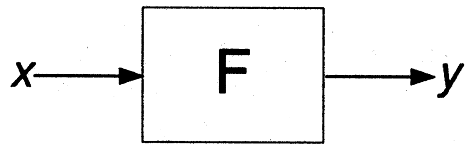
EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 2
Department of Electrical Engineering and Computer Sciences 03 April 2008
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name Philter FIRST Name Eko

Lab Time At all times

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided 8.5" × 11" sheet of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT2.1 (35 Points) The non-zero impulse response values of the discrete-time LTI system F



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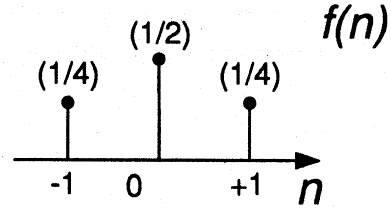


(a) (7 Points) Determine a reasonably simple expression for $F(\omega)$, where F is the frequency response of the filter F.

$$F(\omega) = \sum_{n=-\infty}^{\infty} f(n) e^{-i\omega n} = f(-1)e^{i\omega} + f(0) + f(1)e^{-i\omega} = \frac{1}{4}e^{i\omega} + \frac{1}{2} + \frac{1}{4}e^{-i\omega} \Rightarrow$$

$$F(\omega) = \frac{1}{2} + \frac{1}{2}\cos\omega = \frac{1}{2}(1+\cos\omega)$$

(b) (8 Points) Determine a reasonably simple linear, constant-coefficient difference equation governing the input-output behavior of the LTI system. Explain (briefly) why it is apt to call the system F a *three-point, center-weighted, moving average filter*.

$$y(n) = (f * x)(n) = \sum_{m=-\infty}^{\infty} f(m)x(n-m) = f(-1)x(n+1) + f(0)x(n) + f(1)x(n-1)$$

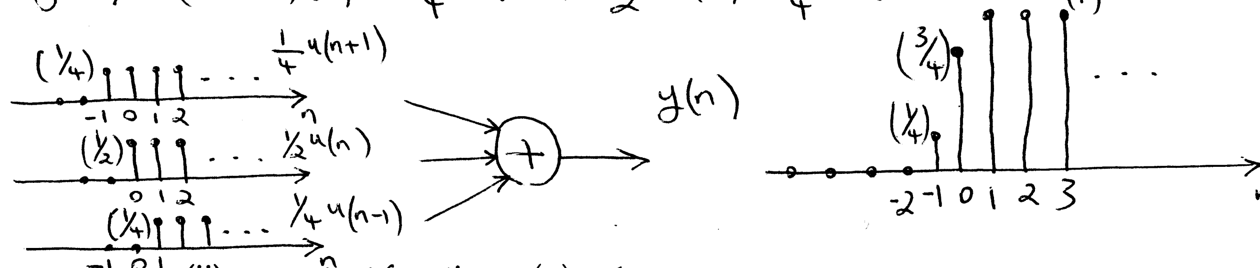
$$\Rightarrow y(n) = \frac{1}{4}x(n+1) + \frac{1}{2}x(n) + \frac{1}{4}x(n-1)$$

To compute the n^{th} sample of the output, the filter uses the input values at samples $n-1$, n, and $n+1$, with the largest weight given to the input at sample n (the center sample).

(c) (12 Points) Determine the output of the system if the input is

(i) the unit-step function: $x(n) = u(n)$.

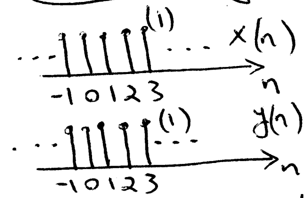
$$y(n) = (f \star x)(n) = \frac{1}{4}u(n+1) + \frac{1}{2}u(n) + \frac{1}{4}u(n-1)$$



(ii) a constant function: $x(n) = 1$.

