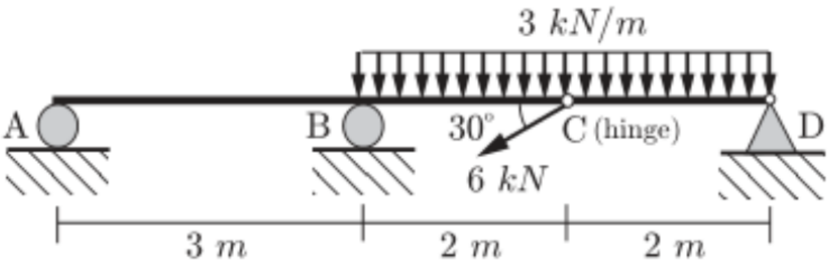
$F_{AB} = P$ (tension)

$F_{BD} = F_{ED} = -P$ (comp.)

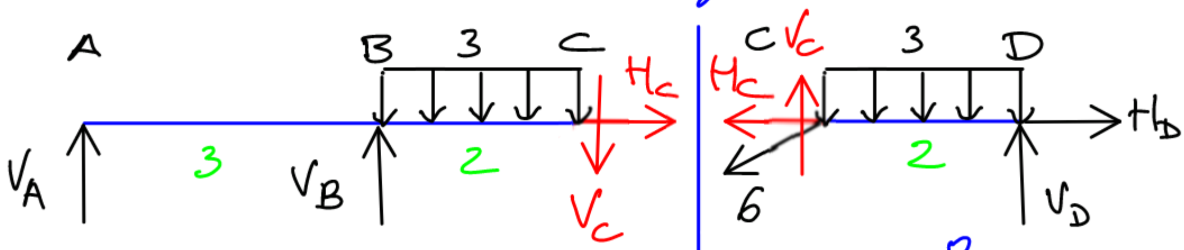
$F_{BE} = \sqrt{2} P$ (tension)

Problem #3 (35%)

Draw the axial force, transversal shear force and bending moment diagrams for the beam shown in the figure. Indicate the characteristic values (min/max values, values at the ends and supports, slopes, linear/parabolic/cubic distributions,...).



Reactions (cut at the hinge C)



$\Rightarrow H_C = 0$

$\sum M_B = 0 \Rightarrow V_A \cdot 3 + \underbrace{V_C}_{=6} \cdot 2 + 3 \cdot 2 \cdot 1 = 0$

$\Rightarrow V_A = -6$

$\Rightarrow V_B = 3 \cdot 2 + V_C - V_A \Rightarrow V_B = 18$

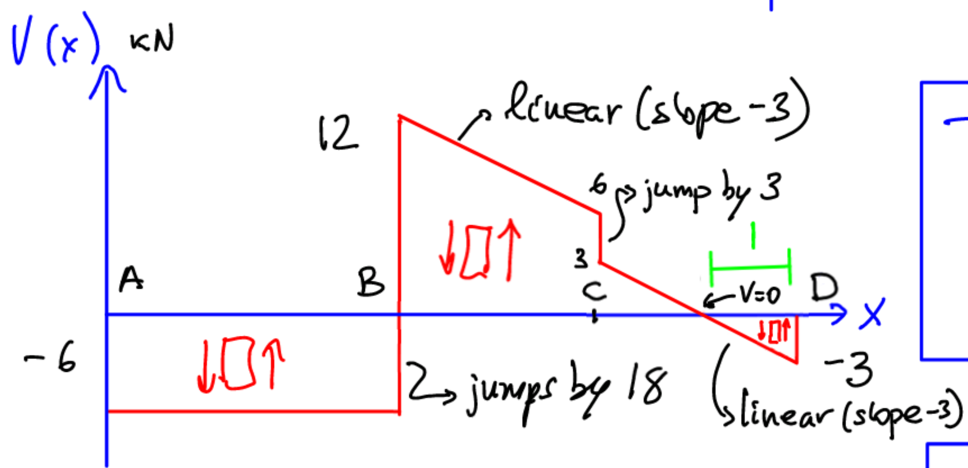
$\Rightarrow H_D = \underbrace{H_C}_{=0} + 6\cos 30 = 3\sqrt{3}$

$V_D \cdot 2 = 3 \cdot 2 \cdot 1 \Rightarrow V_D = 3$

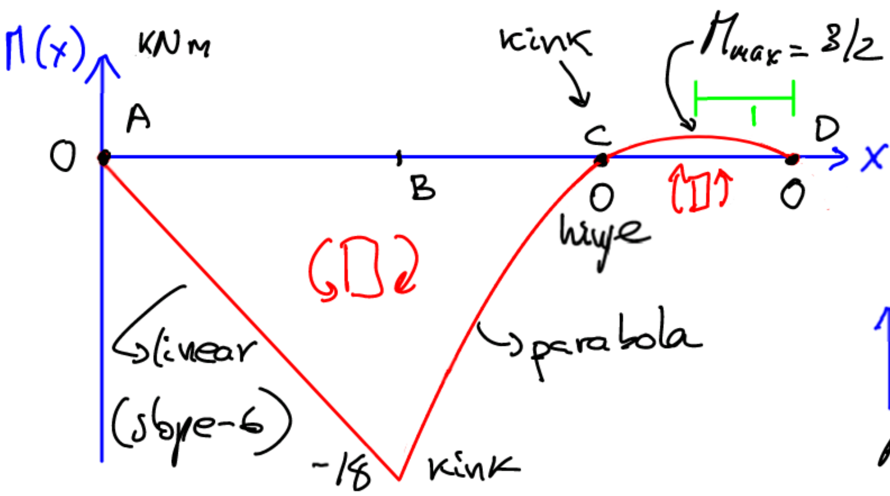
$V_C = 3 \cdot 2 + 6\sin 30 - V_D$

$\Rightarrow V_C = 6$

Transverse shear diagram



Bending moment diagram



Axial