# EECS 16A    Designing Information Devices and Systems I
## Fall 2019                                        Final Exam

Exam Location: Special Proctoring

PRINT your student ID: _____

PRINT AND SIGN your name: _____, _____ _____
                               (last name)              (first name)              (signature)

PRINT time of your Monday section and the GSI's name: _____

PRINT time of your Wednesday section and the GSI's name: _____

Name and SID of the person to your left: _____

Name and SID of the person to your right: _____

Name and SID of the person in front of you: _____

Name and SID of the person behind you: _____

1. **Tell us about something you are proud of this semester. (2 Points)**

2. **What are you looking forward to over winter break? (2 Points)**

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

PRINT your student ID: _____

Extra page for scratchwork.
**Work on this page will NOT be graded.**

PRINT your student ID: _____

3. **Where is the sound coming from? (14 points)** (All subparts of this problem can be solved independently.)

In this problem we will use concepts from the class to determine the angle and position of an incoming audio signal recorded by microphones. All of the microphones and transmitting sources are in the same plane (i.e. the problem is in 2D).

(a) (4 points) A transmitter sends the signal $\vec{t}$. This signal is received by microphone 1 as $\vec{r}_1$ and by microphone 2 as $\vec{r}_2$. The microphones record 1 sample every millisecond ($1ms = 10^{-3}$ s) and the speed of sound is $v_s = 300m/s$. **How far are microphone 1 and microphone 2 from the transmitter?**

*Hint: You do not have to do cross-correlation to solve this question.*
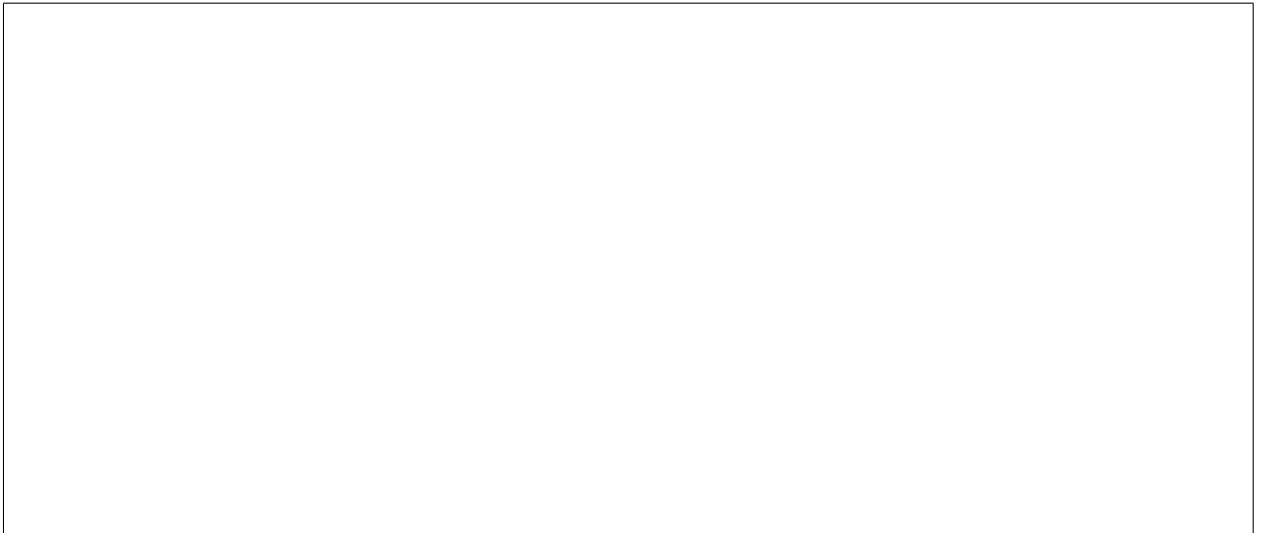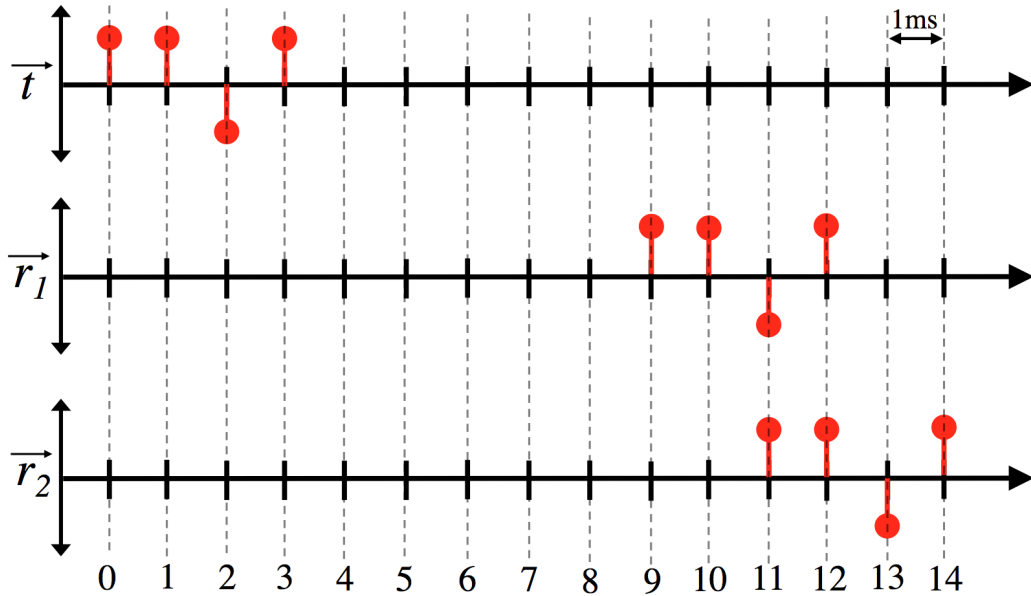
PRINT your student ID: _____

(b) (3 points) Microphone 1 and Microphone 2 receive signals $\vec{r}_1$ and $\vec{r}_2$ respectively from the transmitter. The time delay between $\vec{r}_1$ and $\vec{r}_2$ is related to the perpendicular distance $d$ between Microphone 2 and the incoming signal $\vec{t}$. **You have measured that** $d = 1m$ (i.e. the distance between Microphone 2 and the incoming signal $\vec{t}$) in the figure below. The positions $\vec{p}_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $\vec{p}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ of Microphones 1 and 2, respectively, are shown below. The units are in meters.

**What is the angle of arrival $\alpha$ (see the figure below) between the incoming signal $\vec{t}$ and the line joining the microphones?** You may leave your answer in terms of a trigonometric function.
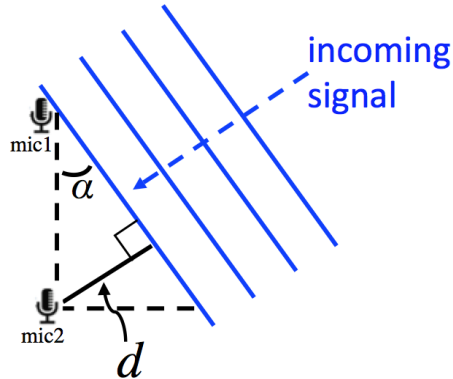
PRINT your student ID: _____

(c) (7 points) **Now you are considering a different setup of the microphones from earlier parts.** You have placed four microphones at $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$, and $\vec{p}_4$ in the plane (see figure). You want to determine the position of the transmitting source. You used cross correlation and determined that the distances $d_i$ between the microphones and the transmitting source as follows:

$\vec{p_1} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$, $\quad d_1 = 1\text{m}$

$\vec{p_2} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $\quad d_2 = \sqrt{5}\text{m}$

$\vec{p_3} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\quad d_3 = \sqrt{10}\text{m}$

$\vec{p_4} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\quad d_4 = \sqrt{17}\text{m}$
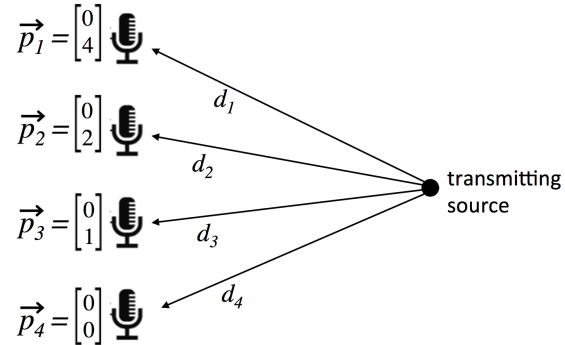


Figure 3.1: Note the microphone schematic is not drawn to scale.

