University of California, Berkeley *Department of Mechanical Engineering*
Summer Session 2020 *Instructor: P. Papadopoulos*

Engineering W7 Final Examination

August 14, 2020

NAME : ______________________________________

SID : ______________________________________

Part	Points	Grade	Part	Points	Grade
A	5		H	5	
B	5		I	6	
C	6		J	6	
D	7		K	4	
E	6		L	6	
F	10		M	6	
G	9		N	4	
			O	6	

TOTAL: /91

Notes:
1. Write your name on the top right corner of each page.
2. Record your answers only in the spaces provided.
3. You may not collaborate with others during the exam.
4. There will be no questions answered by the instructors during the exam

With my signature below, I agree to take this examination without the help of any other individual.

____

Part A: (5 points)

In this problem, you must write a function called `vperms`, which performs a random permutation of the elements of a row vector. Below is an example of the use of `vperm`:

```
>> v = [1 3 9 4];
>> vperm(v)
  ans =
        3 1 4 9
>> vperm(v)
  ans =
        4 9 3 1
```

To code `vperms`, you are allowed to use the built-in MATLAB function `randperm`. Recall that `randperm(N)` generates a vector with elements that are random permutations of the elements of the vector `1:N`. For example:

```
>> randperm(4)
  ans =
        2 1 3 4
>> randperm(4)
  ans =
        4 3 1 2
```

Complete the function `vperms` by adding at most three lines of code below:

```
function vp = vperm(v)
```

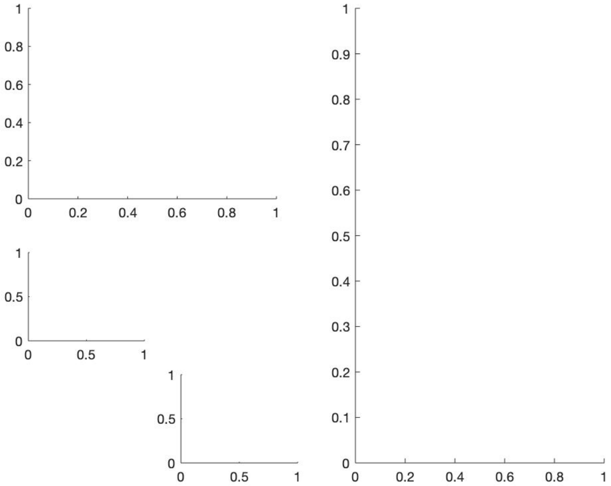
____

____

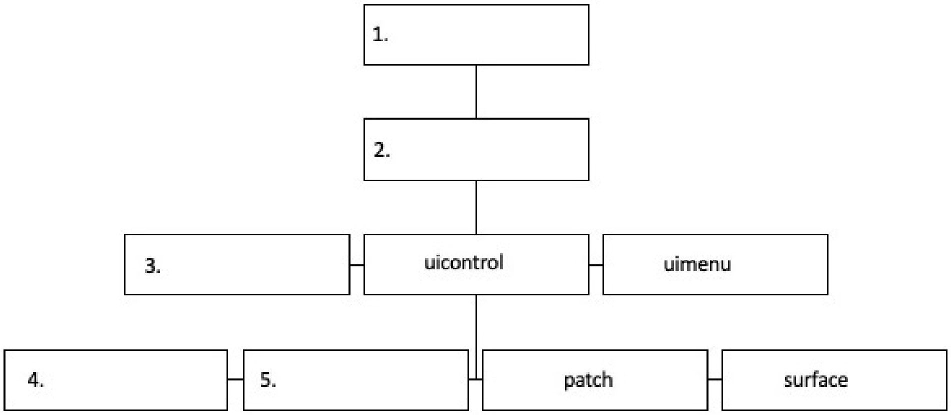
____

Part B: (3+2 points)

B.1 Write the lines of code that would produce the figure shown below. This should take no more than 5 lines of code.



B.2 Recalling Handle Graphics' hierarchy, fill the tree diagram below with object names in the correct hierarchical order: **Axes, text, root, figure, and line.**



Name: ____________________________________

Part C: (2+2+2 points)

C.1 Convert the binary number **1001101** to its decimal representation *(Show all of your work)*.

