1. Super-athlete Baseball

(a) $x = V_{0x} \cdot t$

$y = h + V_{0y} \cdot t - \frac{1}{2} g t^2$ $\quad x = x_0 + V_0 t + \frac{1}{2} a t^2$

(b) $y = h + V_{0y} \cdot t - \frac{1}{2} g t^2 = 2h$

$V_f^2 - V_0^2 = 2a\Delta x$

$0 - V_{0y}^2 = -2g \cdot h$

$V_{0y,min} = \sqrt{2gh}$

(c) $y = \cancel{h} + V_{0y} \cdot t - \frac{1}{2} g t^2 - h = 0$ $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-V_{0y} \pm \sqrt{V_{0y}^2 - 4(-\frac{1}{2}g)(-h)}}{2(-\frac{1}{2}g)}$

$= \frac{-V_{0y} \pm \sqrt{V_{0y}^2 - 2gh}}{-g}$

$t_1 = \frac{V_{0y} - \sqrt{V_{0y}^2 - 2gh}}{g}$ $\quad t_2 = \frac{V_{0y} + \sqrt{V_{0y}^2 - 2gh}}{g}$

(d) $V_y = V_{0y} - g t_2 = V_{0y} - (V_{0y} + \sqrt{V_{0y}^2 - 2gh})$

$= -\sqrt{V_{0y}^2 - 2gh}$

(e) x coordinates @ when athlete catches the ball is

$x = V_{0x} \cdot t_2$

$x = \frac{1}{2} a t_2^2$

so $V_{0x} \cdot \cancel{t_2} = \frac{1}{2} a t_2^{\cancel{2}}$

$a = \frac{2V_{0x}}{t_2} = \frac{2V_{0x} \cdot g}{V_{0y} + \sqrt{V_{0y}^2 - 2gh}}$

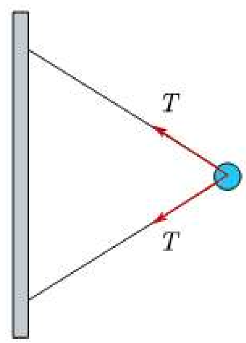
2. Spinny Thing

(a) $\sin 30^\circ = 1/2, \cos 30^\circ = \sqrt{3}/2$

(b) See the figure below. Apply Newton's 2nd law in the horizontal direction:

$$2T\cos(30^\circ) = \frac{mv^2}{L\cos(30^\circ)}$$

which gives $T = 2mv^2/3L$.



(c) At the critical point, the tension in the lower string is zero. This is the point that the lower string is just about to go slack. Then we apply Newton's 2nd law in the vertical direction:

$$T\sin(30^\circ) - mg = 0$$

So the tension in the upper string is $T = 2mg$. In the horizontal direction,

$$T\cos(30^\circ) = \frac{mv^2}{L\cos(30^\circ)}$$

Hence the minimum speed is $v = \sqrt{\frac{3}{2}gL}$.

(d) When both strings begin to go slack, the tension inside them is just zero. The only force acting on the sphere is gravity. Thus the gravity is equal to the centripetal force:

$$mg = \frac{mv^2}{L\cos(30^\circ)}$$

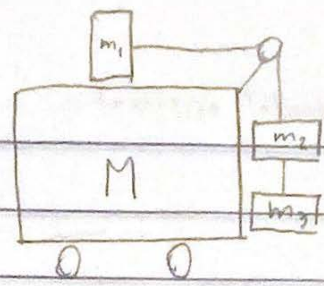
The minimum speed at the highest point is $v = \sqrt{\frac{\sqrt{3}}{2} gL}$

(e) When the speed is doubled, the centripetal force ($\propto v^2$) becomes four times as large as its old value, i.e., 4*mg*. The Newton's 2nd law in the vertical direction gives

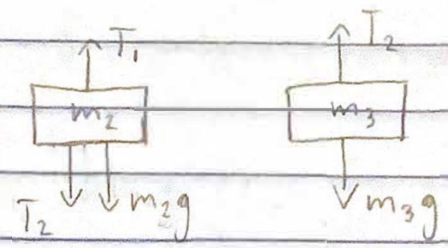
$$2T\cos(30^\circ) + mg = 4mg$$

The tension in each string is $T = \sqrt{3}\,mg$.