**Set up system of linear equations to compute the location of the transmitting source. If you can solve the system to identify the location of the source, solve it. If you cannot identify the location of the source, explain why. Then, propose a design/setup that would make the problem solvable, and explain why your design works.**

PRINT your student ID: _____
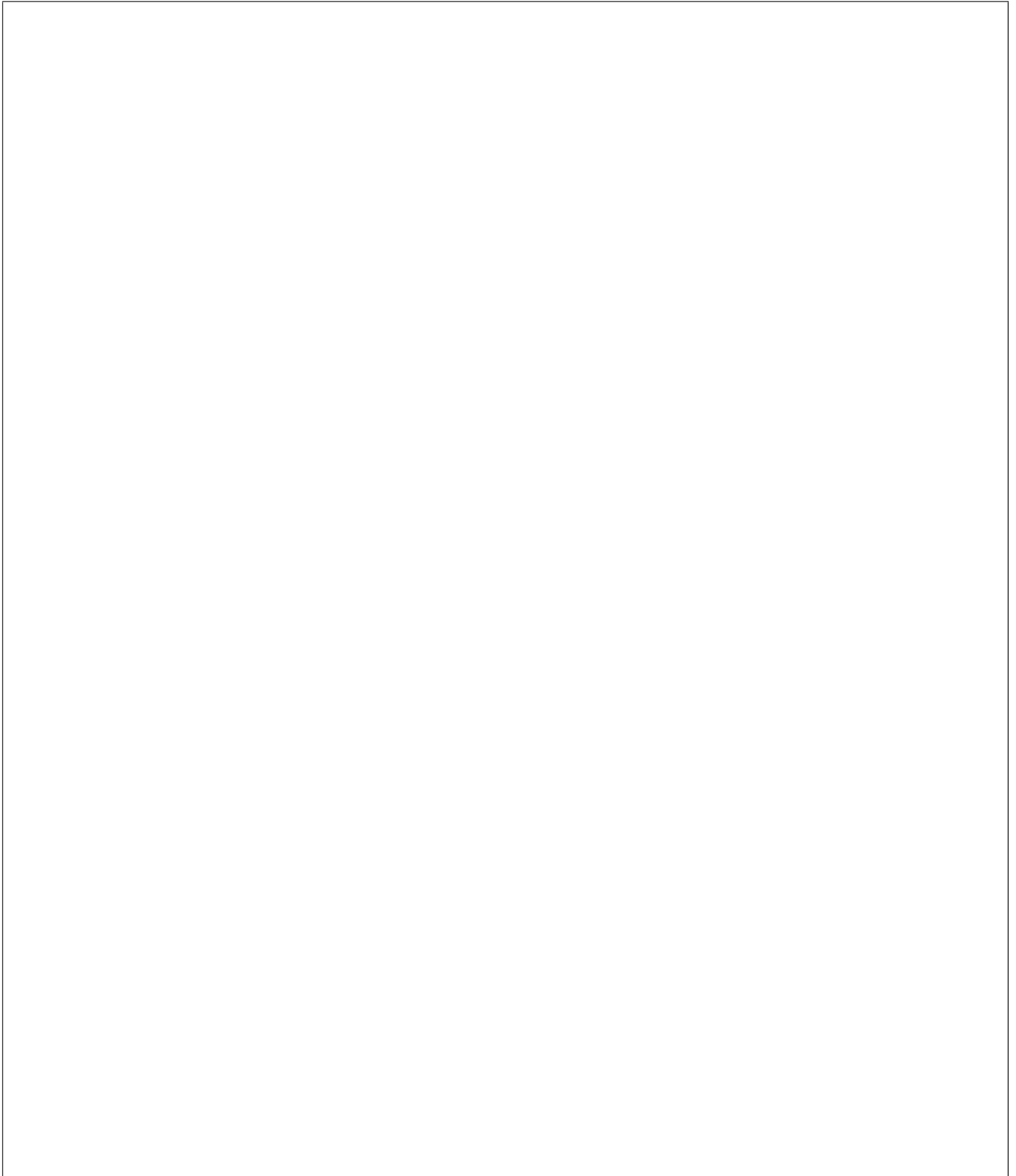
PRINT your student ID: _____

## 4. Building a classifier (22 points)

We used least squares to classify data in the "Labeling Patients Using Gene Expression Data" homework problem. We would like to now develop a classifier to classify points based on their distance from the origin.

You are presented with the following data. Each data point $\vec{d_i}^T = [x_i \; y_i]^T$ has the corresponding label $l_i \in \{-1, 1\}$.

| $x_i$ | $y_i$ | $l_i$ |
|-------|-------|-------|
| -2 | 1 | -1 |
| -1 | 1 | 1 |
| 1 | 1 | 1 |
| 2 | 1 | -1 |

Labels for data you are classifying

(a) (6 points) You want to build a model to understand the data. You first consider a linear model, i.e. you want to find $\alpha, \beta, \gamma \in \mathbb{R}$ such that $l_i \approx \alpha x_i + \beta y_i + \gamma$.
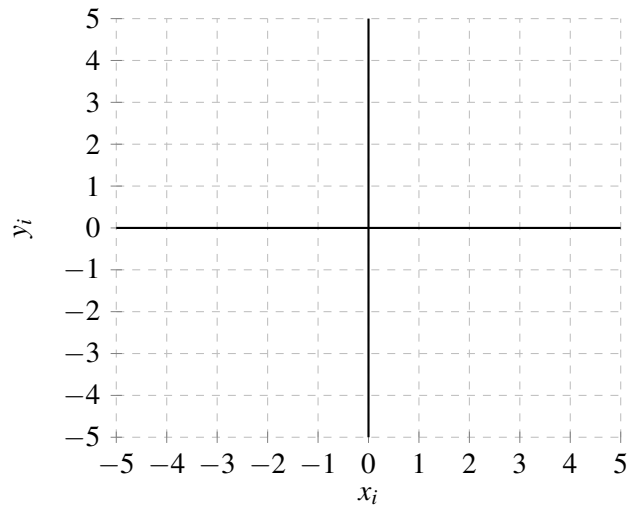
**Set up a least squares problem to solve for $\alpha$, $\beta$ and $\gamma$. If this problem is solvable, solve it, i.e. find the best values for $\alpha, \beta, \gamma$. If it is not solvable, justify why.**

PRINT your student ID: _____

(b) (3 points) **Plot** the data points in the plot below with axes $(x_i, y_i)$. **Is there a straight line such that the data points with a $+1$ label are on one side and data points with a $-1$ label are on the other side? Answer yes or no, and if yes, draw the line.**

| $x_i$ | $y_i$ | $l_i$ |
|---|---|---|
| -2 | 1 | -1 |
| -1 | 1 | 1 |
| 1 | 1 | 1 |
| 2 | 1 | -1 |

Table repeated for your convenience: Labels for data you are classifying

PRINT your student ID: _____

(c) (6 points) You now consider a model with a quadratic term: $l_i \approx \alpha x_i + \beta x_i^2$ with $\alpha, \beta \in \mathbb{R}$. *Read the equation carefully!*

**Set up a least squares problem to fit the model to the data. If this problem is solvable, solve it, i.e, find the best values for $\alpha, \beta$. If it is not solvable, justify why.**

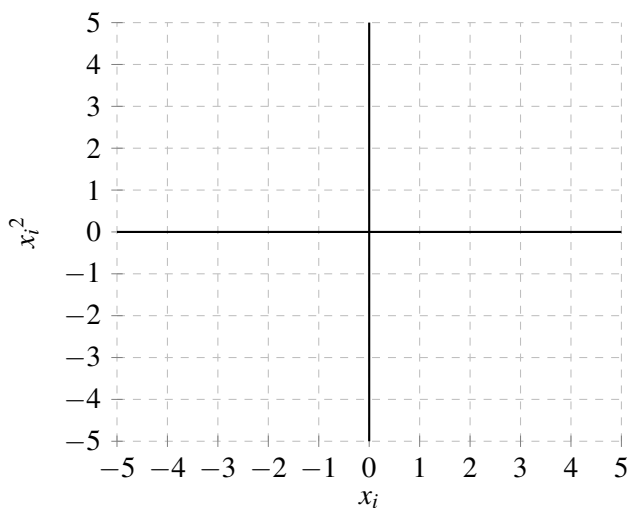| $x_i$ | $y_i$ | $l_i$ |
|-------|-------|-------|
| -2    | 1     | -1    |
| -1    | 1     | 1     |
| 1     | 1     | 1     |
| 2     | 1     | -1    |

Table repeated for your convenience: Labels for data you are classifying

PRINT your student ID: _____

(d) (3 points) **Plot** the data points in the plot below with axes $(x_i, x_i^2)$. **Is there a straight line such that the data points with a $+1$ label are on one side and data points with a $-1$ label are on the other side? Answer yes or no, and if yes, draw the line.**

| $x_i$ | $y_i$ | $l_i$ |
|-------|-------|-------|
| -2 | 1 | -1 |
| -1 | 1 | 1 |
| 1 | 1 | 1 |
| 2 | 1 | -1 |

Table repeated for your convenience: Labels for data you are classifying

PRINT your student ID: _____

(e) (4 points) Finally you consider the model: $l_i \approx \alpha x_i + \beta x_i^2 + \gamma$, where $\alpha, \beta, \gamma \in \mathbb{R}$. Independent of the work you have done so far, **would you expect this model or the model in part (c) (i.e. $l_i \approx \alpha x_i + \beta x_i^2$)** **to have a smaller error in fitting the data? Explain why.**

| $x_i$ | $y_i$ | $l_i$ |
|-------|-------|-------|
| -2    | 1     | -1    |
| -1    | 1     | 1     |
| 1     | 1     | 1     |
| 2     | 1     | -1    |

Table repeated for your convenience: Labels for data you are classifying

PRINT your student ID: _____

5. **Putting on the Pressure: Build your own InstantPot (27 points)** (All subparts of this problem can be solved independently.)