C.2 Convert the decimal number **25** to its binary representation *(Show all of your work)*.

C.3 Convert the hexadecimal number **a59f** to its binary representation *(Show all of your work)*.

Part D (1+2+1+1+1+1 points)

Recall that an n-th degree polynomial interpolation function can be constructed from the general equation of the form

$$y_i \;=\; a_n x_i^n + a_{n-1} x_i^{n-1} + ... + a_1 x_i + a_0 \ ,$$

where (x_i, y_i) are the coordinates of a data point i.

D.1 If there are 10 known data points, what is the highest degree of the polynomial that interpolates between these points?

Answer: ____________________

D.2 Given a set of data points (x_i, y_i), $i = 1, 2, 3, 4$, write the set of equations to construct a polynomial interpolating function as a system of linear equations in the standard $\mathbf{Ax} = \mathbf{b}$ format.

D.3 What are the dimensions of $\mathbf{A}$, $\mathbf{x}$ and $\mathbf{b}$?

$\mathbf{A}$: ________________

$\mathbf{x}$: ________________

$\mathbf{b}$: ________________

D.4 Write a line of MATLAB code that would most efficiently solve for $\mathbf{x}$, assuming $\mathbf{A}$ and $\mathbf{b}$ have already been defined.

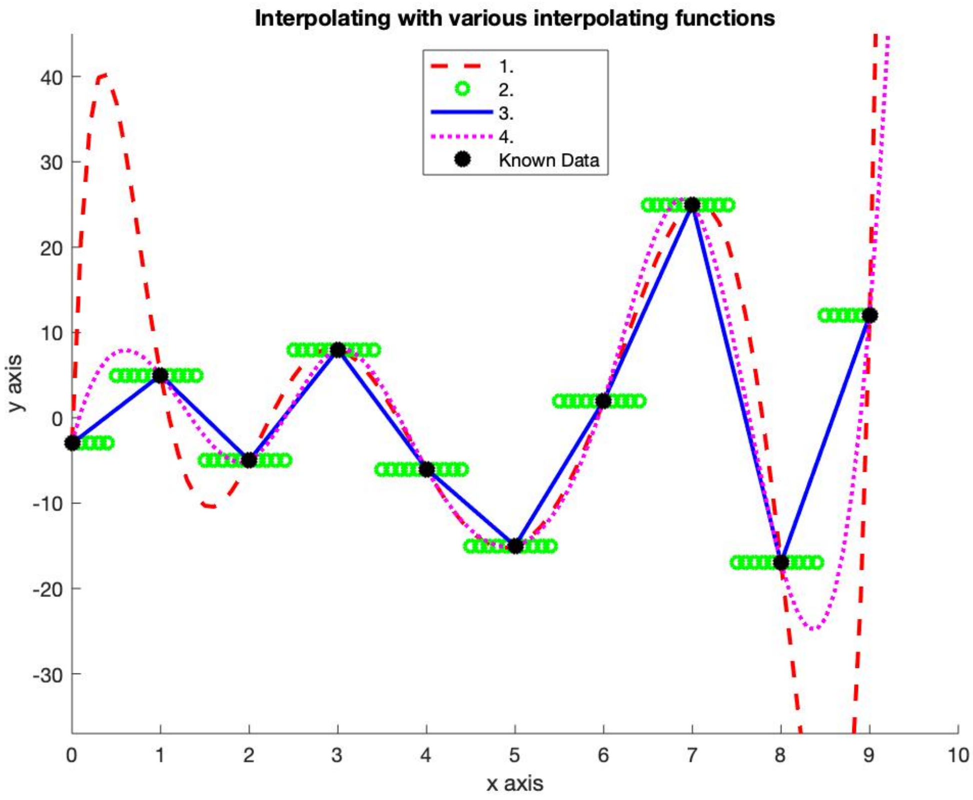
Answer: ____________________

D.5 Write an alternative one-line MATLAB code that would again (albeit, perhaps, less efficiently) solve for $\mathbf{x}$, assuming once more that $\mathbf{A}$ and $\mathbf{b}$ have already been defined.