Method 1: $x(n) = 1 = e^{i0n} \Longrightarrow y(n) = F(0)e^{i0n} = F(0) = 1 \Longrightarrow y(n) = 1 \quad \forall n$

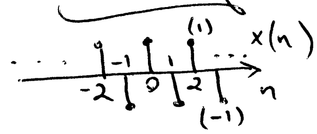
Method 2: $y(n) = \frac{1}{4}x(n+1) + \frac{1}{2}x(n) + \frac{1}{4}x(n-1) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1 \quad \forall n$

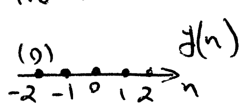


(iii) the sign-alternating signal $x(n) = \cos(\pi n)$.

$x(n) = \cos \pi n = (-1)^n = e^{i\pi n} \Longrightarrow y(n) = F(\pi)e^{i\pi n} = 0 \Longrightarrow y(n) = 0 \quad \forall n$

You can use the difference equation here as well.





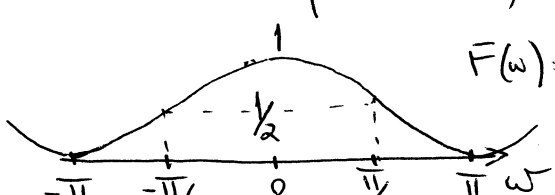
Be sure to provide a well-labeled sketch of all the signals involved.

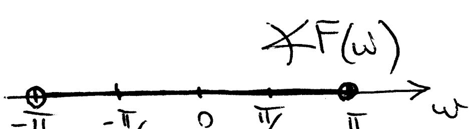
(d) (8 Points) Provide well-labeled sketches of $|F(\omega)|$ and $\angle F(\omega)$, the magnitude and phase responses, respectively, of the filter. Is your magnitude response consistent with your last two results in part (c)?

$F(\omega) = \frac{1}{2}(1 + \cos\omega)$ Note: $F(\omega) \geq 0 \quad \forall \omega$. In fact,

$F(\omega) > 0 \quad \forall \omega \neq$ odd multiple of $\pi \Longrightarrow \begin{cases} |F(\omega)| = F(\omega) & \forall \omega \neq \text{odd mult of } \pi \\ \angle F(\omega) = 0 \end{cases}$

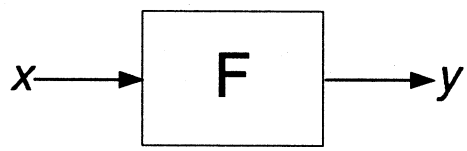
At odd multiples of π, $F(\omega) = 0 = |F(\omega)|$ and $\angle F(\omega)$ is undefined.





Consistent w/ previous responses, even with (c)(i). Smoothing (low-pass filter). Eliminates freq $\omega = \pi$.

MT2.2 (30 Points) Consider a discrete-time LTI filter



The two parts of this problem refer to two different, and mutually independent, filters, which we generically name F. Filter properties disclosed (or inferred) in one part may not be carried over to the other. You may tackle the parts in either order.

(a) (15 Points) Assume F is an ideal high-pass LTI filter with the frequency response

$$F(\omega) = \begin{cases} 0, & |\omega| \le \pi/4 \\ e^{-i\omega/2}, & \pi/4 < |\omega| \le \pi\,. \end{cases}$$

Determine a simple expression for the output of the system, if the input is $x(n) = A + B\cos\left(\frac{\pi}{3}n\right)$, where A and B are constants.

$$x(n) = Ae^{i0n} + \frac{B}{2}e^{-i\pi/3\,n} + \frac{B}{2}e^{i\pi/3\,n} \Longrightarrow y(n) = A\cancel{F(0)}^{\,0}e^{i0n} + \frac{B}{2}F(-\pi/3)e^{-i\pi/3\,n} + \frac{B}{2}F(\pi/3)e^{i\pi/3\,n}$$