Prof. Ranade tells Prof. Boser about her great experience with her automatic pressure cooker, and they decide to try and build one together. The design of the pressure cooker uses a pressure sensor and heating element. Whenever the pressure is below a set target value, an electronic circuit turns on the heating element.

**Pressure Sensor Resistance**

The first step is designing a pressure sensor; the figure below shows your design. As pressure $p_c$ is applied, the flexible membrane stretches.
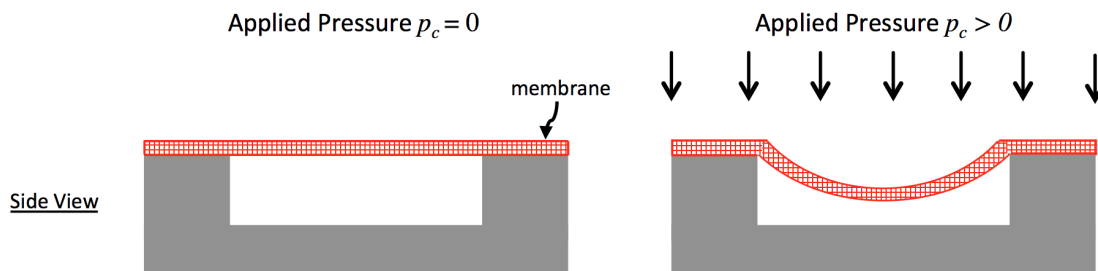
PRINT your student ID: _____

(a) (4 points) Now you attach a resistor layer $R_p$ with resistivity $\rho = 0.1\ \Omega m$, width $W$, length $L$, and thickness $t$ to the pressure sensor membrane, as illustrated in the figure below. When the pressure $p_c = 0kPa$ (i.e. there is no applied pressure), $W = 1mm$, $L = 1cm$, $t = 100\mu m = 100 \times 10^{-6}m$. **Calculate the value of $R_p$ when there is no applied pressure.** Note that direction of current flow in the resistor is from A to B as marked in the diagram. Show all your work.
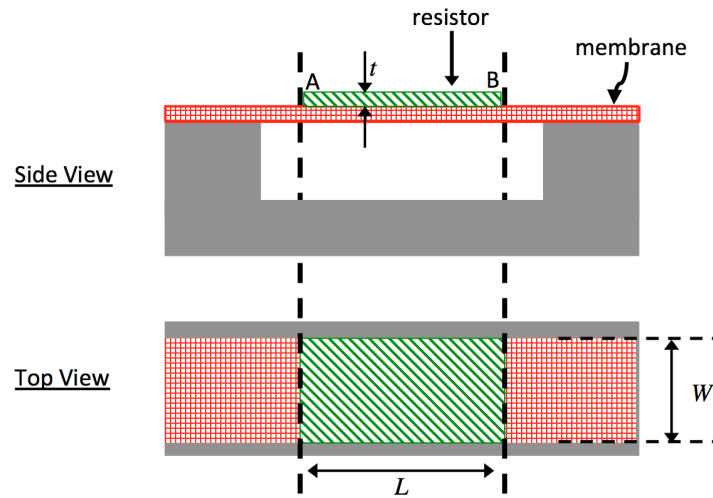
PRINT your student ID: _____

(b) (5 points) When pressure is applied, the length $L$ of the resistor changes and is a function of applied pressure $p_c$, and is given by

$$L(p_c) = L_0 + \beta p_c,$$

where $L_0$ is the length of the resistor with no pressure applied, and $\beta$ is a constant. As a result, the value of resistance $R_p$ changes from its nominal value $R_{p0}$ (the value of $R_p$ with no pressure applied) **Derive an expression for $R_p(p_c)$ as a function of resistivity $\rho$, width $W$, thickness $t$, nominal length $L_0$, constant $\beta$, and applied pressure $p_c$.**

*Note:* The width and thickness of the resistor will also change with applied pressure. However, we ignore this to keep the math simple.

**Extra space for scratch work.**

PRINT your student ID: _____

(c) **Pressure Sensor Circuit** (6 points)

Now assume that the resistance $R_p$ is a function of applied pressure $p_c$ according to the relationship $R_p(p_c) = R_o \times \frac{p_c}{p_{ref}}$ where $R_o = 1\text{k}\Omega$ and $p_{ref} = 100\text{kPa}$.

To complete our sensor circuit, we would like to generate a voltage $V_p$ that is a function of the pressure $p_c$. **Complete the circuit below so that the output voltage $V_p$ depends on the pressure $p_c$ as:**

$$V_p(p_c) = -V_o \times \frac{p_c}{p_{ref}}, \text{ where } V_o = 1V.$$

- **You may add at most one ideal voltage source and one additional resistor to the circuit, but you must calculate their values and mark them in the diagram.**
- **Mark the positive and negative inputs of the operational amplifier with "+" and "-" symbols, respectively, in the boxes provided.**

You may assume that the operational amplifier is ideal.
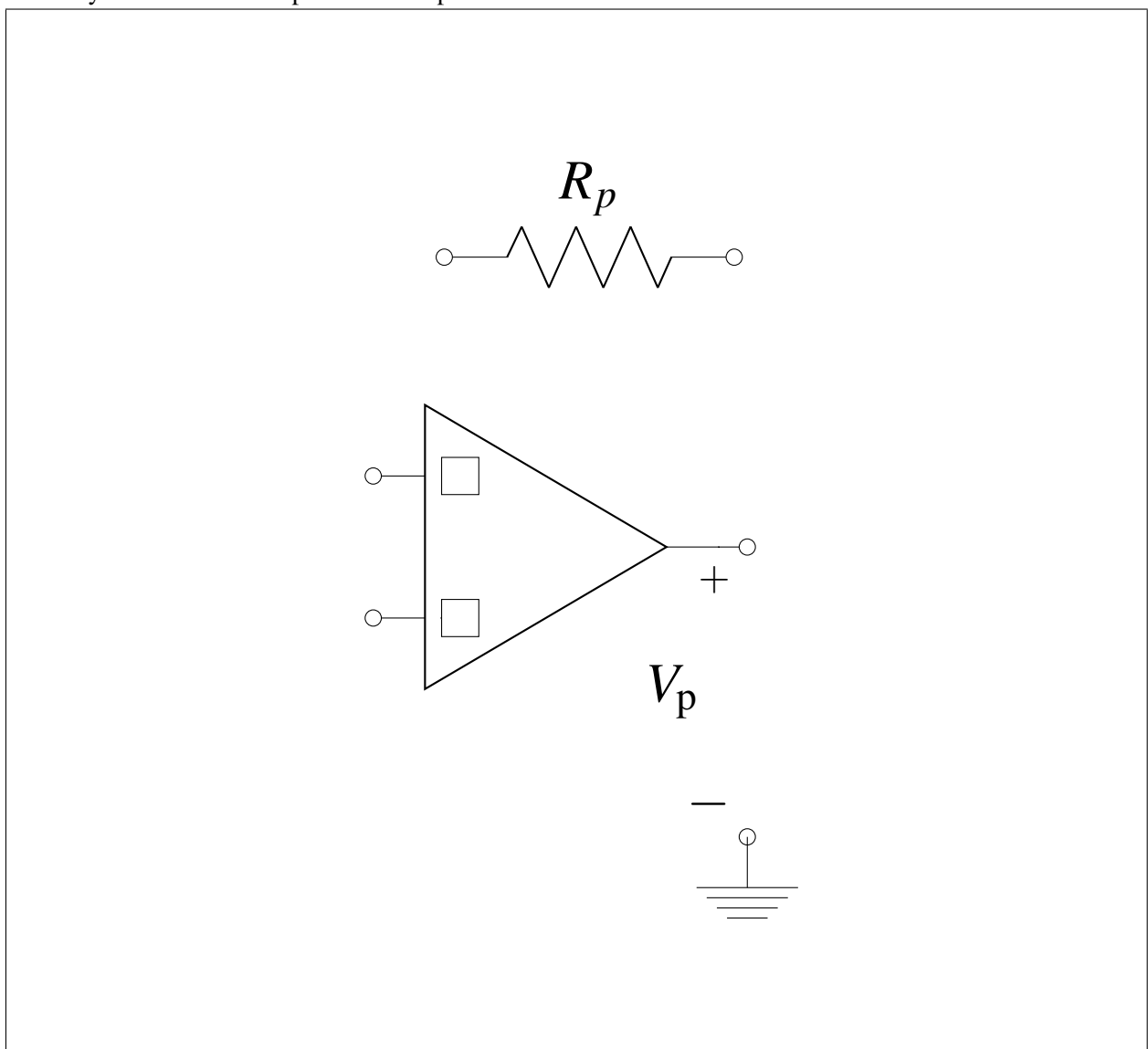
PRINT your student ID: _____