Answer: ____________________

Name: ______________________

D.6 Given a set of data points, various interpolating functions have been implemented and plotted in the following figure including: **Nearest neighbor, linear, cubic-spline, and polynomial interpolation**.



Based on your knowledge of the implemented methods, write the names of the methods corresponding to the lines in the figure. (*Refer to the legend in the figure.*)

1. ____________________

2. ____________________

3. ____________________

4. ____________________

Part E (1+1+1+1+1+1 points)

In Parts E.1 to E.4, you will find some intermediate steps in performing various ascending-order sorting algorithms on the same initial set of numbers. The steps are not necessarily consecutive (*i.e.*, some steps are missing), but they are in the correct order.

For each part, write on the blank line your best guess for the algorithm it illustrates. Select the algorithm from among the following choices: **Bubble sort, mergesort, quicksort (first element of sequence as pivot), and straight insertion**.

Input List: (first line of each algorithm)

2648, 3569, 4932, 5390, 3291, 5012, 9899, 5901, 6257, 1010, 480, 8426

Final List: (sorted in ascending order)

480, 1010, 2648, 3291, 3569, 4932, 5012, 5390, 5901, 6257, 8426, 9899

E.1 ______________________________

2648, 3569, 4932, 5390, 3291, 5012, 9899, 5901, 6257, 1010, 480, 8426
480, 1010, 2648, 5390, 3291, 5012, 9899, 5901, 6257, 3569, 4932, 8426
480, 1010, 2648, 4932, 3291, 5012, 3569, 5390, 6257, 9899, 5901, 8426
480, 1010, 2648, 4932, 3291, 5012, 3569, 5390, 5901, 6257, 9899, 8426
480, 1010, 2648, 4932, 3291, 5012, 3569, 5390, 5901, 6257, 8426, 9899
480, 1010, 2648, 3569, 3291, 4932, 5012, 5390, 5901, 6257, 8426, 9899

E.2 ______________________________

2648, 3569, 4932, 5390, 3291, 5012, 9899, 5901, 6257, 1010, 480, 8426
2648, 3569, 4932, 3291, 5390, 5012, 9899, 5901, 6257, 1010, 480, 8426
2648, 3569, 4932, 3291, 5012, 5390, 9899, 5901, 6257, 1010, 480, 8426
2648, 3291, 3569, 4932, 5012, 5390, 9899, 5901, 6257, 1010, 480, 8426
2648, 3291, 3569, 4932, 5012, 5390, 5901, 9899, 6257, 1010, 480, 8426
2648, 3291, 3569, 4932, 5012, 5390, 5901, 6257, 9899, 1010, 480, 8426
2648, 3291, 3569, 4932, 5012, 5390, 5901, 6257, 9899, 480, 1010, 8426

E.3 ______________________________

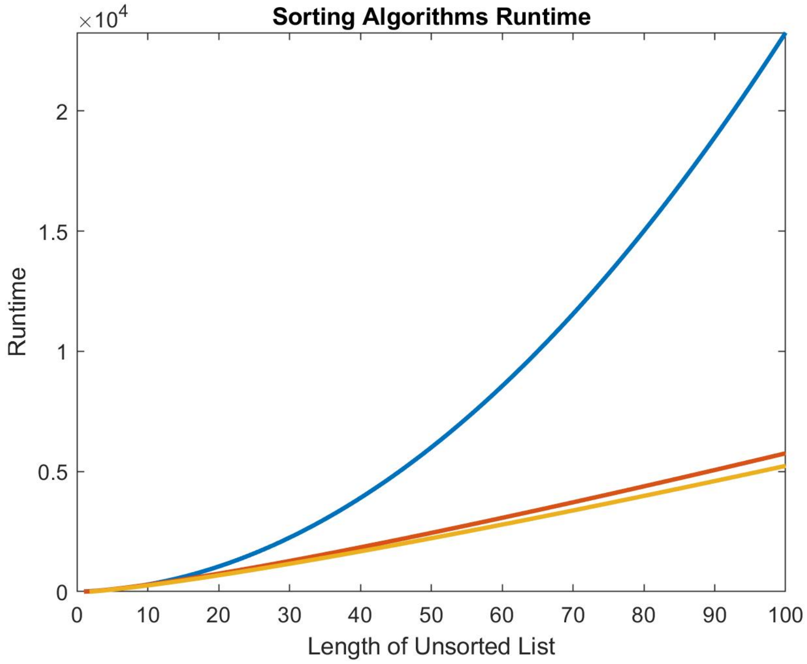
2648, 3569, 4932, 5390, 3291, 5012, 9899, 5901, 6257, 1010, 480, 8426
2648, 3569, 4932, 3291, 5012, 5390, 5901, 6257, 9899, 1010, 480, 8426
2648, 3569, 3291, 4932, 5012, 5390, 5901, 1010, 6257, 480, 8426, 9899
2648, 3291, 3569, 1010, 4932, 480, 5012, 5390, 5901, 6257, 8426, 9899
2648, 3291, 1010, 3569, 480, 4932, 5012, 5390, 5901, 6257, 8426, 9899
2648, 1010, 480, 3291, 3569, 4932, 5012, 5390, 5901, 6257, 8426, 9899
1010, 2648, 480, 3291, 3569, 4932, 5012, 5390, 5901, 6257, 8426, 9899

