University of California, Berkeley
Department of EECS
# EE120: SIGNALS AND SYSTEMS (Spring 2021)
## Midterm 1

**Issued:** 12:15 PM, February 22, 2021 **Due:** 1:45 PM, February 22, 2021

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| **Full Name:** | **SID:** |
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**Berkeley Honor Code:** "As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others."
Your copy:

**Problem 1: (40 pt) Period of signals.** Determine the period of the following continuous time ($x(t)$) or discrete time ($x[n]$) signals. For periodic signals, fill the blank with **fundamental period**, for aperiodic signals, fill the blank with "**N/A**". No proof is needed, nor will it be graded. Use the provided space as draft space.

**Answer:**
For example, for $x(t) = \sin(t)$, the table is:

| signal | fundamental period |
|---|---|
| $x(t) = \sin(t)$ | $2\pi$ |

| signal | fundamental period | signal | fundamental period |
|---|---|---|---|
| $x(t) = \sin(3t - 1)$ | | $x[n] = \sin[3n - 1]$ | |
| $x(t) = \sin(\frac{\pi}{3}t^2 - 1)$ | | $x[n] = \sin[\frac{\pi}{3}n^2 - 1]$ | |
| $x(t) = \sin^2(\frac{\pi}{3}t - 1)$ | | $x[n] = e^{j\frac{\pi}{8}n}\cos[\frac{\pi}{3}n]$ | |
| $x(t) = e^{j(\pi t - 1)}$ | | $x[n] = \cos[\pi + 0.2n]$ | |
| $x(t) = e^{t(\pi j - 1)}$ | | $x[n] = \sum_{k=-\infty}^{\infty} e^{-(2n-k)}u[2n - k]$ | |

**Problem 2: (48 pt) System properties.** Fill in the table for the following systems described by input-output relationships. Specify if the system is 1) linear or not, 2) time-invariant or not, 3) memoryless or not, 4) causal or not, 5) stable or not. *If you identify the system as LTI*, determine the impulse response function. If you identify the system as non-LTI, put "N/A" for the impulse response. For each system, $x(t)$ or $x[n]$ is the input, $y(t)$ or $y[n]$ is the output.

Use "✓" to indicate **yes**, use "$\times$" to indicate **no**, "N/A" for **not applicable**. No proof is needed, nor will it be graded. Use the provided space as draft space.

1. $y(t) = x(t+3) - x(1-t)$
2. $y(t) = x(2t)$
3. $y(t) = \cos(2x(t))$
4. $y(t) = \int_{-\infty}^{t} x(u)e^{-(t-u)}du$
5. $y[n] = \sum_{k=n}^{\infty} x[k]$
6. $y[n] = \begin{cases} (-1)^n x[n] & x[n] \geq 0 \\ 2x[n] & x[n] < 0 \end{cases}$
7. $y[n] = \max\{x[n], x[n+1], \ldots, x[\infty]\}$
8. $y[n] = x[n]x[n-1]$

**Answer:**

For example, for an LTI, not memoryless, non-causal, and stable system $y(t) = x(t+2)$, the table is:

| system | linear | time invariant | memoryless | causal | stable | impulse response h(t) or h[n] |
|---|---|---|---|---|---|---|
| e.g. | ✓ | ✓ | $\times$ | $\times$ | ✓ | $h(t) = \delta(t+2)$ |

| system | linear | time invariant | memoryless | causal | stable | impulse response h(t) or h[n] |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | | | | |

**Problem 3: (80 pts) Multiple choices.** Circle your choice. No proof is needed, nor will it be graded. Use the provided space as draft space.

1. What is the magnitude of $(j-1)e^{j-1}$?

   (a) $\frac{\sqrt{2}}{e}$

   (b) $j-1$

   (c) $1$

   (d) $2$

2. What is the phase of $e^{j-1}$?

   (a) $-1$

   (b) $j$

   (c) $1$

   (d) $\sqrt{2}$

3. A memoryless system must also be

   (a) not sure, because memoryless does not imply other properties of this system

   (b) causal

   (c) stable

   (d) noncausal

4. Which of the following systems is stable?

   (a) $y(t) = \log(x(t))$

   (b) $y(t) = \exp(x(t))$

   (c) an LTI system with impulse response $h(t) = \sin(t)$

   (d) $y(t) = tx(t) + 1$

5. Is the LTI system with impulse response $h(t) = \exp(-t)u(t)$ stable?

   (a) Yes

   (b) No

   (c) It depends on the input.