(d) **Resistive Heating Element** (4 points)

To heat the pressure cooker, you use a heating element with resistance $R_{heat}$. **Calculate the value of $R_{heat}$ such that the power dissipated is $P_{heat} = 1000W$ with $V_{heat} = 100V$ applied across the heating element.**

PRINT your student ID: _____

Extra page for scratchwork.
**Work on this page will NOT be graded.**

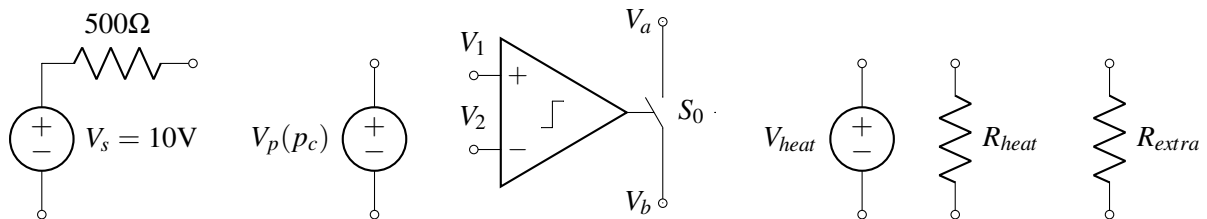PRINT your student ID: _____

(e) **Pressure Regulation** (8 points)

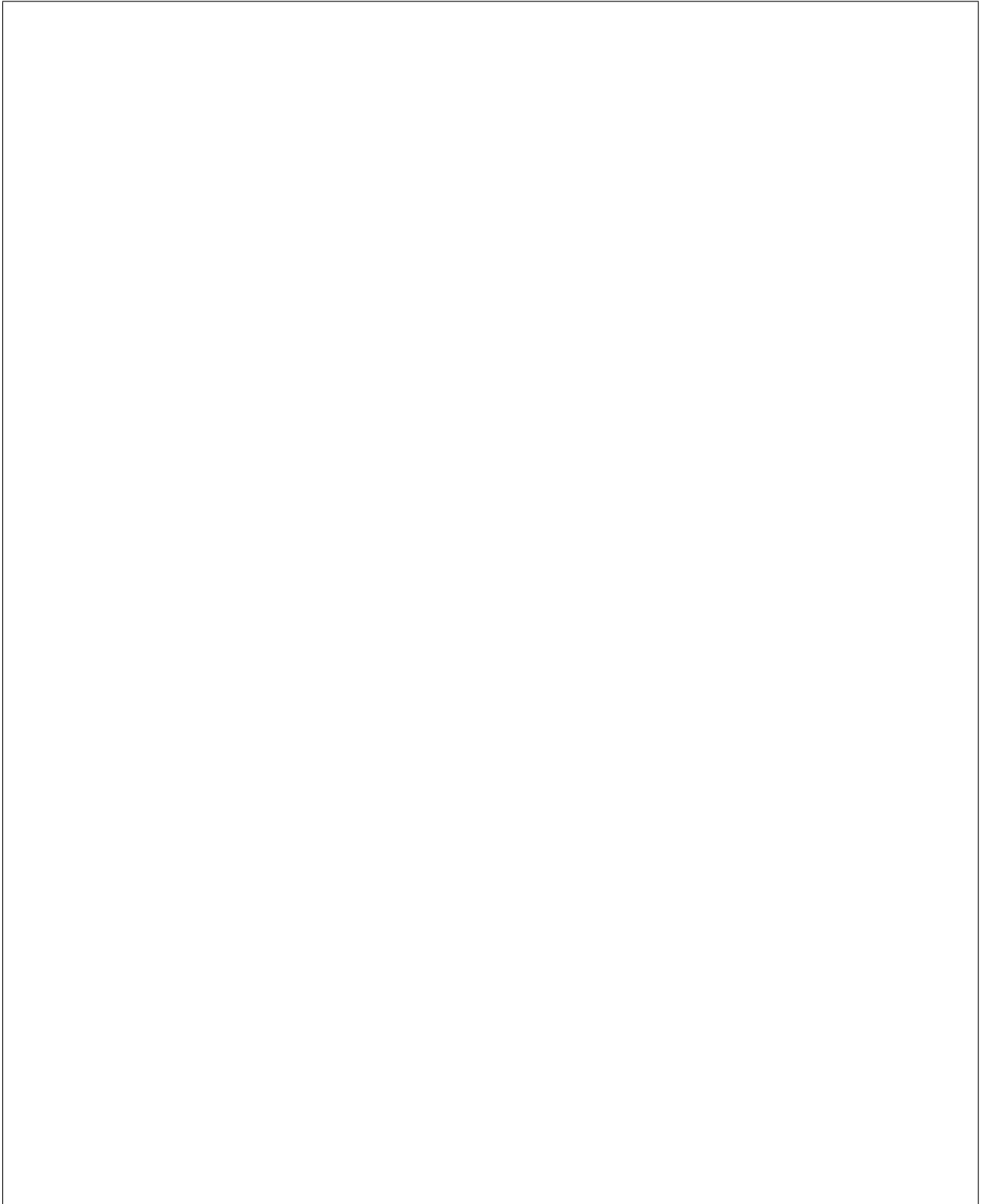You are finally ready to complete the design of your pressure cooker.

**Using the circuit elements below, make a circuit that will turn the heater to on (i.e. current flowing through $R_{heat}$) when the pressure is less than 500 kPa, and off (i.e. no current flowing through $R_{heat}$) when the pressure is greater than 500 kPa.**

The elements are:

- A voltage source $V_s = 10$V with a Thevenin resistance of $500\Omega$.
- A voltage source $V_p(p_c) = V_o \times \frac{p_c}{p_{ref}}$, with $V_o = 1$V and $p_{ref} = 100$kPa. (This is a voltage source whose voltage is a function of pressure $p_c$, unrelated to any previous parts of the question.)
- A comparator that controls switch $S_0$. The switch is normally opened (i.e. an open circuit between nodes $V_a$ and $V_b$), and is closed only when $V_1 > V_2$ (i.e. a short circuit between nodes $V_a$ and $V_b$).
- The heater supply ($V_{heat} = 100$V).
- The heater resistor $R_{heat}$.
- One additional resistor $R_{extra}$. If you use this resistor you must calculate and note its value on your circuit diagram.
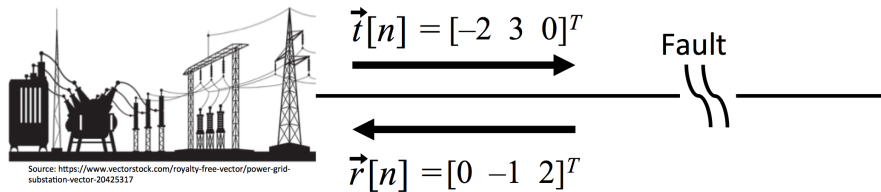- You may assume you have access to a ground node.

PRINT your student ID: _____

PRINT your student ID: _____

**6. Finding faults with PG&E (16 points)** (All subparts of this problem can be solved independently.)

PG&E has been having problems with the grid, and needs to more accurately locate faults in their transmission lines. EECS 16A students decide to use their new design skills to help.