E.4 ____________________________

2648, 3569, 4932, 5390, 3291, 5012, 9899, 5901, 6257, 1010, 480, 8426
2648, - - - - - - - - - - -
2648, 3569, - - - - - - - - - -
2648, 3569, 4932, - - - - - - - - -
2648, 3291, 3569, 4932, 5012, 5390, - - - - - -
2648, 3291, 3569, 4932, 5012, 5390, 5901, 6257, 9899, - - -
480, 1010, 2648, 3291, 3569, 4932, 5012, 5390, 5901, 6257, 9899, -

For parts E.5 and E.6, look at the two runtime plots below. Write in the blank spaces next to the line color what sorting algorithm or what searching method is used based on the average efficiency.

E.5 Sorting Algorithms: Mergesort, Quicksort, and Straight Insertion



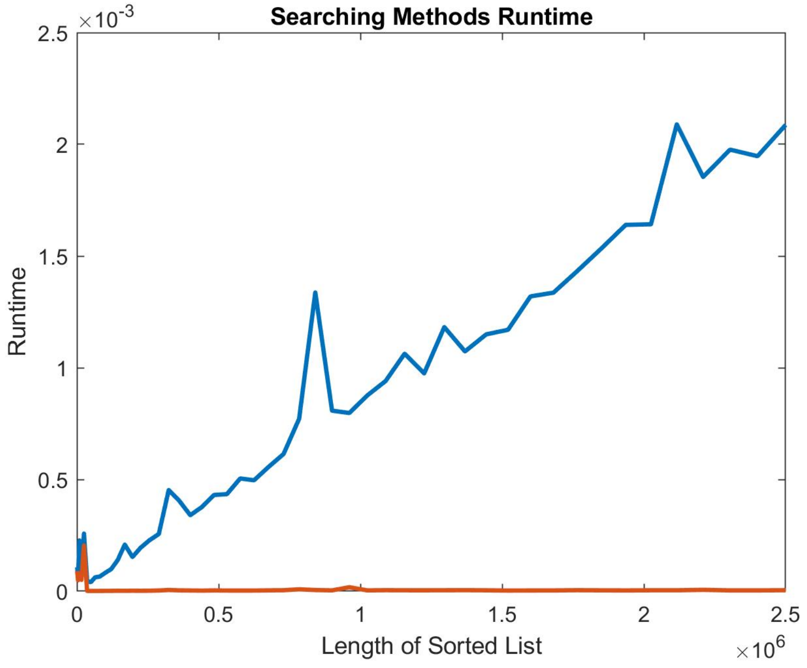
Blue Line (top):____________________________

Red Line (middle):____________________________

Yellow Line (bottom):____________________________

Name: ____

E.6 Searching Methods: Binary Search and Linear Search



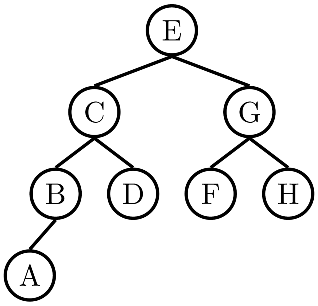
Blue Line (top): ____________________________

Red Line (bottom): ____________________________

Name: ______________________________

Part F: (1+1+1+1+1+1+1+1+1+1 points)

In Parts F.1 to F.3, show the order in which the nodes of the tree shown immediately below are visited. For example if the order is A → B → C, write *ABC*.



