Physics 7B Spring 2019 - Final Exam Formula Sheet

$$PV = nRT = Nk_BT$$

$$\Delta E_{int} = Q - W$$

$$dE_{int} = dQ - PdV$$

$$\Delta S = \int \frac{dQ}{T}$$

$$\eta = \frac{W}{Q_h}$$

$$PV^\gamma = \text{const.}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{E} = -\vec{\nabla}V$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}$$

$$
\begin{aligned}
R &= \rho\frac{\ell}{A} \\
\vec{J} &= \sigma\vec{E} \\
P &= RI^2
\end{aligned}
$$

$$
\begin{aligned}
C &= \frac{Q}{V} \\
C &= \kappa C_0 \\
U &= \frac{Q^2}{2C}
\end{aligned}
$$

$$
\begin{aligned}
L &= \frac{N\Phi_B}{I} \\
\mathcal{E} &= -L\frac{dI}{dt} \\
U &= \frac{1}{2}LI^2
\end{aligned}
$$

$$I = \frac{dQ}{dt}$$

$$U = \int \frac{1}{2}\left(\epsilon_0|\vec{E}|^2 + \frac{1}{\mu_0}|\vec{B}|^2\right)dV$$

$$
\begin{aligned}
d\vec{l} &= dx\hat{x} + dy\hat{y} + dz\hat{z} \\
&\quad \text{(Cartesian Coordinates)} \\
d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z} \\
&\quad \text{(Cylindrical Coordinates)} \\
d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi} \\
&\quad \text{(Spherical Coordinates)}
\end{aligned}
$$

$$\int \frac{dx}{(x^2 + y^2)^{1/2}} = \log(\sqrt{x^2 + y^2} + x) + c$$

$$\int \frac{dx}{(x^2 + y^2)} = \frac{\tan^{-1}(x/y)}{y} + c$$

$$\int \frac{dx}{(x^2 + y^2)^{3/2}} = \frac{x}{y^2\sqrt{x^2 + y^2}} + c$$

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$

$$\text{solves } \frac{dy}{dt} = -Ay + B$$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$

$$\text{solves } \frac{d^2y}{dt^2} = -Ay$$

$$\log(2) = 0.69$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$k_B = 1.4 \times 10^{-23} J/K$$

$$R = 8.3 J/(\text{mol} \cdot K)$$

$$\epsilon_0 = 8.9 \times 10^{-12} C^2/(N \cdot m^2)$$

$$\mu_0 = 4\pi \times 10^{-7} T \cdot m/A$$

$$c = 3 \times 10^8 m/s$$

Final Exam

- You **may** use your notes, textbook, lectures, and other physical and online materials while taking this exam.
- You **may not** use another person as a resource either online or in person. No collaboration of any kind!
- Please **start each problem on a new page.**
- Put a **box around your final answer** and cross out any work you wish the grader to disregard.
- Try to be **neat** and **organized**.

Problems are weighted as indicated. Remember to look over your work. Good luck!

When you are finished with the exam….

1) **On a new page, you must <u>write</u> and <u>sign</u> the following honor statement:**

   "I swear on my honor that I have not given aid to another person on this exam, received aid from another person on this exam, and will not discuss this exam with another student who has not yet taken this exam."

   Make this the first page of your exam.

2) **Scan your exam with <u>each problem on a new page</u>, create a <u>PDF</u>, and upload at www.<u>gradescope</u>.com.**

1) (20 pts) A square parallel plate capacitor of sides L and separation of D has a dielectric of dielectric constant K inserted partially between the plates a distance x.

a) (5 pts) Find an expression for the capacitance of this capacitor in terms of L, K, x, D and constants.

b) (5 pts) If the capacitor is connected directly to a battery of potential difference V, allowed to charge completely, **stays connected** to the battery and then the dielectric is inserted an additional distance dx, find an expression for the change in the energy stored by the capacitor in terms of V, K, L, D, constants, and the additional distance dx.