All the transmission lines are connected to substations. To find a fault, a substation sends a signal $\vec{t}$ down a transmission line. If the line has a break in it, then a reflection occurs and the substation receives back a signal $\vec{r}$.

$$\vec{t}[n] = [-2 \ 3 \ 0]^T \quad \text{Fault}$$

$$\vec{r}[n] = [0 \ -1 \ 2]^T$$

Source: https://www.vectorstock.com/royalty-free-vector/power-grid-substation-vector-20425317

(a) (6 points) Assume that a substation sends the signal $\vec{t}$ and receives back a signal $\vec{r}$. The received signal $\vec{r}$ is a delayed, scaled version of $\vec{t}$ with added noise.

$$\vec{t}[n] = \begin{bmatrix} -2 & 3 & 0 \end{bmatrix}^T \tag{1}$$

$$\vec{r}[n] = \begin{bmatrix} 0 & -1 & 2 \end{bmatrix}^T \tag{2}$$

**Use the axes below to plot the cross-correlation $corr_{\vec{r}}(\vec{t})$, and use this to identify the index corresponding to the peak (maximum magnitude) of the cross-correlation.**
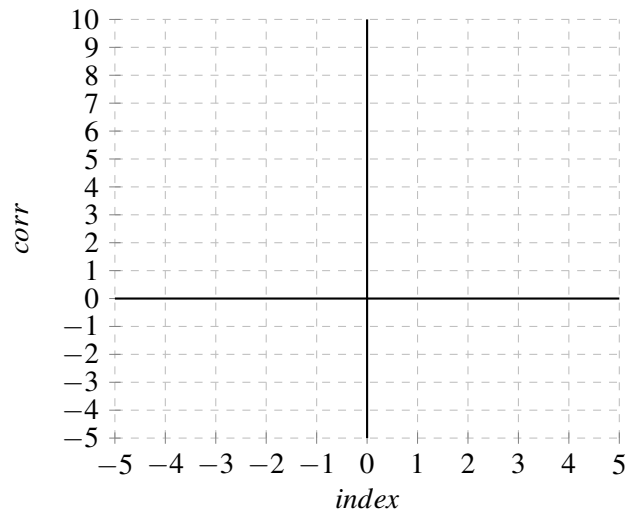
Index of maximum cross-correlation magnitude:

PRINT your student ID: _____

Extra page for scratchwork.
**Work on this page will NOT be graded.**

PRINT your student ID: _____

(b) (10 points) Now the substations send signals along the transmission lines to send information about the state of the power grid. **Each substation has a unique code that it uses to transmit its information, $\vec{s}_1$, $\vec{s}_2$, $\vec{s}_3$, and $\vec{s}_4$. You receive the signal $\vec{r}[n] = \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \end{bmatrix}^T$ and you know it contains signals from two different substations.**

Since the locations of the various substations are known, we can compute the delay with which the signals are received. The delays corresponding to the max correlation of $\vec{s}_1$, $\vec{s}_2$, $\vec{s}_3$, and $\vec{s}_4$ are: 1 unit, 2 units, 1 unit, and 2 units respectively. **We have provided shifted versions of the signals** $(\vec{u}_1[n], \vec{u}_2[n], \vec{u}_3[n], \vec{u}_4[n])$ that correspond to these distances, and included appropriate zeros to the signals **to make your calculations easier.**

$$\vec{s}_1[n] = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T \xrightarrow{\text{delayed by 1}} \vec{u}_1[n] = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \end{bmatrix}^T, \tag{3}$$

$$\vec{s}_2[n] = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^T \xrightarrow{\text{delayed by 2}} \vec{u}_2[n] = \begin{bmatrix} 0 & 0 & 1 & -1 & 1 \end{bmatrix}^T, \tag{4}$$

$$\vec{s}_3[n] = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}^T \xrightarrow{\text{delayed by 1}} \vec{u}_3[n] = \begin{bmatrix} 0 & 1 & 1 & -1 & 0 \end{bmatrix}^T, \tag{5}$$

$$\vec{s}_4[n] = \begin{bmatrix} -1 & -1 & 1 \end{bmatrix}^T \xrightarrow{\text{delayed by 2}} \vec{u}_4[n] = \begin{bmatrix} 0 & 0 & -1 & -1 & 1 \end{bmatrix}^T. \tag{6}$$

**Determine which two unique signals are contained in the received signal $\vec{r}[n]$. What are the weights on the two signals? Show all of your work.**
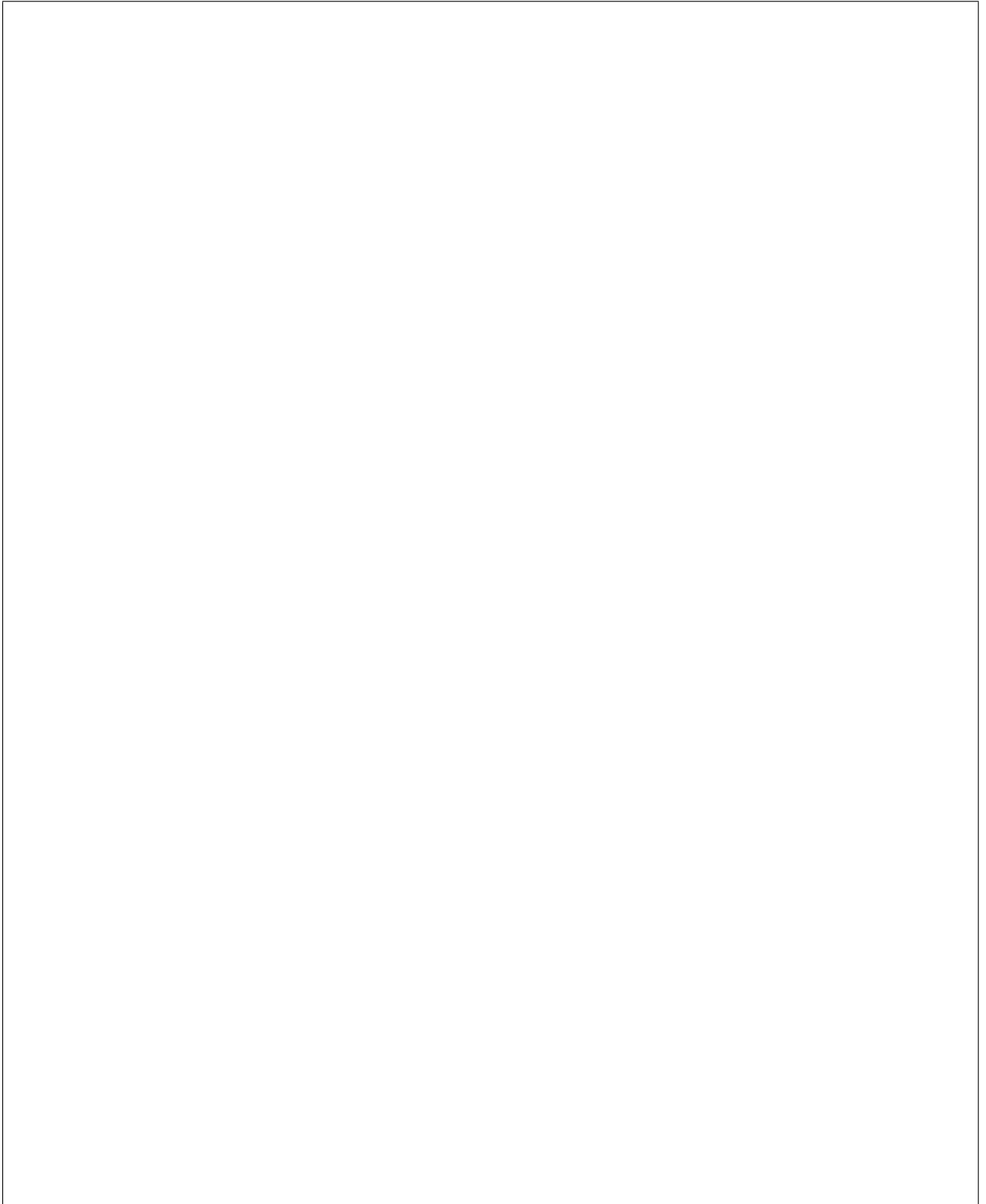
*Some calculations that might be useful:*

| | | | |
|---|---|---|---|
| $< \vec{r}[n], \vec{u}_1[n] > = 5$ | $< \vec{u}_1[n], \vec{u}_2[n] > = 0$ | $< \vec{u}_2[n], \vec{u}_3[n] > = 2$ | $< \vec{u}_3[n], \vec{u}_4[n] > = 0$ |
| $< \vec{r}[n], \vec{u}_2[n] > = 0$ | $< \vec{u}_1[n], \vec{u}_3[n] > = 1$ | $< \vec{u}_2[n], \vec{u}_4[n] > = 1$ | |
| $< \vec{r}[n], \vec{u}_3[n] > = 1$ | $< \vec{u}_1[n], \vec{u}_4[n] > = -2$ | | |
| $< \vec{r}[n], \vec{u}_4[n] > = -2$ | | | |

*This might also help:*

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{7}$$
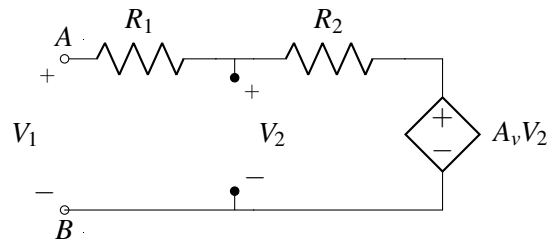
PRINT your student ID: _____

PRINT your student ID: _____

7. **Fun With Circuits (14 points)** (All subparts of this problem can be solved independently.)