F.1 Pre-order traversal:

____

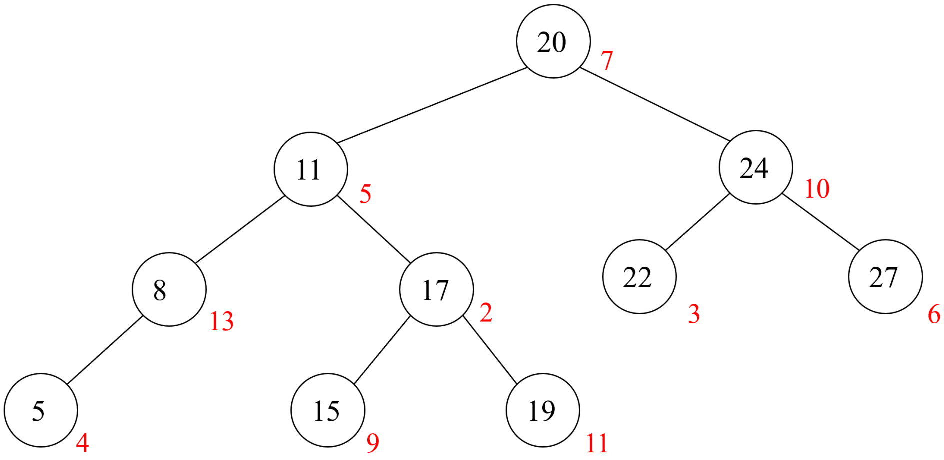
F.2 Post-order traversal:

____

F.3 In-order traversal:

____

In parts F.4 to F.7 refer to the tree shown below.



F.4 What property makes this tree a binary search tree? ______________________

F.5 Which type of traversal will visit the nodes in ascending order? ______________

F.6 If this tree is used to arrange numbers in ascending order, where would you add a new node with index 8 and containing the number 16? Show the placement of the new node directly on the sketch of the tree.

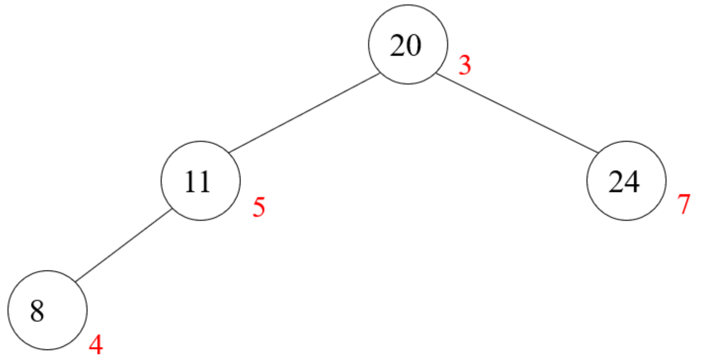
F.7 Following from Part F.6, what are the indices for the left and right child of the newly added node?

Left : ____________________

Right : ____________________

Name: ______________________________

F.8 Write MATLAB code to generate the tree shown below. The name of the tree should be `finalTree`. You should use no more than 12 lines for this task.