c) (5 pts) Instead, what if the original capacitor is connected directly to a battery of potential difference V, allowed to charge completely, and then the battery is **disconnected**, what is the magnitude of charge on each plate?

d) (5 pts) If the charged capacitor from part c had the dielectric then inserted an additional distance dx, what would the difference in stored energy be?
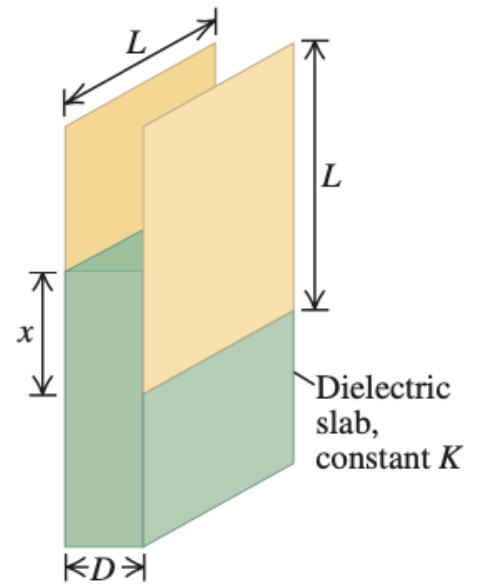
2) (15 pts) Given the circuit shown below

1.00 Ω 12.0 V

2.00 Ω   *a*   *b* 1.00 Ω 10.0 V   1.00 Ω

2.00 Ω   1.00 Ω 8.0 V   3.00 Ω   2.00 Ω
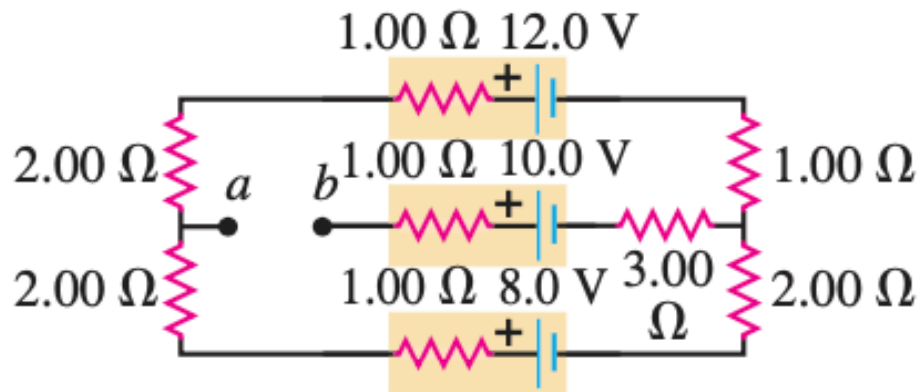
a) (8 pts) What is the potential difference between points a and b? Which point is at a higher potential?

b) (7 pts) If a wire was used to connect points a and b, what would the current through the top branch be?

3) (15 pts) Consider a circular loop of wire which is in the xy-plane and centered on the origin. The loop has an area A and a current I which is flowing around the loop in the clockwise direction as viewed from above the loop and looking in the $-z$-direction. An external magnetic field $\vec{B}$ passes through the loop such that it causes a torque on the loop given by $\vec{\tau} = X(5\hat{\imath} - 3\hat{\jmath})$ where X is a positive constant. In this particular orientation the magnetic potential energy is negative. The magnitude of the magnetic field is given by B_0 = *10X/IA*.

a) (5 pts) Find an expression for the vector magnetic moment of the loop in terms of I, A, and any necessary constants.

b) (10 pts) Find expressions for the components B_x, B_y, and B_z of the magnetic field $\vec{B}$ in terms of X, I, A, and any necessary constants.