In his spare time Professor Boser invents new circuits. The circuit schematic below shows his latest creation that uses a voltage controlled voltage source.



(a) **Equivalent Resistance** (8 points)

**Analyze the circuit above and model it with an equivalent resistance $R_{eq}$ as illustrated below. The I/V curves of the original and equivalent circuit should be identical.** Use the following values: $A_v = 3$, $R_1 = 1\ k\Omega$, and $R_2 = 4\ k\Omega$. Show your calculations.
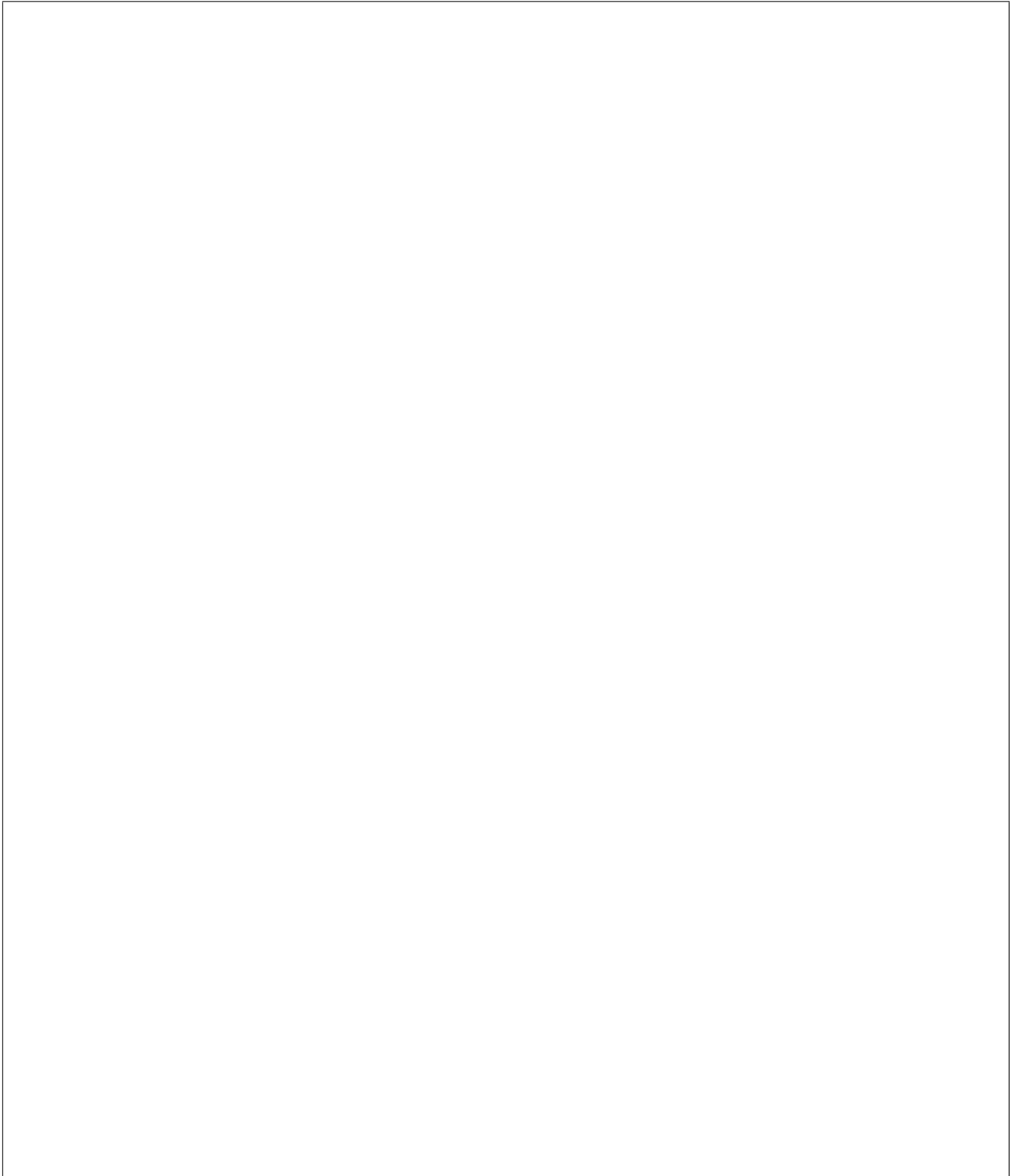
Recall that a resistor is just a model of the IV dependence of a circuit element; you may get a positive or a negative answer for your equivalent resistance.

*Hint: Apply a test voltage (or current) and compute the resulting current (or voltage).*
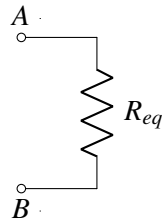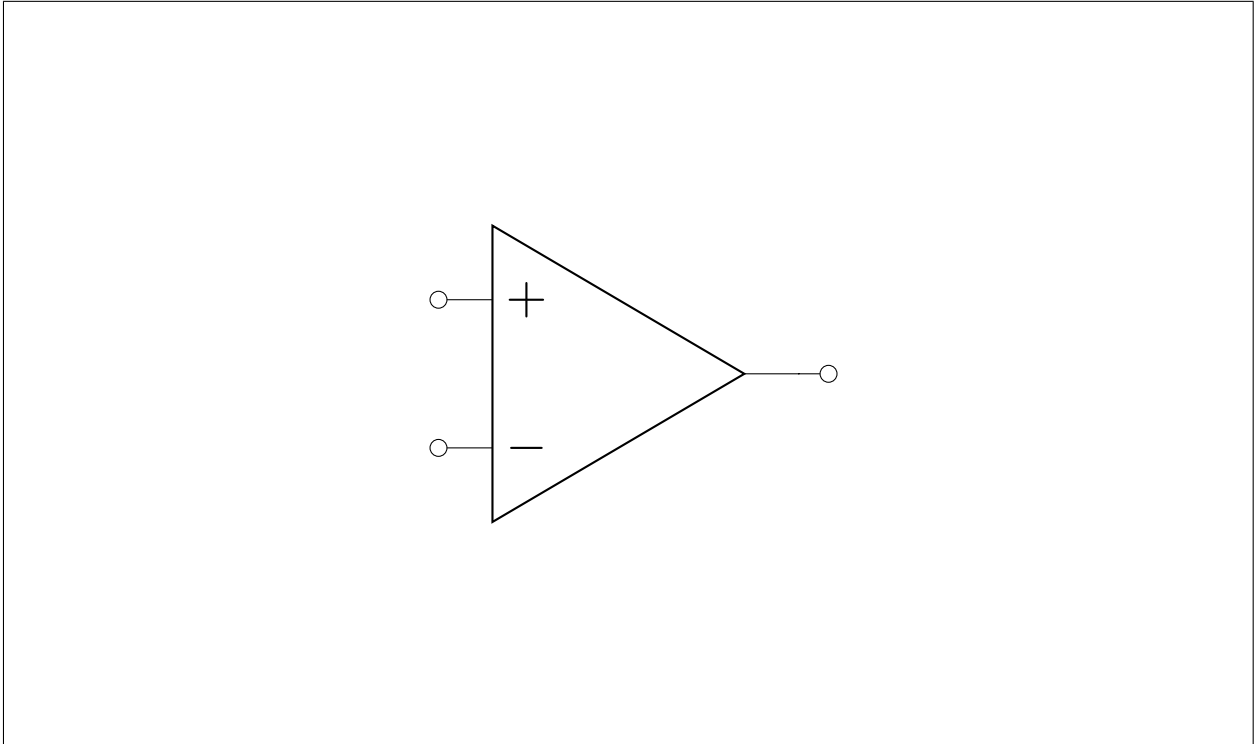
PRINT your student ID: _____

PRINT your student ID: _____

(b) **Amplifier Design** (6 points) This part is independent of the previous part.

