NAME:

ID # :

# 1	# 2	# 3	# 4	# 5	#6	#7	#8	TOTAL
14	10	9	10	4	8	14	12	81

Instructions:

1 Write your name and student ID number.

2 Read the questions carefully.

3 This exam has 8 questions worth 81 points.

4 Please write your solution clearly.

Problem # 1 (4 +4+3+3 = 14 points)

Explain your answers.

(a) Find eigenvalues (including multiplicity) of the matrix $A = \begin{bmatrix} I_4 & B \\ 0 & -I_3 \end{bmatrix}$.

Here I_4 is the 4×4 identity matrix and I_3 is the 3×3.

Block upper triangular

Answer: $1, 1, 1, 1 \quad -1, -1, -1$

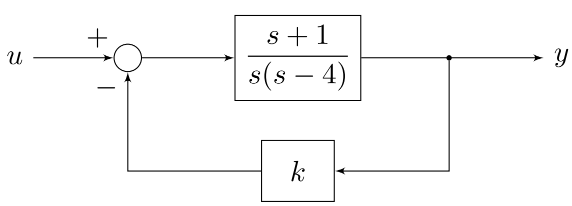
(b) Compute A^{100} for the matrix $A = \begin{bmatrix} I_4 & B \\ 0 & -I_3 \end{bmatrix}$.

$$A = T \begin{bmatrix} I_4 & 0 \\ 0 & I_3 \end{bmatrix} T^{-1} \Rightarrow A^{100} = T \begin{bmatrix} I^{100} & 0 \\ 0 & I^{100} \end{bmatrix} T^{-1}$$

$$= I_7$$

Answer: I_7

(c) Consider the block diagram shown below.



Closed loop TF from $u \to y$

$$= \frac{P}{1+Pk} = \frac{s+1}{s^2+(k-4)s+k}$$

- Find the value of the gain k so that the free response of the closed loop system exhibits **undamped** sinusoidal oscillations, i.e. the free response looks like $M\cos(\omega t + \phi)$ where M and ϕ are constants.

For this, need poles on imaginary axis

So, $k = 4$

$k =$ 4

- What is the frequency ω of these oscillations in radians/second?

poles @ $s^2 + 4 = 0$

$\pm 2j$

$\omega =$ 2 rad/sec

Problem # 2 (4+4+2 = 10 points)

No partial credit.

(a) Find the natural frequency ω_n and damping ξ of the transfer function $H(s)$ with realization Σ.

$$H(s) \sim \Sigma \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -1 & -1 & 1 \\ \hline 4 & 5 & 6 \end{array}\right]$$

controllable canonical form

$$= \frac{*}{s^2+s+1}$$

$\omega_n^2 = 1$

$2\xi\omega_n = 1$

$\omega_n =$ 1 rad/sec	$\xi =$ 0.5

(b) Find the controllable canonical form realization for $H(s) = \dfrac{2s^3+3}{s^3}$.

$$\frac{2s^3+3}{s^3} = 2 + \frac{3}{s^3}$$

$A =$	$B =$	$C =$	$D =$
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 3 & 0 & 0 \end{bmatrix}$	2

(c) Consider two vectors in $\mathbb{R}^2$ given by

$$v = \begin{bmatrix} \alpha \\ 1 \end{bmatrix}, \quad w = \begin{bmatrix} 1 \\ \alpha \end{bmatrix}$$

Find all values of α for which v and w are linearly dependent.

Answer:
$\alpha = \pm 1$

$$\det \begin{bmatrix} \alpha & 1 \\ 1 & \alpha \end{bmatrix} = 0$$

$$\alpha^2 - 1 = 0 \qquad \alpha = \pm 1$$

Problem # 3 (3+3 +3= 9 points)

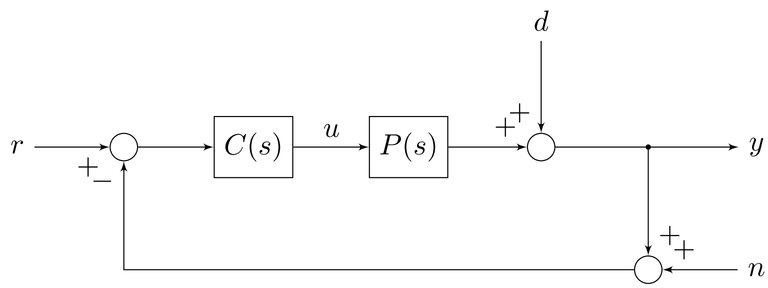
Show your work for partial credit.

