UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Sciences

EE 42 / 100 | Midterm 2 | Spring 2008

Name: Solutions	
SID:	
Section:	
Name of student left of you:	
Name of student right of you:	

Problem	Score
1	
2	
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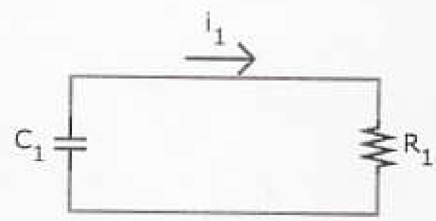
Total	

- Check the units of your results.
- Closed book, closed notes.
- No calculators.
- Leave pack, books, and electronic devices (e.g. cell phones) in isle.
- Take off caps or hats.
- Copy your answers into marked boxes on exam sheets.
- Simplify numerical and algebraic results as much as possible.
- Be kind to the graders and write legibly. No credit for illegible results.
- No credit for multiple differing answers to the same question.
- The UC rules on dishonesty apply.

1. [25 points] A capacitor C_1 is used to power a model airplane, represented in the circuit diagram below by resistor R_1. Initially the capacitor is charged to voltage V_1. Calculate the fraction r of the initial energy remaining on the capacitor when the current i_1 has decreased to half its initial value.



$r = \frac{1}{4}$

The energy stored in a capacitor is given by

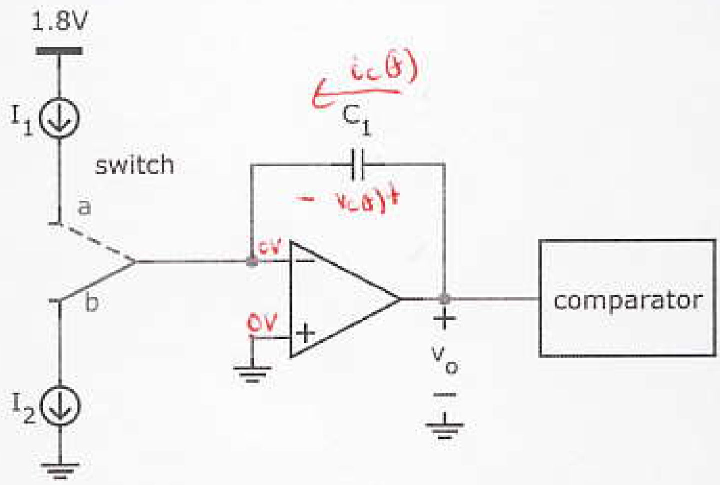
$$E_{stored} = \frac{1}{2} C V^2$$

$$E_0 = \frac{1}{2} C_1 V_0^2 = \frac{1}{2} C_1 V_1^2$$

$$E_f = \frac{1}{2} C_1 V_f^2 = \frac{1}{2} C_1 (I_f R_1)^2 = \frac{1}{2} C_1 \left(\frac{I_1}{2} R_1\right)^2 = \frac{1}{2} C_1 \left(\frac{V_1}{2}\right)^2$$

$$r = \frac{E_f}{E_0} = \frac{\frac{1}{2} C_1 \left(\frac{V_1}{2}\right)^2}{\frac{1}{2} C_1 V_1^2} = \frac{1}{4}$$

2. [25 points] In the circuit below C_1 represents a touch sensor. The comparator controls the position of the switch as follows: Whenever V_o reaches the value V_{ref}, the switch is set to position a. After V_o drops to zero, the switch is set to position b. Touch is detected by measuring the frequency f_o at which the switch position changes.
Derive an analytical expression for f_o. Assume that the operational amplifier is ideal.
Suggestion: sketch $v_o(t)$ and mark the knows and unknown in the graph.



$$f_o = \boxed{\frac{I_1 \cdot I_2}{C \cdot V_{ref}\,(I_1 + I_2)}}$$

The op-amp is in negative feedback.

$i_p = i_n = 0$ $\quad v_p = v_n = 0\,V$

Let's define the current & voltage through C_1 as $i_c(t)$ & $V_c(t)$

$$i = C\frac{dV}{dt} \Rightarrow V_c(t) = \frac{1}{C}\int_0^t i_c(x)\,dx + V_c(0)$$

Charge Cycle (switch is set to b)

$$V_{ref} = \frac{1}{C}\int_0^{t_a} i_c(x)\,dx + V_c(0)$$

$$V_{ref} = \frac{1}{C} I_2 T_a + 0V$$

$$T_a = \frac{C \cdot V_{ref}}{I_2}$$

Discharge Cycle (switch is set to a)