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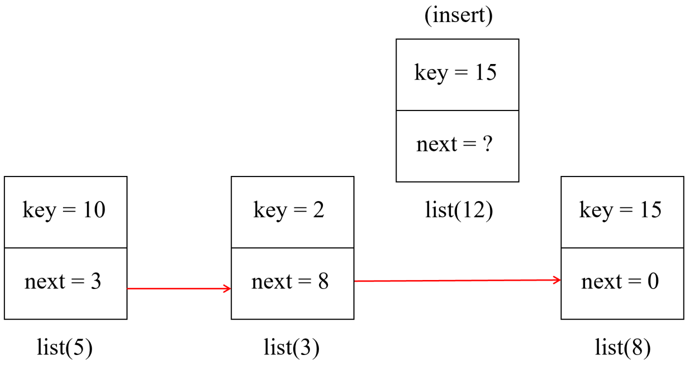
F.9 What is `finalTree`'s class? ______________________________

F.10 What is `finalTree`'s size? ______________________________

Name: ________________________________

Part G: (2+1+1+2+1+2 points)

A linked list called `list` is shown below and comprises the items connected by the red arrows.



G.1 Describe in words how you would insert the data item `list(12)` into `list` between data items `list(3)` and `list(8)`. The new linked list will be called `list1`.

G.2 What is the class of `list1`? ____________________

G.3 What is the size of `list1`? ____________________

G.4 Describe in words how you would delete the element after index 5 from `list1` and close the gap. The new linked list will be called `list2`.

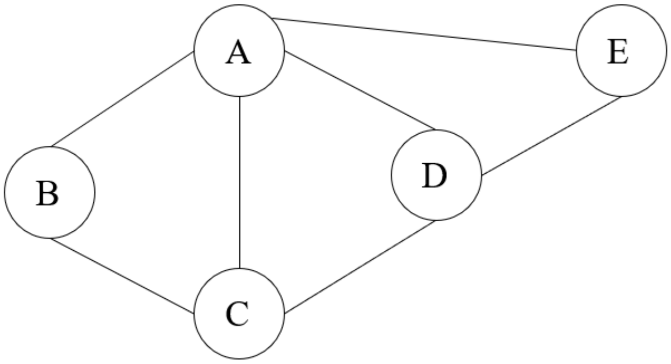
G.5 Is the deleted element still accessible in the MATLAB workspace? ______________

G.6 Describe in words how you would delete the element after index 8 from `list2`. The new linked list will be called `list3`. How are `list2` and `list3` related?

Name: ______________________________

Part H: (1+2+2 points)

Consider the directed graph shown below. In this graph, the order of the elements is A-B-C-D-E.



H.1 Is this an acyclic graph? ____________

H.2 What is the adjacency array that represents the graph?

H.3 What is the Depth-First Search of A, DFS(A)? *(Show all of your work.)*

Part I: (4+2 points)

A supermarket has n aisles, with each aisle containing m_i different product types, with $i = 1, 2, ..., n$. For each product type p, there are b_p different brands of that product, each with a different price. For simplicity, the supermarket chooses to represent the customers' shopping carts using structure arrays.

Each cart is to be modelled as a structure array (n by 1) called `cart`, with each element of that array representing an aisle. The i^{th} structure array `cart(i)` has 2 fields: the first is called `Aisle` which is the name of the corresponding aisle as a row char array, while the second, called `Product`, is a structure array (m_i by 1) representing the m_i products in aisle i.

The `Product` struct has 4 fields: `Name`, representing the name of that product as a row char array, `Brands` which is a cell array, with each container containing the names of the brands for that product, `Prices`, which is a row double array (with the same size as the `Brands` cell array for that element) containing the unit prices for the corresponding brands, and `Sales`, which is a row double array (with the same size as the previous two fields) representing the quantity of each brand of that product in the customer's cart.

I.1 Consider a function called `AddItem` with the following declaration line:

```
function newCart = AddItem(cart,aisle,prod,type,q)
```

Here, `cart` is a structure array as described above, `aisle`, `prod`, and `type` are each row char arrays, and `q` is a scalar double. The output newCart is an "updated" version of the cart structure.

This function should add `q` items of the brand `type` of product `prod` from aisle `aisle` to the shopping cart `cart` and return the updated shopping cart `newCart`. In other words, the only difference between `cart` and `newCart` is the `Sales` field of the `Product` field.

To that end, follow the steps here to complete the function:

Step 1: In one line, find the index of the element of `cart` that corresponds to the given item to be added, and store it in a variable called `i`.

```
if
```

____

Step 2: In one line, find the index of the element of the `Product` field struct that corresponds to the given item to be added, and store it in a variable called `j`.