Consider the plant $P(s) = \dfrac{1}{s+1}$. Design a controller $C(s) = \dfrac{K_1 s + K_2}{s+p}$ such that

(a) The feedback system system is stable

(b) The feedback system rejects constant disturbances. $\longrightarrow$ need integral control, so $p=0$

(c) The feedback system is critically damped with a natural frequency of 1 rad/sec.



closed loop TF from r to y

$$= \frac{PC}{1+PC} = \frac{K_1 s + K_2}{(s+1)(s+p) + K_1 s + K_2}$$

(a) stable

(b) integral control so $p = 0$

(c) $$TF = \frac{K_1 s + K_2}{s^2 + (1+K_1)s + K_2}$$

$= 1 + K_1$

$2\xi\omega_n = 2$

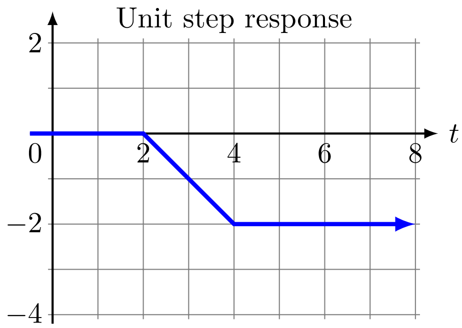
crit damped $\Rightarrow \xi = 1$, $\omega_n = 1 \Longrightarrow K_2 = 1$

$p =$	$K_1 =$	$K_2 =$
0	1	1

Problem # 4 (2 + 4 + 4 = 10 points)

Show your work for partial credit.

Suppose $H(s)$ is a linear time invariant system.
Its **unit** step response with zero initial conditions is plotted below.
Find $H(s)$.

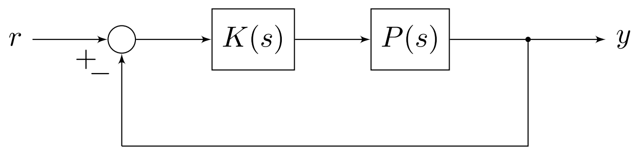


$H(s) = \dfrac{e^{-4s}}{s} - \dfrac{e^{-2s}}{s}$

$$-e^{-2s}\left[\frac{1}{s} - \frac{e^{-2s}}{s}\right]$$

$$= \frac{e^{-4s}}{s} - \frac{e^{-2s}}{s}$$

Problem # 5 (4 points)