**Design a circuit which implements a voltage controlled voltage source with gain** $A_v = 3$**, i.e.** $V_{out} = 3V_{in}$**.** Use the ideal operational amplifier shown below and up to three additional $1\ k\Omega$ resistors. Label $V_{in}$ and $V_{out}$.

PRINT your student ID: _____

8. **Projections and eigenvectors** (32 points) (All subparts of this problem can be solved independently.)

Consider two $n$-dimensional vectors $\vec{x} \in \mathbb{R}^n$ and $\vec{y} \in \mathbb{R}^n$. Consider the matrix $\mathbf{M} = \vec{x}\,\vec{y}^T$. **Note the order of the multiplication, this is distinct from** $< \vec{x}, \vec{y} > = \vec{x}^T \vec{y}$.

(a) (2 points) What are the dimensions of matrix $\mathbf{M}$?

(b) (6 points) Let $\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. **Find the eigenvalues and eigenvectors of M.** Show your work.

PRINT your student ID: _____

(c) (6 points) Now let $\vec{x} \in \mathbb{R}^n$ and $\vec{y} \in \mathbb{R}^n$. Let $\mathbf{M} = \vec{x}\,\vec{y}^T$. **Find the projection of $\vec{x}$ onto the columnspace of M.** Show all your work. *Hint: Write out the columns of* **M**.

PRINT your student ID: _____

(d) (8 points) $\vec{x} \in \mathbb{R}^n$ and $\vec{y} \in \mathbb{R}^n$. Let $\mathbf{M} = \vec{x}\,\vec{y}^T$. Let $\vec{z} \in \mathbb{R}^n$ be a vector such that $<\vec{z}, \vec{y}> = 0$. **Prove that** $\vec{z} \in Null(\mathbf{M})$. You must prove this from first principles. No theorems can be used in the proof. Show all your work.

PRINT your student ID: _____

(e) (10 points) $\vec{x} \in \mathbb{R}^n$ and $\vec{y} \in \mathbb{R}^n$. Let $\mathbf{M} = \vec{x}\,\vec{y}^T$. Further, assume that $<\vec{x},\vec{y}> \neq 0$. **Find an eigenvector of M corresponding to a non-zero eigenvalue.** Show all your work and justify your answer.

PRINT your student ID: _____

9. **Electronic Level (24 points)** (All subparts of this problem can be solved independently.)

A one-wheel scooter (as shown below) is fun to ride, but unfortunately quite expensive to buy. Inspired by the success of the pressure cooker, you decide to build your own.
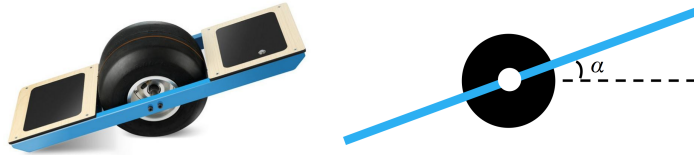


One challenge of building a one-wheel scooter is designing a circuit that measures the angle $\alpha$ of the riding platform, as shown above. You are considering the design of an angle sensor consisting of two capacitors $C_a$ and $C_b$ whose values depend on the angle $\alpha$ (measured in degrees) as follows:

$$C_a(\alpha) = C_o\left(100 + \frac{\alpha}{\alpha_{ref}}\right), \tag{8}$$

$$C_b(\alpha) = C_o\left(100 - \frac{\alpha}{\alpha_{ref}}\right), \tag{9}$$

where $\alpha_{ref} \neq 0$ and $C_o$ are properties of the sensor.

The circuit in Fig. 9.1 produces a voltage $V_{out}(k)$ at time $k$ that depends on $\alpha$ through the capacitances $C_a(\alpha), C_b(\alpha)$. The timing diagram (Fig. 9.2) shows the state of the switches as a function of time $k$. **At time $k = 0$ all of the capacitors are completely discharged, meaning that $\mathbf{V_{out}(0) = 0V}$.**
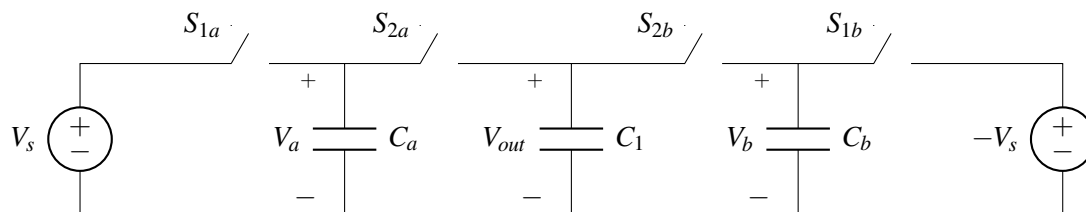


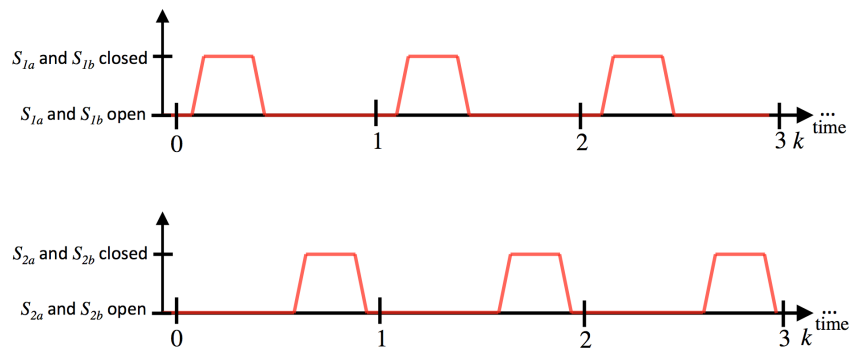Figure 9.1: The circuit for the electronic level