$$\Longrightarrow y(n) = \frac{B}{2}e^{+i\frac{\pi}{6}}e^{-i\frac{\pi}{3}n} + \frac{B}{2}e^{-i\frac{\pi}{6}}e^{i\frac{\pi}{3}n} = \frac{B}{2}e^{-i\left(\frac{\pi}{3}n-\frac{\pi}{6}\right)} + \frac{B}{2}e^{i\left(\frac{\pi}{3}n-\frac{\pi}{6}\right)}$$

$$\Longrightarrow y(n) = B\cos\left(\frac{\pi}{3}n - \frac{\pi}{6}\right)$$

(b) (15 Points) The impulse response of the filter is $f(n) = a^{|n|}$, where $0 < |a| < 1$. Determine an expression for $F(\omega)$, the frequency response of the filter; simplify as much as you can.

$$F(\omega) = \sum_{n=-\infty}^{\infty} a^{|n|}e^{-i\omega n} = \sum_{n=-\infty}^{-1} a^{-n}e^{-i\omega n} + \sum_{n=0}^{\infty} a^{n}e^{-i\omega n} = \sum_{n=-\infty}^{0}\left(a e^{i\omega}\right)^{-n} - 1 + \sum_{n=0}^{\infty}\left(ae^{-i\omega}\right)^{n}$$

for adding the $n=0$ term to the first sum.

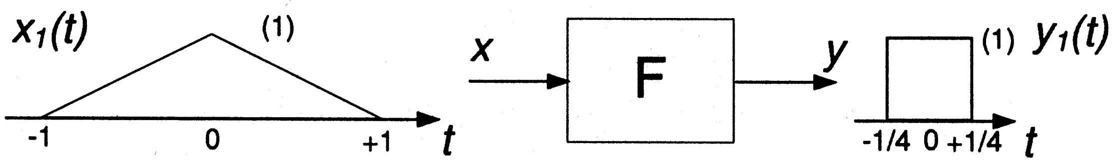
$$= \sum_{m=0}^{\infty}\left(ae^{i\omega}\right)^{m} - 1 + \sum_{n=0}^{\infty}\left(ae^{-i\omega}\right)^{n} =$$

$m=-n$ in first sum in the previous expression.

$$F(\omega) = \frac{1}{1-ae^{i\omega}} - 1 + \frac{1}{1-ae^{-i\omega}} = \frac{1-ae^{-i\omega} - (1-ae^{i\omega})(1-ae^{-i\omega}) + 1 - ae^{i\omega}}{(1-ae^{i\omega})(1-ae^{-i\omega})}$$

$$F(\omega) = \frac{\cancel{1}-\cancel{ae^{-i\omega}}-\cancel{1}+\cancel{2a\cos\omega}-a^2+1-\cancel{ae^{i\omega}}}{1-2a\cos\omega+a^2} \Longrightarrow F(\omega) = \frac{1-a^2}{1-2a\cos\omega+a^2}$$

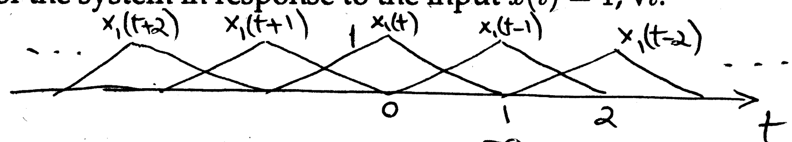
MT2.3 (15 Points) Consider a continuous-time LTI system F for which one input-output behavior is depicted below:



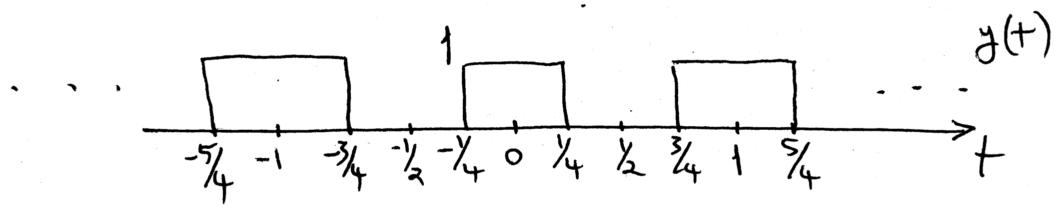
The input and output values are zero in the ranges of t not shown in the figure.