6. Compute the convolution $(\exp(-at)u(t)) * u(t))$, where $u(t)$ is the unit step function.

   (a) $(\frac{1}{a} - \frac{1}{a}\exp(at))u(-t)$

(b) $(\frac{1}{a} - \frac{1}{a}\exp(-at))u(-t)$

(c) $(\frac{1}{a} - \frac{1}{a}\exp(at))u(t)$

(d) $(\frac{1}{a} - \frac{1}{a}\exp(-at))u(t)$

7. Compute the convolution $h[n] * \delta[n-5]$.

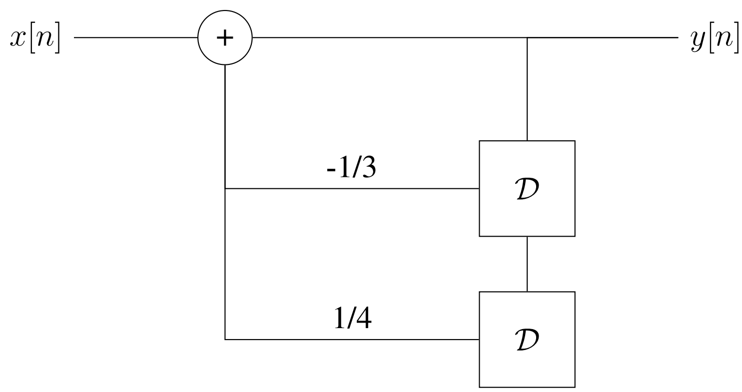
   (a) $h[n+5]$

   (b) $h[n-5]$

   (c) $h[5]$

   (d) $h[-5]$

8. Write an LCCDE characterizing the following system:




   (a) $y[n] = x[n] - \frac{1}{3}x[n-1] + \frac{1}{4}x[n-1]$

   (b) $y[n] = x[n] - \frac{1}{3}x[n-1] + \frac{1}{4}x[n-2]$

   (c) $y[n] = x[n] - \frac{1}{3}y[n-1] + \frac{1}{4}y[n-1]$

   (d) $y[n] = x[n] - \frac{1}{3}y[n-1] + \frac{1}{4}y[n-2]$

9. What's the Fourier transform of $\text{rect}(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$?

   (a) $\sin(\pi f)$

   (b) $\sin(\omega)$

   (c) $\frac{\sin(\omega)}{\omega}$

   (d) $\frac{\sin(\pi f)}{\pi f}$

10. What's the Fourier transform of $\cos(\omega_0 t + \phi)$?

   (a) $\sin(\omega t + \phi)$

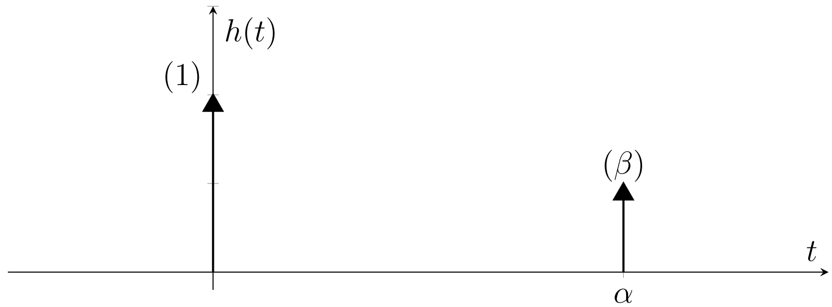
   (b) $\cos(\pi f)$

   (c) $\pi e^{i\phi}\delta(\omega - \omega_0) + \pi e^{-i\phi}\delta(\omega + \omega_0)$

   (d) $\pi e^{i\phi}(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$

**Problem 4: (72 pt) Echoed Signal.** You're recording sound in a room with an echo, so your microphone picks up both the original signal and a delayed, attenuated version of the signal. You decide to model this process as an LTI system with impulse response

$$h(t) = \delta(t) + \beta\delta(t - \alpha).$$




In this problem, we will examine the behavior of this system in the frequency domain.

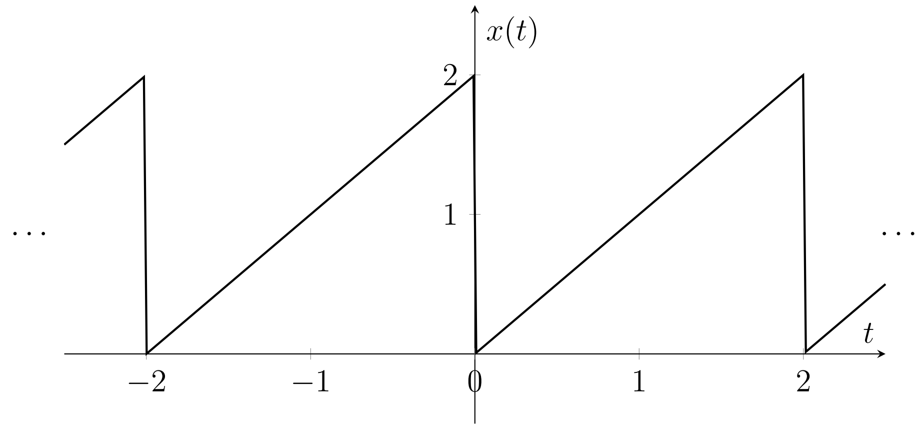
1. (10 pt) Find the Fourier Transform of the impulse response, $H(\omega)$.