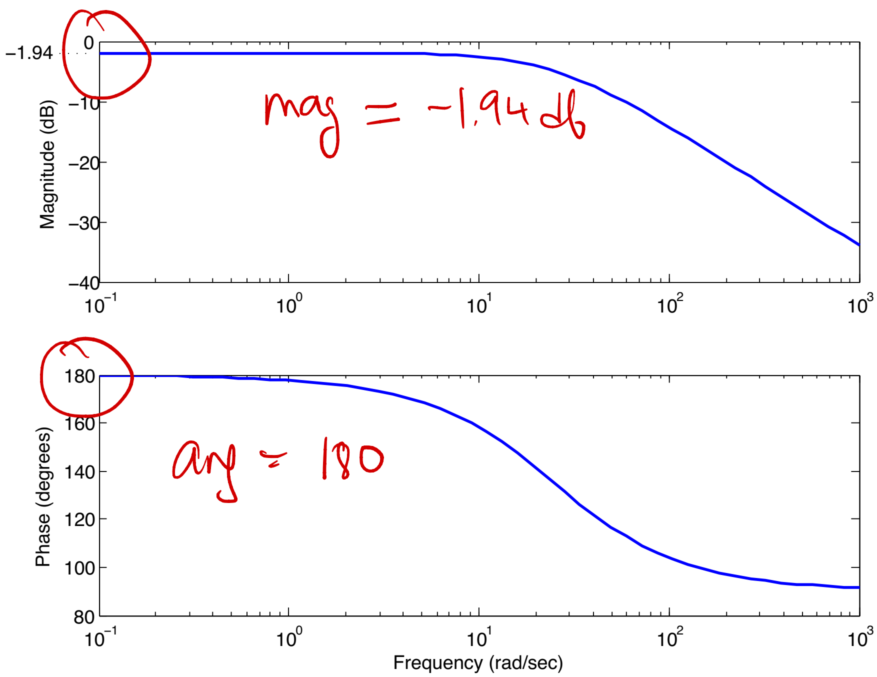


Consider the closed loop system shown above.
Here $P(s)$ and $K(s)$ are single-input single-output transfer functions.
Assume the feedback system is stable.

Plotted below is the magnitude and phase frequency response of the **loop gain** $L(s) = P(s)K(s)$.
Calculate the DC gain of the **closed-loop system** from r to y.

No partial credit.

$-1.94\text{ db} = 0.8$



mag $= -1.94$ db

ang $= 180$

$L(0) = -0.8$

$$\text{Closed loop } T(s) = \frac{PK}{1+PK} = \frac{L}{1+L} \Rightarrow T(0) = \frac{-0.8}{1-0.8} = -4$$

DC gain =

-4

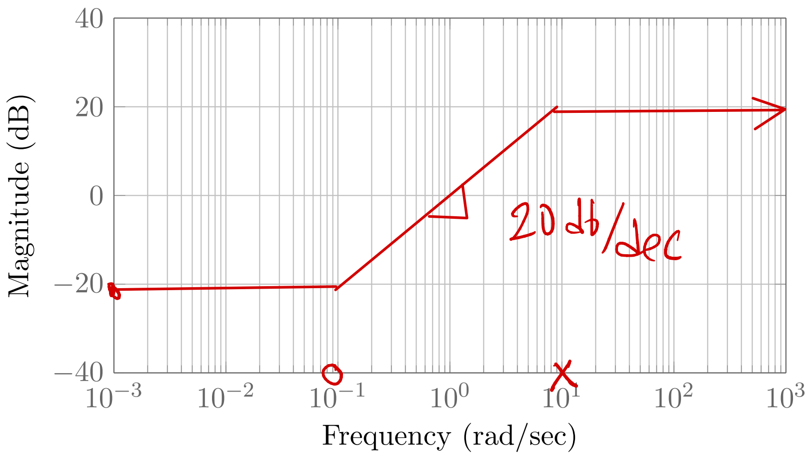
Problem 6 (4+4 = 8 points)

Consider the first-order LTI system

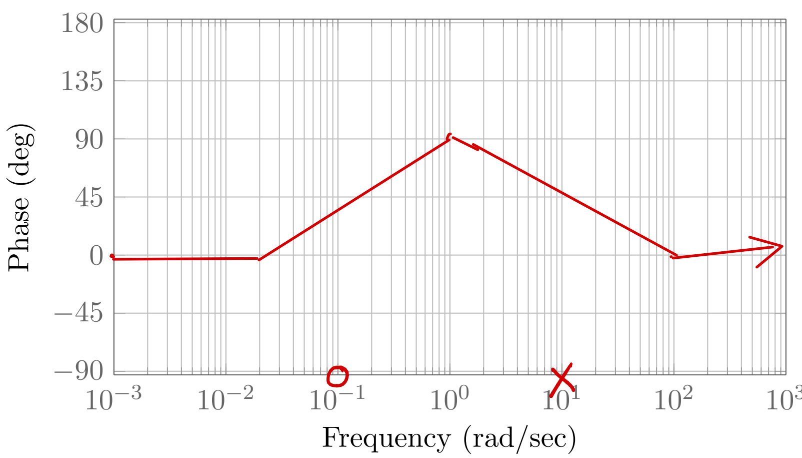
$$C(s) = \frac{10s + 1}{(s + 10)}$$

@ DC $\quad C(0) = \frac{1}{10} = -20\text{db}$

(a) Sketch the straight-line approximation of the magnitude frequency response plot of $C(s)$ on the graph paper below.



