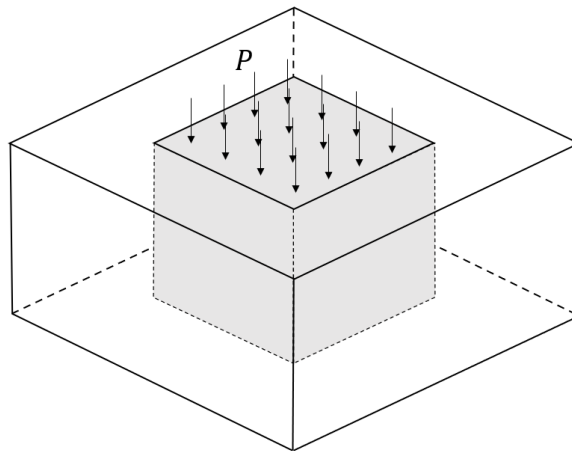


MecEng 127/227 Midterm
(March 18 at 9:40-11 AM PST)

Each question is worth 10 points

1. A cube of isotropic material is inserted into a cubical hole of the same size that has been carved into a rigid block. Contact between the cube and the walls of the hole is frictionless. The top face of the cube is exposed to a uniform pressure of magnitude p . (see the Figure).



(a) Let ϵ be the extensional strain in the inserted material in the direction perpendicular to its exposed face. Find the constant C in the relation

$$\epsilon = Cp,$$

in terms of E and ν for the isotropic material. Assume uniform states of stress and strain in the cube.

(b) Discuss the case of an incompressible material ($\nu = 1/2$).

2. The stiffness matrix of an orthotropic material is given, in Voigt notation, by

$$(C_{ij}) = \begin{pmatrix} -0.673 & -1.827 & -1.058 & 0 & 0 & 0 \\ -1.827 & -0.673 & -1.442 & 0 & 0 & 0 \\ -1.058 & -1.442 & 0.481 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.5 \end{pmatrix} GPa.$$

Find:

- The stresses generated by the strains $\epsilon_{11} = .0010$, $\epsilon_{22} = 0$, $\epsilon_{33} = 0$, $\epsilon_{23} = 0$, $\epsilon_{13} = 0$, $\epsilon_{12} = .0030$.
- The value of the strain-energy function W .
- The components of the traction vector acting on the plane with unit normal

$$\mathbf{n} = \frac{1}{\sqrt{3}}(\mathbf{e}_1 + \mathbf{e}_2 - \mathbf{e}_3).$$

3. Consider an orthotropic lamina with the strength properties (all numbers in units of MPa)

$$t_1 = 1400, \quad c_1 = -1400, \quad t_2 = 50, \quad c_2 = -50 \quad \text{and} \quad S = 100.$$

Suppose the fibers are oriented at $\theta = 60^\circ$ counter-clockwise from the x - axis.

- Determine the numerical values of the coefficients $F_1, F_2, F_6, F_{11}, F_{22}$ and F_{66} in the Tsai-Wu criterion. Assume $F_{12} = 0$.
- If $\sigma_{yy} = 0$ and $\sigma_{xy} = 0$, what are the possible values of the stress σ_{xx} at failure?
- If $\sigma_{xx} = 0$, find the equation satisfied by σ_{yy} and σ_{xy} at incipient failure.

4. An isotropic glass fiber material (with Young's modulus $E_f = 85 GPa$ and Poisson ratio $\nu_f = 0.2$) is combined with an isotropic epoxy material (with Young's modulus $E_m = 3.4 GPa$ and Poisson ratio $\nu_m = 0.3$) to fabricate an off-axis lamina with $\theta = 45^\circ$ and fiber volume fraction $c_f = .70$.

- Compute the effective lamina shear modulus relative to the material axes using the refined estimate

$$G_{12} = G_m \frac{(1 + c_f)G_f + (1 - c_f)G_m}{(1 + c_f)G_m + (1 - c_f)G_f}.$$

- What is the shear strain γ_{12} , relative to the *material* axes, produced by the stress state $\sigma_{xx} = 1000 MPa$, $\sigma_{yy} = 0$, $\sigma_{xy} = 0$ relative to the *global* axes?