Determine the output of the system in response to the input $x(t) = 1, \forall t$.

$$x(t) = \sum_{k=-\infty}^{\infty} x_1(t-k)$$



Since the system is LTI, then $y(t) = \sum_{k=-\infty}^{\infty} y_1(t-k)$



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MT2.4 (25 Points) The continuous-time *unit doublet* is defined as the "derivative" of the Dirac delta:

$$\dot{\delta}(t) \triangleq \frac{d\delta(t)}{dt}.$$

(a) (15 Points) Let x be a continuous-time function. Show that

$$(x * \dot{\delta})(t) = \dot{x}(t) \triangleq \frac{dx(t)}{dt}$$

for every value of t at which x is "well-behaved." In particular, note that

$$(x * \dot{\delta})(t) = \int_{-\infty}^{+\infty} x(\tau)\, \dot{\delta}(t-\tau)\, d\tau\,,$$

and employ the method of integration by parts to show that the integral evaluates to $\dot{x}(t)$. An LTI system whose impulse response is $\dot{\delta}$ is called an *ideal differentiator*.

Let $u = x(\tau) \Rightarrow du = \dot{x}(\tau)\,d\tau$

$dv = \dot{\delta}(t-\tau)\,d\tau \Rightarrow v = -\delta(t-\tau)$

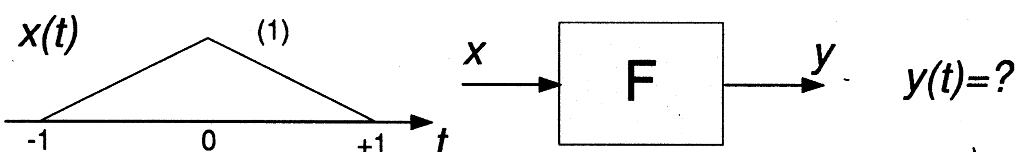
$$(x * \dot{\delta})(t) = \int_{-\infty}^{\infty} x(\tau)\,\dot{\delta}(t-\tau)\,d\tau$$

$$= uv\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} v\,du = \cancel{-x(\tau)\delta(t-\tau)\Big|_{-\infty}^{\infty}} + \underbrace{\int_{-\infty}^{\infty} \delta(t-\tau)\,\dot{x}(\tau)\,d\tau}_{} = \dot{x}(t) \Rightarrow \boxed{(x * \dot{\delta})(t) = \dot{x}(t)}$$

0 b/c $\delta(t-\tau) = 0$ at $\tau = \pm\infty$

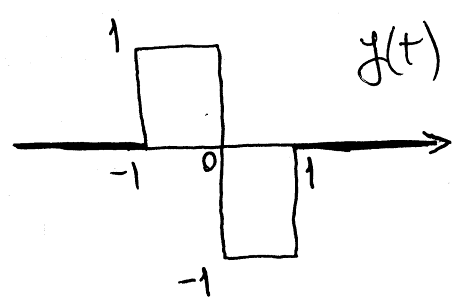
by the sifting property of the Dirac delta

(b) (10 Points) Consider a continuous-time LTI system F whose impulse response is $f(t) = \dot{\delta}(t)$. Use the result of part (a) to determine the output of the system in response to the input shown in the figure below. Briefly explain your work.



F is an ideal differentiator $\Rightarrow$ $y(t)$ is the instantaneous slope of $x(t)$

$$\Rightarrow y(t) = \begin{cases} 0 & t < -1 \\ 1 & -1 \le t \le 0 \\ -1 & 0 < t \le 1 \\ 0 & 1 < t \end{cases}$$



It doesn't matter where you place the equalities.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

LAST Name Philter FIRST Name Eko

Lab Time At all times

Problem	Points	Your Score
Name	10	10
1	35	35
2	30	30
3	15	15
4	25	25
Total	**115**	115