Problem 1

(1)
$$Z_{in,3a} = Z_{03}\frac{Z_L + jZ_{03}\tan\beta l_3}{Z_{03} + jZ_L\tan\beta l_3}, \quad \tan\beta l_3 = \tan 5\pi = 0$$

$$= Z_L = Z_{in,3b}$$

$$Z_{in,3} = Z_{in,3a} \,//\, Z_{in,3b} = Z_L/2 = 35\Omega$$

$$Z_{in,AA'} = Z_{02}\frac{Z_{in,3} + jZ_{02}\tan\beta l_2}{Z_{02} + jZ_{in,3}\tan\beta l_2}, \quad \tan\beta l_2 = \tan\frac{\pi}{2} = \infty$$

$$= \frac{Z_{02}^2}{Z_{in,3}} = \frac{(50\Omega)^2}{35\Omega} = 71.43\Omega$$

$$\Gamma = \frac{Z_{in,AA'} - Z_{01}}{Z_{01} + Z_{in,AA'}} = \frac{71.43 - 50}{71.43 + 50} = 0.17$$

$$\tilde{V}_1(z) = (e^{-j\beta z} + 0.17e^{j\beta z})\,V$$

$$\tilde{I}_1(z) = \frac{e^{-j\beta z} - 0.17e^{j\beta z}}{50}\,A. \quad \beta = \frac{2\pi f}{c} = 20\pi\ m^{-1}$$

$$V_1(z,t) = [\cos(\omega t - \beta z) + 0.17\cos(\omega t + \beta z)]\,V$$

$$I_1(z,t) = \frac{1}{50}[\cos(\omega t - \beta z) - 0.17\cos(\omega t + \beta z)]\,A$$

$$\omega = 2\pi f = 6\pi\ GHz$$

(2)
$$Z_{AA'} = \frac{Z_{02}^2}{Z_{in,3}} = Z_{01} = 50\Omega$$

$$Z_{02} = \sqrt{Z_{01}Z_{in,3}} = \sqrt{35\cdot 50}\,\Omega = 41.83\Omega$$

(3) $\Gamma_{12} = 0. \Rightarrow$ no standing wave in TL1.

$$\Gamma_{23} = \frac{Z_3 - Z_{02}}{Z_3 + Z_{02}} = \frac{35 - 41.83}{35 + 41.83} = -0.089$$

$$\tilde{V}(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z})$$
$$= V_0^+ (e^{-j\beta z} - 0.089 e^{j\beta z})$$

$\Rightarrow$ standing wave in TL2

(4) $Z_{in,3a} = Z_L = Z_{in,3b} = 70\Omega - j20\Omega$

$$Z_{in,2} = \frac{Z_{02}^2}{Z_{in,3a}/2} = \frac{50^2}{35 - 10j} = (66.03 + 18.87j)\Omega$$

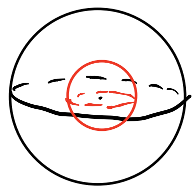
$$\frac{1}{Z_{in,2} + jX} + \frac{1}{R} = \frac{1}{Z_{01}}$$

X is real $\Rightarrow$ $X = -18.87\Omega$

$$\Rightarrow \frac{1}{R} = \frac{1}{50\Omega} - \frac{1}{66.03\Omega} \Rightarrow R = 205.96\Omega$$

Problem 2

(1) Take a spherical Gauss's surface as shown in the left figure.



$$\varepsilon \iint_A \vec{E}(\vec{r}) \cdot d\vec{s} = Q_{enc}$$

For $r<R$:

$$\varepsilon E(r) \cdot 4\pi r^2 = \frac{4}{3}\pi r^3 \rho_v$$

$$\Rightarrow E(r) = \frac{\cancel{\frac{4}{3}}\pi r^3}{\varepsilon \cdot 4\pi r^2} \cdot \frac{Q}{\cancel{\frac{4}{3}\pi} R^3}$$

$$= \frac{Q \cdot r}{4\pi\varepsilon R^3}$$

$$\Rightarrow \vec{E}(r) = \frac{Q \cdot r}{4\pi\varepsilon \cdot R^3}\hat{r}$$

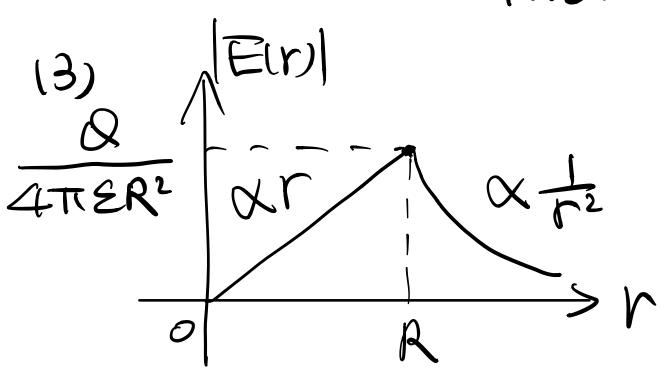
(2) For $r>R$:

$$\varepsilon E(r) \cdot 4\pi r^2 = Q$$

$$E(r) = \frac{Q}{4\pi\varepsilon r^2}$$

$$\Rightarrow \vec{E}(r) = \frac{Q}{4\pi\varepsilon r^2}\hat{r}$$

(3)