(b) Sketch the straight-line approximation of the phase frequency response plot of $C(s)$ on the graph paper below.



@ DC

$\angle C(0) = 0$

LHP zero : $0 \to 90^\circ$

LHP pole $0 \to -90^\circ$

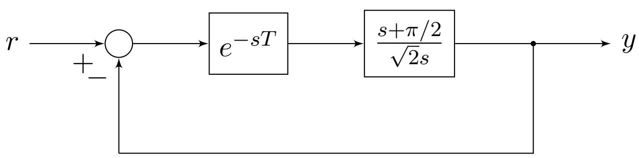
Problem # 7 (14 points)

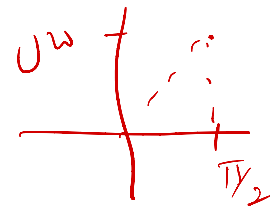
Show your work for partial credit.

Find the delay margin for the system with nominal loop gain

$$L^o(s) = \frac{s + \pi/2}{\sqrt{2}s}$$

In other words, find the maximum delay $T_{\max}$ for which the feedback system shown below is stable. **You answer must be expressed in closed-form. So you have to do this problem by hand, and Matlab won't help.**





$$L^o(j\omega) = \frac{j\omega + \pi/2}{j\sqrt{2}\,\omega}$$

$$|L^o(j\omega)| = \frac{1}{\sqrt{2}\,\omega}\sqrt{\omega^2 + \frac{\pi^2}{4}} \qquad \angle L^o(j\omega) = -90^\circ + \tan^{-1}\left(\frac{\omega}{\pi/2}\right)$$

TD margin : $|L^o(j\omega)| = 1 \Rightarrow \sqrt{2}\,\omega = \sqrt{\omega^2 + \frac{\pi^2}{4}}$

$$2\omega^2 = \omega^2 + \frac{\pi^2}{4}$$

$$\boxed{\omega_c = \frac{\pi}{2}}$$

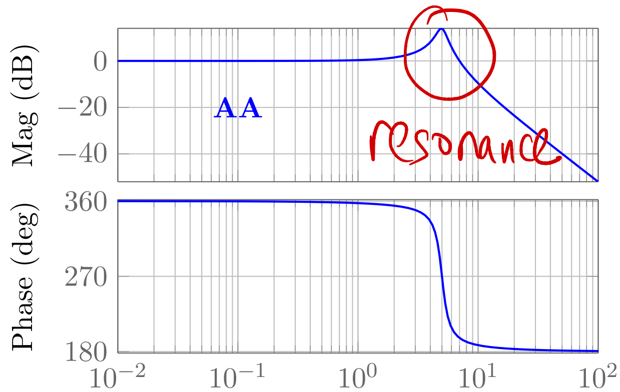
$$\angle L^o(j\omega_c) = -90^\circ + \tan^{-1}\left(\frac{2\omega_c}{\pi}\right)$$
$$= -90^\circ + \tan^{-1}(1)$$
$$= -45^\circ = -\frac{\pi}{4} = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

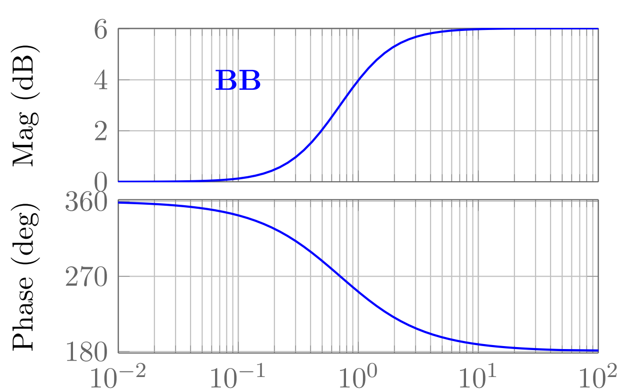
$$\text{TD margin} = \left(\frac{7\pi}{4} - \pi\right)\frac{1}{\omega_c} = \frac{3\pi}{4}\,\frac{2}{\pi} = \frac{6}{4} = 1.5 \text{ sec}$$

