Q1 Honor Code

0 Points

Handwrite the following text on a blank sheet of paper, sign and date it, and upload a scan/photo of it.

"As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. All of the work submitted here is mine, and I have not discussed the content of this exam with any other person. I have not consulted any resources beyond the course materials and my notes during the exam. I have not used calculators, computers, or the internet to do calculations or look up answers."

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Instructions For the remainder of the questions, you must ***neatly handwrite*** each answer on a separate piece of paper and upload a ***separate*** scan/photo for each question. This means you will need *at least ten* sheets of paper (one for the cheat sheet, one each for 4.1-4.4, and one each for Q5-Q9). Feel free to use more than one sheet for an answer if needed. Note that both PDF and picture/photo format (JPG) are accepted, so use whichever is most convenient. After uploading, be sure to check that your file uploaded correctly. Do not wait until the last minute to upload your files.

If you have a question during the exam, send me or one of the GSIs a *private* message on Piazza, and we will respond. If there is a clarification which may be useful to everyone, we will post it on Piazza.

Please show all of your work. When citing a theorem, it is best to simply write the statement of the theorem. Good luck!

Q2 Cheat Sheet

10 Points

Attach your handwritten 1-3 page cheat sheet below.
(update 8:26pm 2/17: in a previous announcement I said "handwritten or typed", so if you already typed it that's also acceptable. If not, handwritten is preferred.)

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Q3 True or False

20 Points

Select True (i.e., always true) or False (i.e., sometimes false) for each statement. No need to provide an explanation. All matrices mentioned are real unless noted otherwise.

Q3.1

2 Points

If the linear system $Ax = 0$ has at least one solution then $Ax = b$ must have at least one solution.

Q3.2

2 Points

If the linear system $Ax = 0$ has at most one solution then $Ax = b$ must have at most one
solution.

Q3.3

2 Points

If $x, y, z \in \mathbb{R}^3$ are vectors such that $\{x, y\}$ are linearly independent and $\{y, z\}$ are linearly independent, then $\{x, y, z\}$ must be linearly independent.

Q3.4

2 Points

If $v_1, v_2 \in \mathbb{R}^3$ are linearly independent vectors and $v_3 \notin span\{v_1, v_2\}$ then $\{v_1, v_2, v_3\}$ must be linearly independent.

(clarification 2:21pm 2/18: you may assume $v_3 \in \mathbb{R}^3$ above)

Q3.5

2 Points

If R is the reduced row echelon form of an $m \times n$ matrix A and $Ax = b$ is consistent for some vector $b \in \mathbb{R}^m$, then $Rx = b$ must also be consistent.

Q3.6

2 Points

If A and B are $n \times n$ matrices then $\det(A+B) = \det(A) + \det(B)$.

Q3.7

2 Points

If A is a square matrix such that A^2 is invertible, then A must be invertible.

Q3.8

2 Points

If b_1, b_2, b_3 is a basis of $\mathbb{R}^3$ and A, B are 4×3 matrices satisfying $Ab_i = Bb_i$ for $i = 1, 2, 3$, then $A = B$.

Q3.9

2 Points

If H is a subspace of $\mathbb{R}^5$, $v_1, \ldots v_4 \in H$, and $v_1, \ldots, v_4$ are linearly independent then $\dim(H) \geq 4$.

Q3.10

2 Points

If H is a subspace of $\mathbb{R}^5$ and $span\{v_1, \ldots v_4\} = H$ then $\dim(H) \geq 4$.

Q4 Examples

20 Points

Give an example of each of the following, explaining why it has the required property,
or explain why no such example exists.

Q4.1 Linear Systems

5 Points

Two vectors $b_1, b_2 \in \mathbb{R}^3$ and a 3×3 matrix A such that the linear system $Ax = b_1$ has exactly one solution and the linear system $Ax = b_2$ is inconsistent.

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Q4.2 PEMDAS

5 Points

Two nonzero 2×2 matrices A and B such that

$$(A+B)^2 = A^2 + B^2.$$

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Q4.3 Onto linear transformation

5 Points

An onto linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that

$$T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and

$$T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ -2 \end{bmatrix}.$$

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Q4.4 Nullspace

5 Points

A 2×4 matrix A such that $Nul(A)$ has dimension equal to 3.

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Q5 Outside Span

10 Points

Consider the vectors

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, v_4 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \in \mathbb{R}^3.$$

Find the **first** vector in this list which is **not** in the span of the other vectors. Explain your reasoning.

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(clarification 9:33pm 2/17: "first" means the v_i with the lowest index $i = 1, 2, 3, 4$, and "other" means all of the vectors besides v_i.)

Q6 Inverse

10 Points

Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}.$$

(a) Is A invertible? If so, compute its inverse. If not, explain why.
(b) Find a solution $x \in \mathbb{R}^3$ to the linear system

$$Ax = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

(c) Is the solution you found unique? Explain why or why not.

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Q7 Determinant

8 Points

Find the determinant of the matrix

$$A = \begin{bmatrix} 0 & 2 & 3 & 4 & 5 \\ 1 & 0 & 3 & 4 & 5 \\ -1 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}.$$

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Q8 Both Subspaces

10 Points

Consider the matrices

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ -1 & 2 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -1 & -3 & 1 \end{bmatrix}.$$

(a) Find a nonzero vector $v \in \mathbb{R}^3$ which is an element of **both** of the subspaces $Nul(A) \subseteq \mathbb{R}^3$ and $Col(B) \subseteq \mathbb{R}^3$ (i.e., $v \in Nul(A) \cap Col(B)$). Explain your reasoning.

(b) Are the columns of the product AB linearly independent? Explain why or why not based on your answer to (a), without doing any matrix multiplication.

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(update 8:42pm 2/17: the entry $A(2,3)$ was mistakenly -4, which was a typo. It is now fixed to -2).

Q9 Rotation and Reflection

12 Points

Let $T_{\pi/6} : \mathbb{R}^2 \to \mathbb{R}^2$ denote the linear transformation which rotates a vector in $\mathbb{R}^2$ counterclockwise by $\pi/6$ radians. Let $T_{ref} : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which reflects a vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ across the line $x_1 = x_2$.

(a) Sketch a cartoon illustrating what these linear transformations do to the vector $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(b) Find the standard matrices of $T_{\pi/6}$ and T_{ref}.

(c) Find a nonzero vector $v \in \mathbb{R}^2$ such that

$$T_{\pi/6} \circ T_{ref}(v) = T_{ref} \circ T_{\pi/6}(v)$$

or explain why no such vector exists.

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(clarification 2:21pm 2/18: for part (a), your sketch should show what each transformation by itself does to the vector e_1)