The field is continuous @ $r=R$.

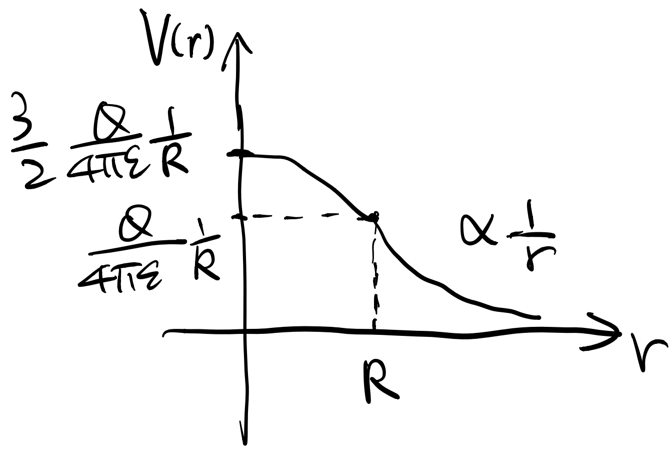
$$V(r) = \int_r^{\infty} \vec{E}\, d\vec{l}$$

$$= \int_r^{\infty} \frac{Q}{4\pi\varepsilon r'^2}\, dr' \qquad (r>R)$$

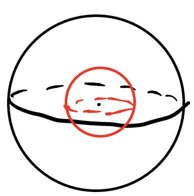
$$= \frac{Q}{4\pi\varepsilon}\left(-\frac{1}{r'}\right)\Big|_r^{\infty} = \frac{Q}{4\pi\varepsilon}\frac{1}{r} \qquad (r>R)$$

$$V(r) = \int_r^{R} \frac{Qr}{4\pi\varepsilon R^3}\, dr + V(R)$$

$$= \frac{Q}{4\pi\varepsilon R^3}\,\frac{1}{2}(R^2-r^2) + \frac{Q}{4\pi\varepsilon}\frac{1}{R} \qquad (r<R)$$



(4)



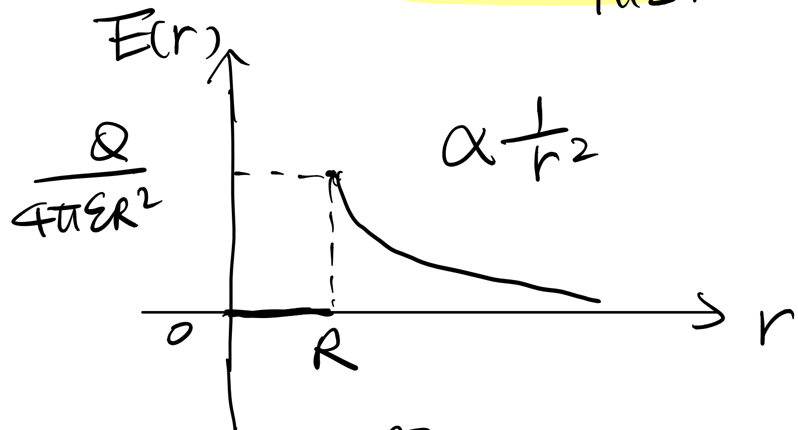
$$\varepsilon \iint_A \vec{E}(\vec{r}) \cdot d\vec{s} = Q_{enc}$$

For $r<R$: $Q_{enc} = 0 \Rightarrow \vec{E}(r) = 0$

For $r>R$: $Q_{enc} = Q$

$$\varepsilon |E(r)| \cdot 4\pi r^2 = Q$$

$$\vec{E}(r) = \frac{Q}{4\pi\varepsilon r^2}\hat{r}$$



$$V(r) = \int_r^\infty \vec{E}\cdot d\vec{l}$$

For $r > R$: $= \frac{Q}{4\pi\varepsilon r}$ similar to Q @ origin.

For $r < R$: $V(r) = V(R) = \frac{Q}{4\pi\varepsilon R}$