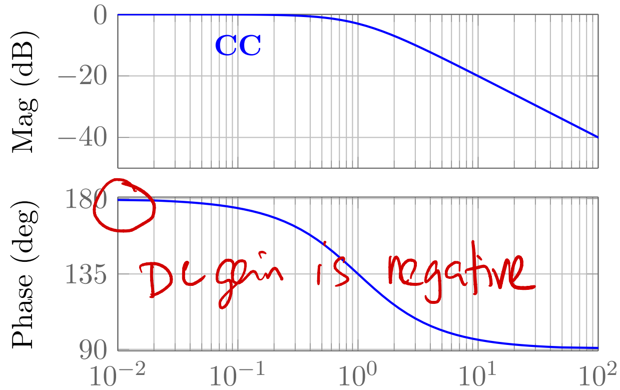
$T_{\max} =$ 1.5 sec.

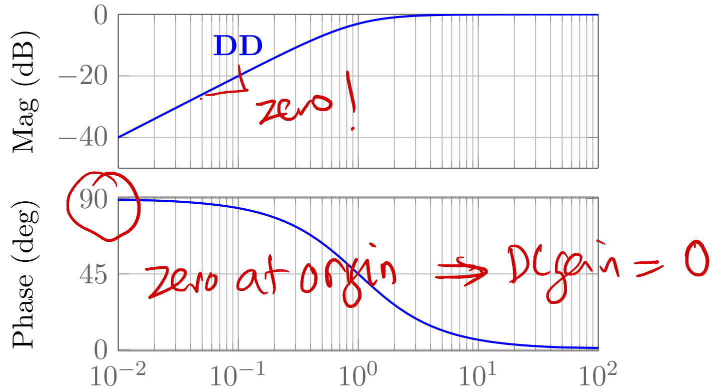
Problem # 8 (12 points)

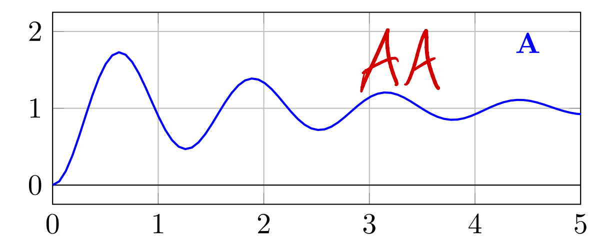
Match the correct unit step response for each frequency response plot given below.
Correct answers get 3 points. Incorrect answers receive −2 points, so you should not guess.
No explanations are necessary.

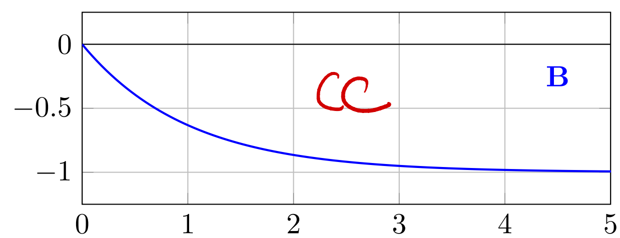


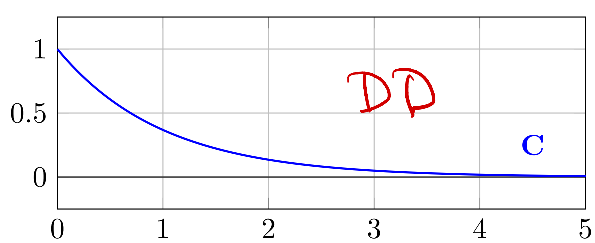


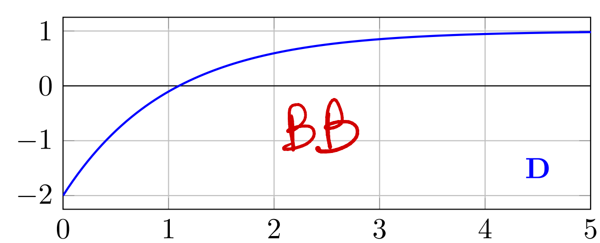












A = AA	B = CC	C = DD	D = BB