3. Many Moving Masses



a)

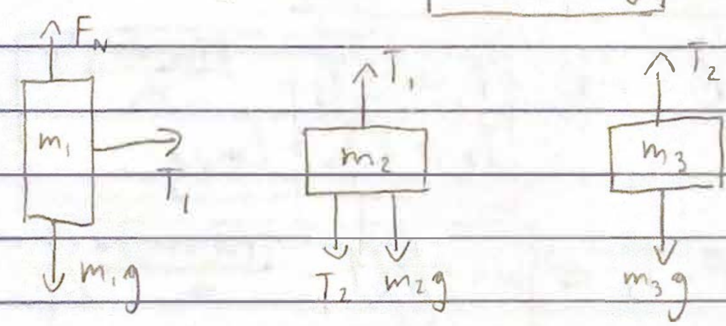


$a_2 = a_3 = 0$

$T_2 = \boxed{m_3 g}$

$T_1 = T_2 + m_2 g \Rightarrow T_1 = \boxed{(m_2 + m_3) g}$

b)



$a_1 = a_2 = a_3$

1) $m_1 a = T_1$

2) $m_2 a = T_2 + m_2 g - T_1$

3) $m_3 a = m_3 g - T_2 \Rightarrow T_2 = m_3 g - m_3 a$

4) $1 \to 2$: $m_2 a = T_2 + m_2 g - m_1 a$

5) $3 \to 4$: $(m_2 + m_1) a = m_3 g - m_3 a + m_2 g$

$$\boxed{a = \frac{(m_2 + m_3) g}{(m_1 + m_2 + m_3)}}$$

horizontal acceleration

c) $F = (M + m_1 + m_2 + m_3)\,a$

$m_1: m_1 a = T_1$

$\left.\begin{array}{l} m_2: m_2 g + T_2 = T_1 \\ m_3: m_3 g = T_2 \end{array}\right\}$ ~~vertical acceleration is 0~~

$m_1 a = (m_2 + m_3) g$

$a = \frac{(m_2 + m_3) g}{m_1}$

$F = \boxed{\frac{(M + m_1 + m_2 + m_3)(m_2 + m_3) g}{m_1}}$

d)

Let $F' = 2F$, where F is the force found in part (c). Let a be the horizontal acceleration of the cart (which is also the horizontal acceleration of m_2 and m_3 since they are being pushed by the cart). The forces acting on the cart of mass M are F' (to the right), as well as several reaction forces (to the left) according to Newton's third law. The reaction forces come from the normal forces which push m_2 and m_3 to the right, and the tension force T_1 which pulls m_1. We can then write

$$Ma = F' - T_1 - (m_2 + m_3)a.$$

Now that we have doubled the force, the small masses have some acceleration relative to the cart. Let us call this relative acceleration a'; m_1 moves left relative to the cart and the other two masses move upwards relative to the cart. To someone observing this system, the net acceleration of m_1 is then $a - a'$ (positive is to the right). According to Newton's second law we then have

$$m_1(a - a') = T_1.$$

The tension T_1 in the upper string must also pull m_2 and m_3 with this relative acceleration a'. Therefore we also have

$$(m_2 + m_3)a' = T_1 - (m_2 + m_3)g.$$

We can eliminate a' from the above two equations in order to find T_1 in terms of a:

$$\begin{aligned} T_1 &= \frac{m_1(m_2 + m_3)(a + g)}{m_1 + m_2 + m_3} \\ T_1 &= K(a + g) \end{aligned}$$

where we introduce the constant $K = m_1(m_2+m_3)/(m_1+m_2+m_3)$ to save some writing. Going back to the first equation, we have

$$\begin{aligned} Ma &= F' - K(a + g) - (m_2 + m_3)a. \\ (M + K + m_2 + m_3)a &= F' - Kg \\ a &= \frac{F' - Kg}{M + K + m_2 + m_3} \end{aligned}$$

e)