____

Step 3: In one line, find the index of the element of the `Brands` field cell that corresponds to the brand of the given item to be added, and store it in a variable called `k`.

____

Step 4: In one line, add the quantity `q` to the correct value in the `Sales` field of the `Product` field corresponding to the given item to be added.

____

Step 5: This last command simply sets `newCart` to `cart`.

```
newCart=cart;
```

I.2 Consider a function called `recMultItems` with the following declaration line:

```
[newCart] = recMultItems(cart,aisles,prods,types,qs)
```

Here, `cart` is a structure array as in the previous part, `aisles`, `prods`, and `types` are cell arrays of the same length, and `qs` is a double array of the same length as the former cell arrays.

This function adds more than one item to the `cart` structure array using ***recursion***. Each product's information is stored in the individual arrays; that is, for the i^{th} set of items, the name of its aisle is contained in the i^{th} container of `aisles`, the name of the product is contained in the i^{th} container of `prods`, the name of the brand is contained in the i^{th} container of `types`, and the quantity of that item is the i^{th} element of `qs`.

To that end, follow the instructions below:

Step 1: In one line, complete the base case below (*i.e.*, add the first item) .

____

Step 2: In the recursive relation, add the rest of the items recursively. This should be done in 1 line.

```
if numel(aisles)~=1
```

____

```
end
```

Part J (3+3 points)

Consider the initial-value problem (IVP)

$$\frac{dy}{dx} = xy^{\frac{1}{2}}$$

on the domain $[0, 2]$, with the initial condition $y(0) = 0$.

The solution of interest for this initial value problem is $y = x^4/16$.

J.1 What is the solution of this IVP using the forward Euler method? Is it a good approximation for the solution of interest? If not, explain why. Show your work below. *(Hint: the step size Δx does not matter.)*

J.2 To solve the preceding IVP with a different initial condition $y(2) = 1$, consider the use of `ode45` in MATLAB. Specifically, complete the expressions for `fHan`, `tSpan`, and `init` to completely solve the IVP:

`fHan =` ______________________________

`tSpan =` ______________________________

`init =` ______________________________

in preparation for using `ode45` using the following command:

```
[tSol,ySol] = ode45(fHan,tSpan,init);
```

Part K: (1+1+1+1 points)

Use one or more of the MATLAB functions `round`, `ceil`, `floor`, `fix` to perform the following tasks:

K.1 Round a double `a` to the nearest integer toward 0

Answer: ____________________

K.2 Round a double `b` to the nearest integer less than or equal to `b`

Answer: ____________________

K.3 Chop a double `c` to two significant digits

Answer: ____________________

K.4 Round a double `d` to the nearest integer toward $+\infty$

Answer: ____________________

Part L: (1+1+3+1 points)

Consider a standard least-squares problem of the form

$$\min_{\mathbf{x}\in\mathbb{R}^n} ||\mathbf{A}\mathbf{x} - \mathbf{b}||$$

L.1 Which norm || || is used? *(Hint: 1-norm, 2-norm, p-norm, or infinity-norm)*

Answer: ______________________

L.2 Write the normal matrix equation for the above least-squares problem.

Answer: ______________________

L.3 In a regression problem, if the function space has m basis functions $(f_1(z), f_2(z), \ldots, f_m(z))$, and there are n input data points $(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \ldots, \langle x_n, y_n\rangle)$, what is the dimension of the matrix $\mathbf{A}$ when constructing a regression equation?

Answer: ______________________

What does $A(i, j)$ represent? Use the basis functions and the input data only in your answer.

Answer: ______________________

What does $b(j)$ represent? Again, use the basis functions and the input data only in your answer.

Answer: ______________________

L.4 Suppose that a cubic curve-fitting of the sampled data is desired. What are the basis functions in your function space? Write all of them in the space below.

Answer: ______________________

Part M: (2+2+2 points)

Suppose that we wish to find the sum of the absolute value of all even-indexed components in a vector. For example, if $\mathbf{x} = [x_1, x_2, x_3, x_4]$, the result would be $|x_2| + |x_4|$. We will use linear programming to achieve such a goal.