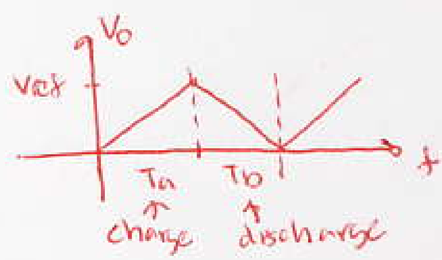
$$0V = \frac{1}{C}\int_{t_a}^{t_b+t_a} i_c(x)\,dx + V_c(t_a)$$

$$0V = \frac{1}{C}(-I_2)T_b + V_{ref}$$

$$T_b = \frac{C \cdot V_{ref}}{I_1}$$

$$T = T_a + T_b \Rightarrow f = \frac{1}{T_A + T_B}$$

$$f = \frac{1}{\frac{C \cdot V_{ref}}{I_1} + \frac{C \cdot V_{ref}}{I_2}} = \boxed{\frac{I_1 \cdot I_2}{C \cdot V_{ref}\,(I_1 + I_2)}}$$



Note: C_1 charges through I_2 & discharges through I_1.

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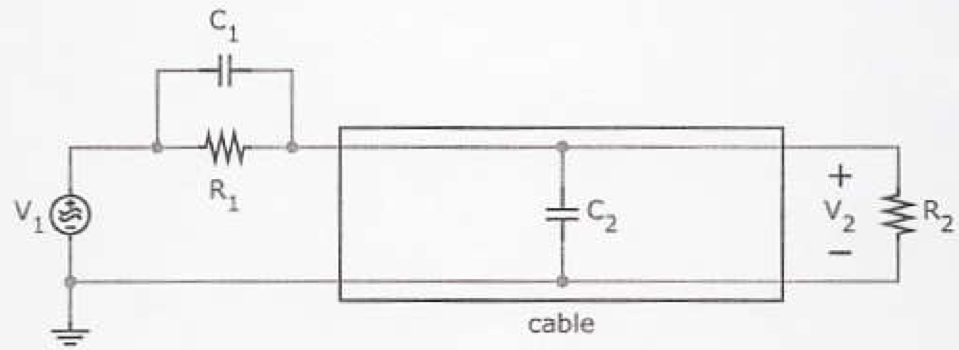
3. [25 points] The circuit below shows a simple model of a wired Ethernet connection. V_1 is the signal source and R_1 represents its impedance. The cable is modelled by capacitor C_2 and R_2 represents the receiver's input impedance. The capacitor C_1 has been added to enable higher frequency transmission at low error rates.

Derive an expression for C_1 as a function of R_1, R_2, and C_2 for which $\frac{V_2(j\omega)}{V_1 j\omega}$ has a constant value that is independent of frequency.

Note: This technique is used in many transceivers including 100 Mbit Ethernet.



$C_1 = \frac{C_2 R_2}{R_1}$

Short Solution

The impedance at low frequencies should equal the impedance at high frequencies. Since the caps are open at low freqs & ~~ot~~ shorts at high freqs

$$Z(lo) = Z(hi)$$

$$\frac{R_2}{R_1+R_2} = \frac{\frac{1}{sC_2}}{\frac{1}{sC_1}+\frac{1}{sC_2}}$$

$$\frac{R_2}{R_1+R_2} = \frac{C_1}{C_1+C_2}$$

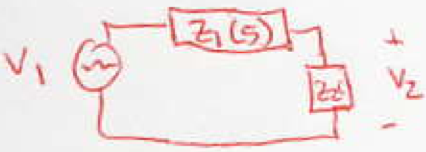
$$R_2(C_1+C_2) = C_1(R_1+R_2)$$

$$\boxed{C_1 = \frac{C_2 R_2}{R_1}}$$

Prob 5)

"Long" Solution

- Let's find the transfer function $V_2(s)/V_1(s)$



$$\frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$ Voltage Divider

Where $Z_1(s) = \dfrac{R_1 \cdot \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \dfrac{R_1}{1+sR_1C_1}$ $\quad Z_2(s) = \dfrac{R_2 \cdot \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \dfrac{R_2}{1+sR_2C_2}$

$$\frac{V_2(s)}{V_1(s)} = \frac{\dfrac{R_2}{1+sR_2C_2}}{\dfrac{R_1}{1+sR_1C_1} + \dfrac{R_2}{1+sR_2C_2}} = \frac{R_2(1+sR_1C_1)}{R_1(1+sR_2C_2) + R_2(1+sR_1C_1)}$$

$$= \frac{R_2}{R_1+R_2} \cdot \frac{(1+sR_1C_1)}{1+s\frac{R_1R_2}{R_1+R_2}(C_1+C_2)}$$

In order for the transfer to be independent of frequency, the coefficients of $j\omega$ or s should be equal.