Figure 9.2: The timing diagram for the circuit

PRINT your student ID: _____

(a) (5 points) We define $Q_a = C_aV_a$, $Q_b = C_bV_b$ and $Q_1 = C_1V_{out}$, per the labels in Figure 9.1. **Find an expression for the total charge on the capacitors $Q_{tot}(k) = Q_a + Q_b + Q_1$ at time $k = 1$ as a function of $\alpha$, $\alpha_{ref}$ $C_o$, $C_1$, and $V_s$.** At time $k = 1$, all switches have been closed and opened once and are now in the open state. Show all your work.
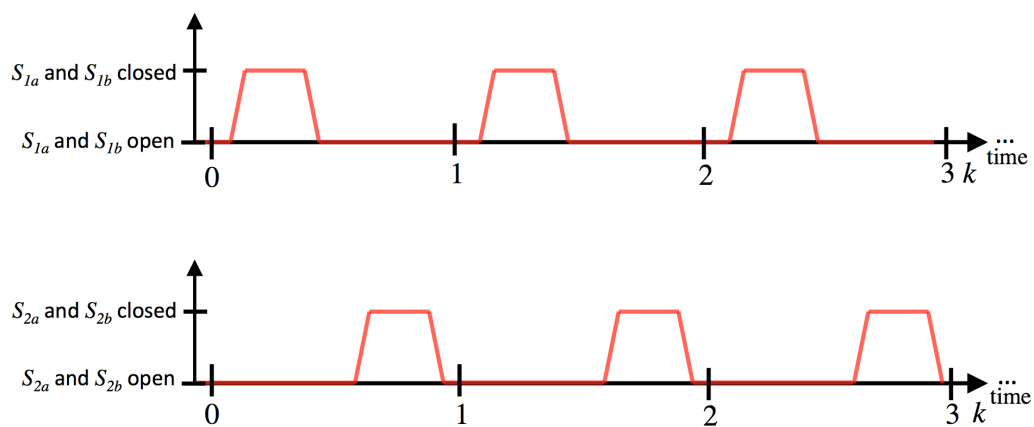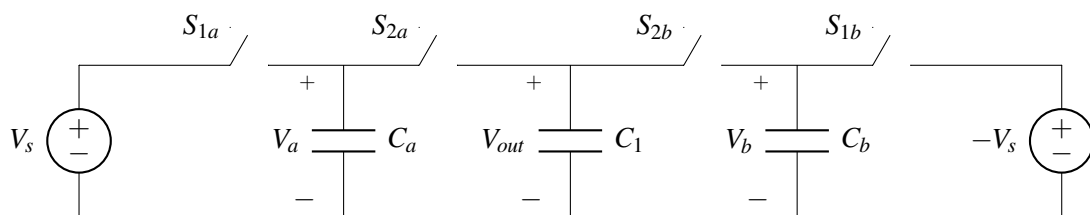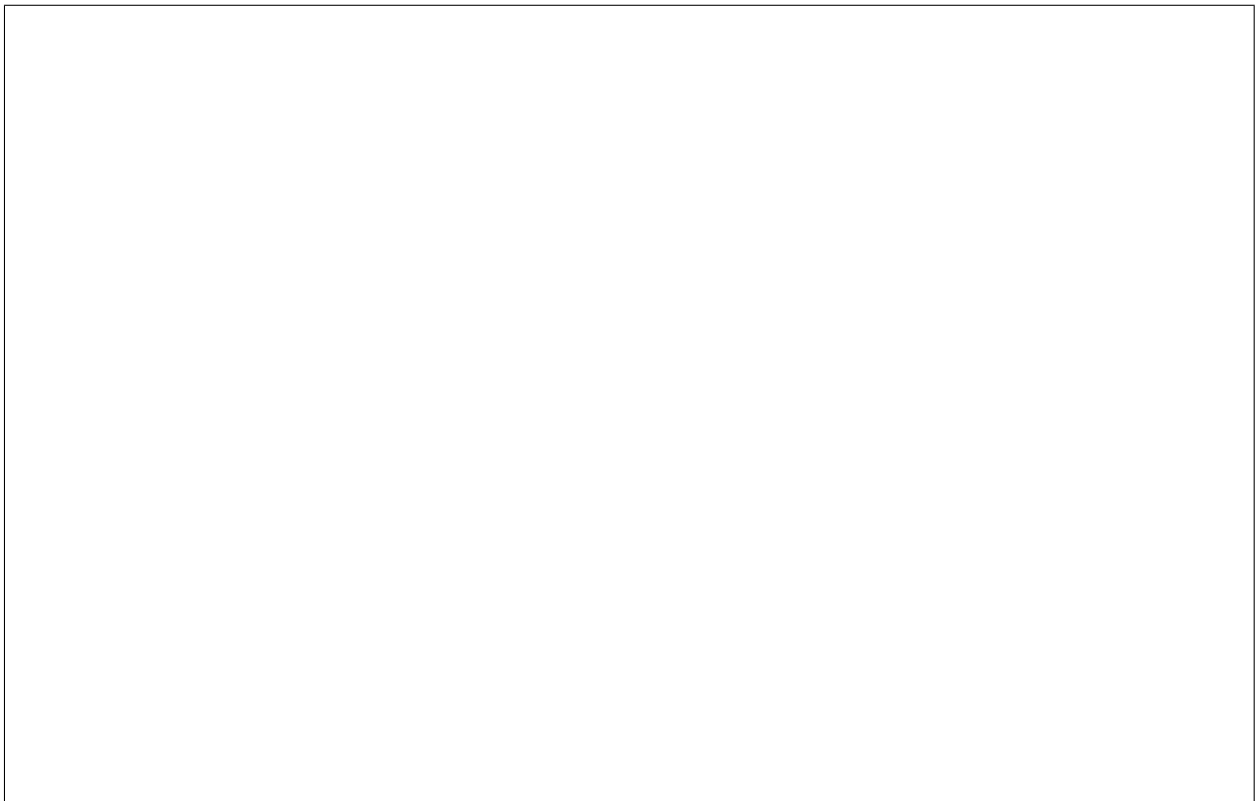
PRINT your student ID: _____
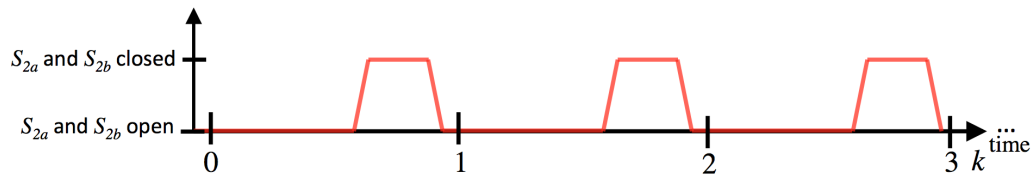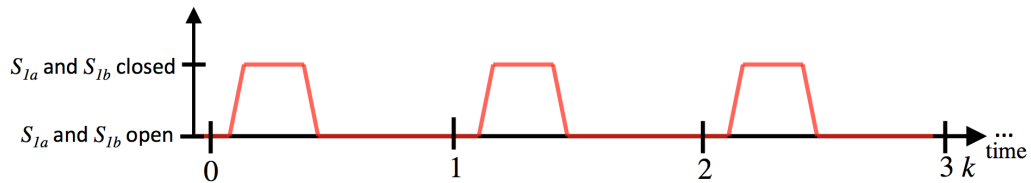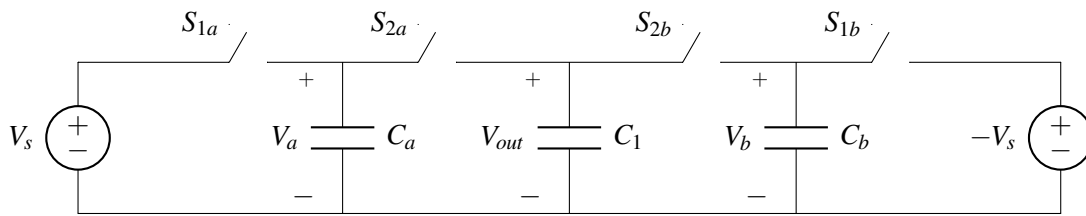


The same circuit and timing diagram from earlier, reproduced for your convenience.

(b) (5 points) **Calculate $V_{out}(1)$ as a function of $Q_{tot}(1)$, $C_o$, and $C_1$.** Show your work.

PRINT your student ID: _____

(c) (8 points) **Find an expression for $V_{out}(k)$ as a function of $V_{out}(k-1)$, $\alpha$, $\alpha_{ref}$, $C_o$, $C_1$, and $V_s$.** It is important to have an exact formula. Vague answers will receive no credit. Show your work. *Hint: You might try to first find $V_{out}(2)$ as a function of $V_{out}(1)$.*

PRINT your student ID: _____



The same circuit and timing diagram from earlier, reproduced for your convenience.

PRINT your student ID: _____

(d) (6 points) It turns out that the behavior of this circuit is exactly like the linear control systems you modeled in Module 1, and can be analyzed similarly.

Assume

$$V_{out}(k) = \gamma V_{out}(k-1) + \beta.$$

Both $\gamma, \beta \in \mathbb{R}$, and $0 < \gamma < 1$. Recall $V_{out}(0) = 0$. **What does $V_{out}$ converge to as $k \to \infty$, i.e., what is $V_{out}(\infty)$?** Your answer should be in terms of $\beta, \gamma$ and numbers. Show your work. (*Hint: The infinite series $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$ if $0 < x < 1$.*)

PRINT your student ID: _____

Extra page for scratchwork.
**Work on this page will NOT be graded.**

PRINT your student ID: _____

Extra page for scratchwork.
**Work on this page will NOT be graded.**

PRINT your student ID: _____

Doodle page!
Draw us something if you want or give us suggestions, compliments, or complaints.
You can also use this page to report anything suspicious that you might have noticed.

# EECS 16A    Designing Information Devices and Systems I
# Fall 2019                                    Final Exam Instructions

**There are 9 problems of varying numbers of points.** Not all subparts of a question are related to each other. You have 180 minutes for the exam. The problems are of varying difficulty, so pace yourself accordingly and avoid spending too much time on any one question until you have gotten all of the other points you can.

**There are 40 pages on the exam, so there should be 20 sheets of paper in the exam.** The exam is printed double-sided. Do not forget the problems on the back sides of the pages! Notify a proctor immediately if a page is missing. **Do not tear out or remove any of the pages. Do not remove the exam from the exam room.**

**No collaboration is allowed, and do not attempt to cheat in any way. Cheating will not be tolerated.**

**Write your student ID on each page before time is called. If a page is found without a student ID, we are not responsible for identifying the student who wrote that page.**

You may consult ONE handwritten $8.5'' \times 11''$ note sheet (front and back). No phones, calculators, tablets, computers, other electronic devices, or scratch paper are allowed.

**Please write your answers legibly in the boxed spaces provided on the exam.** The space provided should be adequate. **If you still run out of space, please use a blank page and clearly tell us in the <u>original problem space</u> where to look for your solution.**

Show all of your work in order to receive full credit. Partial credit will be given for substantial progress on each problem.

If you need to use the restrooms during the exam, bring your student ID card, your phone, and your exam to a proctor. You can collect them once you return from the restrooms.

**Our advice to you:** if you can't solve the problem, state and solve a simpler one that captures at least some of its essence. You might get some partial credit, and more importantly, you will perhaps find yourself on a path to the solution.

**Good luck!**

Do not turn this page until the proctor tells you to do so.