Assume the desired variable is $\mathbf{t} = [t_1, t_2]^T$. In order to use the MATLAB `lpsolver()`, we need to define a vector $\mathbf{c}$, a matrix $\mathbf{A}$, and another vector $\mathbf{b}$, such that our problem becomes:

$$\min_{\mathbf{t} \in \mathbb{R}^2} \mathbf{c}^T \cdot \mathbf{t}$$

$$\text{subject to } \mathbf{At} \leq \mathbf{b}$$

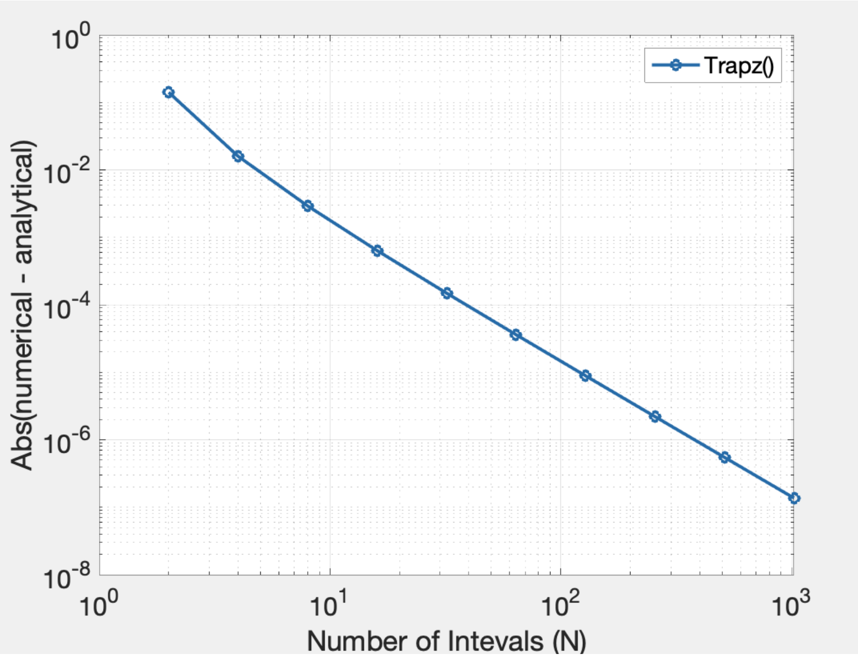
What is the vector $\mathbf{c}$ equal to in this case? Write in a row vector form.

What is matrix $\mathbf{A}$ equal to in this case? Write in matrix form.

What is matrix $\mathbf{b}$ equal to in this case? Write in a column vector form.

Part N: (2+2 points)

N.1 To study the convergence of `trapz()`, we integrate a given function in the same domain with different number N of intervals and plot the difference between the numerical and analytical integral. The figure below shows a log-log plot of the error vs. the interval lengths.



From the figure, what is the order of convergence of `trapz()` in terms of N (*e.g.,* linear O(N), quadratic O(N^2), *etc.*)?

Answer: ____________________

N.2 If we used Simpson's method instead, what change would you expect to happen the order of convergence to be?

Answer: ____________________

<u>Part O</u>: (1+1+1+2+1 points)

You are asked to debug a MATLAB function, called *bisection*, which is supposed to employ the bisection method to determine the root of a real-valued function. The declaration line of this function is

```
function y = bisection(h,a,b)
```

where `h` is a function handle, and `a`, `b` are the two brackets of the root. State (in words) what you may conclude from the results of the following MATLAB runs of this function if `h = @(x) sin(x)`:

O.1

```
>> bisection(h,pi/2,3*pi/2)

   ans =
         3.141592653589972
```

Answer: ____

O.2

```
>> bisection(h,3*pi/2,pi/2)

   ans =
         3.141592653589972
```

Answer: ____

O.3

```
>> bisection(h,0,pi)
>>
```

No output at all!

Answer: ____

O.4

```
>> bisection(h,pi/2,2*pi/3)
```

Leads to an infinite loop.

Answer: ____

O.5

Based on the previous results, what do you expect the output to be for the following function call?

```
>> bisection(h,pi/2,pi/2)
```

Answer: ____