$$\therefore \quad R_1C_1 = \frac{R_1R_2}{R_1+R_2}(C_1+C_2)$$

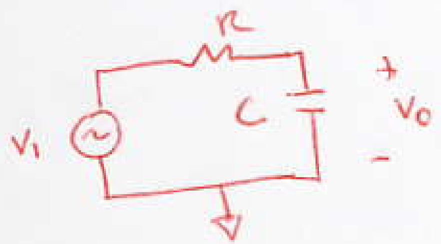
$$R_1C_1(R_1+R_2) = R_1R_2(C_1+C_2)$$

$$C_1\frac{R_1}{R_1+R_2} = C_2\frac{R_2}{R_1+R_2}$$

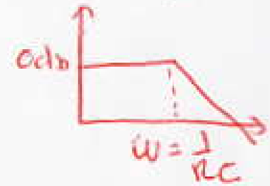
$$\boxed{C_1 = \frac{C_2R_2}{R_1}}$$

4. [25 points] An audio system suffers from high frequency interference. Design a circuit consisting of a resistor R and capacitor C that passes low frequency signals ($\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right|_{\omega=0} = 1$) and attenuates high frequency signals ($\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right|_{\omega>0} < 1$).

(a) Draw a diagram of a circuit with these characteristics, consisting of R and C. Clearly mark the input and output voltages V_i and V_o with plus and minus signs.



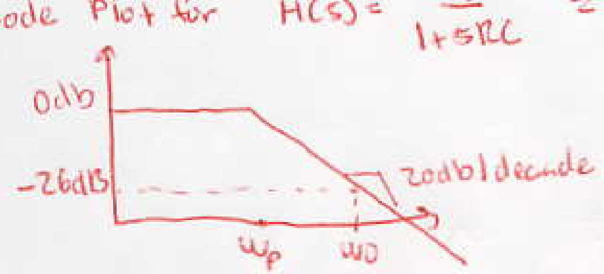
$$\frac{V_o}{V_1} = \frac{\frac{1}{sC}}{\frac{1}{sC}+R} = \frac{1}{1+sRC}$$



(b) Derive a symbolic expression for the value of C as a function of R that results in 26 dB attenuation at a given frequency f_o.
Hint: draw the Bode plot (piece-wise linear approximation of magnitude response) of the circuit. Mark what is known and unknown in the graph.

$C = \dfrac{10}{\pi f_o R}$ (assume f_o is in Hertz) $= \dfrac{20}{f_o R}$ (assume f_o is in rad/s)

Bode Plot for $H(s) = \dfrac{1}{1+sRC} = \dfrac{1}{1+s/\omega_p}$ where $\omega_p = \dfrac{1}{RC}$



The transfer function of a first-order system is given by $H(j\omega) = \dfrac{1}{1+j\omega RC} = \dfrac{1}{1+j\omega/\omega_p}$. If $\omega \gg \omega_p$ (large), the imaginary term in the denominator dominates, and we have

$$H(j\omega) \approx \frac{1}{j\omega/\omega_p}$$

$$Gain_{dB}(\omega) = 20\log_{10}|H(j\omega)|$$

Prob 4 cont.

The $gain_{dB}$ difference at a w_0 (frequency) 10 times w_p is

$$Gain_{dB}(10w_p) - Gain_{dB}(w_p)$$
$$= -20\log_{10}(10w_p) + 20\log_{10}(w_p)$$
$$= -20\log_{10}(10) - \cancel{20\log_{10}(w_p)} + \cancel{20\log_{10}(w_p)}$$
$$= -20\log_{10}(10) = \underline{-20\,dB} \text{ (at 10 times } w_p)$$

The $gain_{dB}$ difference at a w_0 2 times w_p is

$$Gain_{dB}(2w_p) - Gain_{dB}(w_p)$$
$$= -20\log_{10}(2w_p) + 20\log_{10}(w_p)$$
$$= -20\log(2) - \cancel{20\log(w_p)} + \cancel{20\log(w_p)}$$
$$= -2\log(2) = -6\,dB \text{ (at 2 times } w_p)$$

Now for our problem, we have 26 dB attenuation

$$26dB = \underset{10x}{\underset{\uparrow}{20dB}} + \underset{2x}{\underset{\uparrow}{6dB}}$$
$$= Gain_{dB}(10x) + Gain_{dB}(2x)$$
$$= Gain_{dB}(20 \cdot w_p)$$

$$w_0 = 20 \cdot w_p = 20 \cdot \frac{1}{RC}$$

$$\Rightarrow C = \frac{20}{R \cdot w_0}$$

Since $w_0 = 2\pi f_0$

$$C = \frac{20}{2\pi f_0 R} = \boxed{\frac{10}{\pi f_0 R}}$$