**Answer:**

2. (12 pt) For $\alpha = 2$ and $\beta = 1$, find expressions for and plot the magnitude and phase of $H(\omega)$. Your expressions should be closed-form, but piecewise expressions are allowed.

**Answer:**

3. (12 pt) Let the input signal, $x(t)$ be a triangle wave, as pictured below. **Note that all subsequent parts of this question will use this definition of** $x(t)$.




$x(t) = t, \forall t \in [0, 2]$, and $x(t)$ repeats with a period of 2.
Find the CTFS coefficients $a_k$ in the complex exponential representation of $x(t)$.

**Answer:**

4. (12 pt) Show that, if the CTFS coefficients of an arbitrary signal $g(t)$ are represented by $b_k$ in complex exponential representation, then the CTFT of $g(t)$ is

$$G(\omega) = A \sum_k b_k \delta(\omega - k\omega_0),$$

and find a value for $A$.

**Answer:**

5. (12 pt) Find the CTFT $X(\omega)$ of the input signal $x(t)$ defined in part 3. Find an expression for and plot (with carefully labeled ticks on both axes) the magnitude $|X(\omega)|$ in the range $\omega \in [-4\pi, 4\pi]$. Leave your expressions in summation form.
*Hint: the magnitude of a series of delta functions with different shifts can be written as*

$$\left| \sum_i A_i \delta(t - T_i) \right| = \sum_i |A_i| \delta(t - T_i).$$

**Answer:**

6. (14 pt) If $y(t) = x(t) * h(t)$, find the CTFT of $y(t)$, $Y(\omega)$.
Find an expression for and plot the magnitude $|Y(\omega)|$ in the interval $\omega \in [-4\pi, 4\pi]$ for the following values of $\alpha$ and $\beta$:

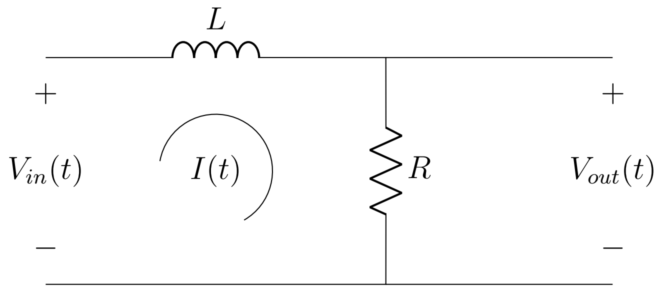
(a) $\alpha = 3$, $\beta = 1/2$.

(b) $\alpha = 2$, $\beta = 1/2$.

*Hint: Convolution in time corresponds to multiplication in frequency.*

**Answer:**

**Problem 5: (60 pt) RL circuits.**
Consider the following RL circuit




The system can be represented by the equation

$$RI(t) + L\frac{dI(t)}{dt} = V_{in}(t).$$

$V_{in}(t)$ is the input, and the voltage across the resistor is the output, $V_{out}(t) = V_R(t) = RI(t)$. The system is causal and starts at rest.

1. (14 pt) Find the impulse response $h(t)$ for the voltage across the resistor. Assume the current through the resistor is of the form $Ae^{st}u(t)$.

**Answer:**

**Answer:**
(Continued)

2. (12 pt) Assume the system is at rest, and at $t = 1$ s and $t = 1.1$ s, it receives two sudden impulse shocks from $V_{in}$. This can be modeled as two impulse inputs as $V_{in}(t) = \delta(t - 1) + \delta(t - 1.1)$. Calculate $V_{out}(t)$ for input $V_{in}(t) = \delta(t - 1) + \delta(t - 1.1)$ and sketch $V_{out}(t)$.
   For sketching, you can use the following set of parameters: $R = 1$ m$\Omega$, and $L = 100$ uH, label the location and heights of peaks.

**Answer:**

3. (10 pt) Find the frequency response $H(\omega)$ for the voltage on the resistor.

**Answer:**

4. (8 pt) Sketch a well-labeled magnitude plot of the frequency response.

**Answer:**

5. (16 pt) Find the output when the input is $V_{in}(t) = e^{-t}u(t)$.

**Answer:**