T_1, the tension in the upper string, can be found by substituting the expression for a in the equation $T_1 = K(a + g)$. T_2, the tension in the lower string, requires us to consider m_2 and m_3 separately. Applying Newton's second law to m_3 gives us

$$m_3a' = T_2 - m_3g$$

and applying it to m_2 gives us

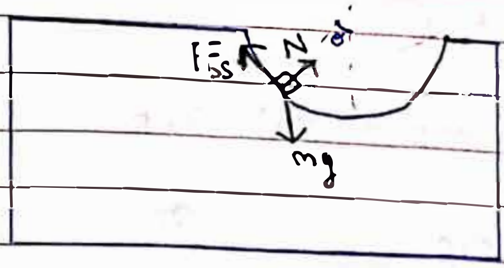
$$m_2 a' = T_1 - T_2 - m_2 g.$$

Eliminating a', we find

$$T_2 = \frac{m_3 T_1}{m_2 + m_3}.$$

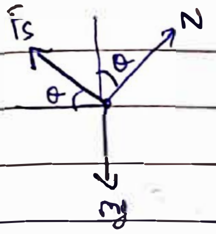
4. Half-pipe Block

4 a



N = Normal Force
mg = weight
F_s = Static Friction

b



$x: \; N \sin\theta = F_s \cos\theta$ (1)
$y: \; N \cos\theta + F_s \sin\theta = mg$ (2)

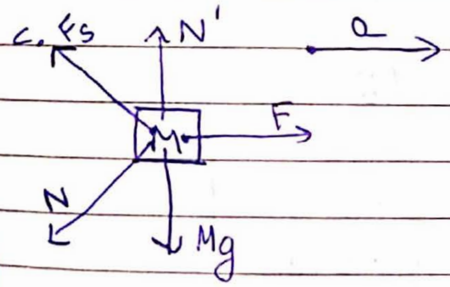
$F_s \leq \mu_s N$ (3) $\mu_s N$ = max static friction

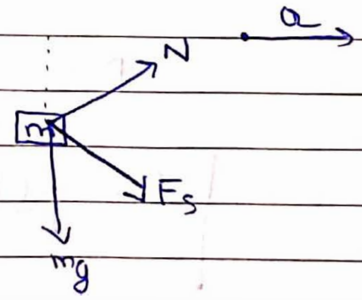
Using (1) + (3)

$$\sin\theta = \mu_s \cos\theta$$

$$\therefore \boxed{\theta = \tan^{-1} \mu_s}$$

c.





N = Normal b/w the 2 blocks
N' = Normal b/w M & floor
Fs acts on M as well

Fs switches direction to prevent it from slipping on top.

d. Variable force here is $F_s \leq \mu_s N$
$\therefore$ Max F occurs at max F_s

M

$x: \; F - F_s\cos\theta - N\sin\theta = Ma$ (1)
$y: \; Mg + N\cos\theta = N' + F_s\sin\theta$ (2)

m

$x: \; N\sin\theta + F_s\cos\theta = ma$ (3)
$y: \; N\cos\theta = mg + F_s\sin\theta$ (4)

$F_s = \mu_s N$ (5)
$F = (M+m)a$ (6)

classmate

Date

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(4) + (5)

$$N\cos\theta = mg + \mu_s N \sin\theta$$

$$N = \frac{mg}{(\cos\theta - \mu_s \sin\theta)} \quad (7)$$

(3) + (5)

$$N\sin\theta + \mu_s N\cos\theta = ma$$

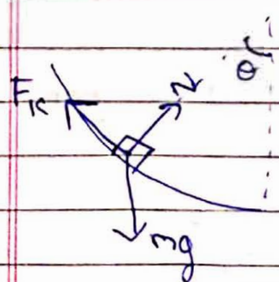
$$\frac{N}{m}(\sin\theta + \mu_s\cos\theta) = a \quad (8)$$

Put (7) in (8)

$$a = \frac{g(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} \quad (9)$$

Putting (9) in (6)

$$\boxed{F = \frac{(M+m)g(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}}$$

c. 

F_K = Kinetic Friction

Radial: $\boxed{a_r = V^2/R}$ (1)

$$N - mg\cos\theta = \frac{mv^2}{R} \quad (2)$$

$$\therefore N = mg\cos\theta + ma_r$$

$$mg\sin\theta - \mu_K N = ma_t \quad (3)$$

(3) + (2)

$$a_t = \boxed{g\sin\theta - \mu_K g\cos\theta - \frac{V^2}{R}\mu_K}$$