4) (15 pts) Consider a set of infinitely long straight conductors placed side by side in the xy-plane and parallel to the y-axis as shown in the figure. Each has a square cross-sectional area and carries a current I in the $+y$-direction. Let n represent the number of conductors per unit length along the x-axis.
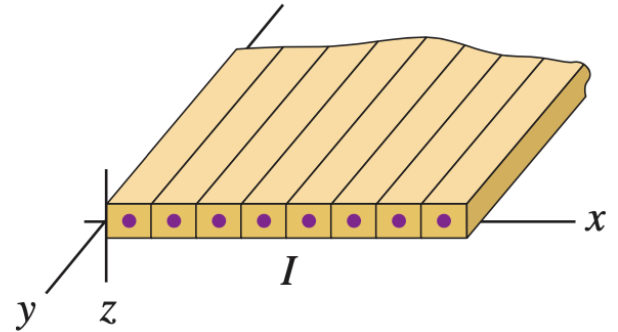

a) (8 pts) Find an expression for the magnitude and direction of the magnetic field just above the conductors in terms of I, n, and any necessary constants.

b) (7 pts) Now consider a new but similar situation - an insulating surface which is very wide and long and has a uniform positive surface charge density σ. The surface moves to the right at constant speed v. Find an expression for the magnitude and direction of the magnetic field just above the surface produced by the moving surface in terms of σ, v, and any necessary constants.

5) (20 pts) A long straight wire has current i flowing in the direction shown in the figure which is decreasing at a rate $R = di/dt$.

a) (7 pts) Find an expression for the induced emf in the loop to the right of the wire in terms of d, b, a, R, N, and any necessary constants. (While not pictured, the loop actually consists of N loops of wire.) Also, state the direction of the induced current in the loop. Explain completely for credit!!
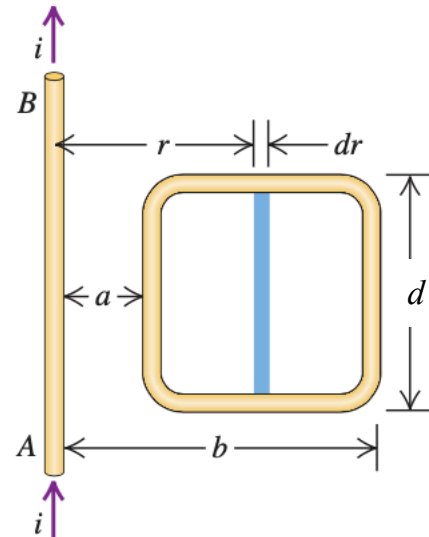
b) (6 pts) Now consider this figure to be part of a larger circuit as shown below. In this circuit $C = 25$ μF and is charged initially to 130 V with the positive side as shown in the figure. The resistor $R_0 = 12$ Ω. The larger circuit is 3 m wide and 6 m tall. At $t = 0$, the switch is closed. Find the current in the large circuit at $t = 225$ μs after the switch is closed.
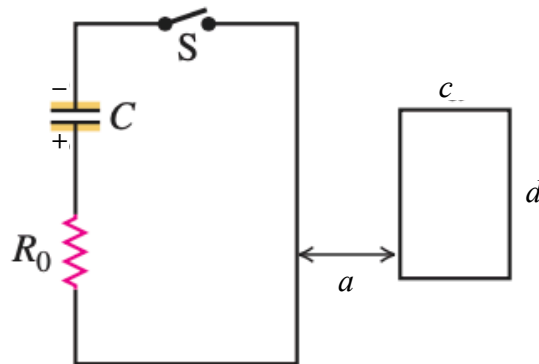
c) (7 pts) The small loop has dimensions $c = 12$ cm by $d = 24$ cm, a resistance of 1.2 Ω/m, contains 30 loops, and sits a distance $a = 4$ cm from the larger circuit. (This is the same small loop from part a.) Assuming that only the wire on the right side of the circuit contributes significantly to the magnetic field through the small loop, find the current in the small loop at the same time, $t = 225$ μs after the switch is closed.

6) (15 pts) Consider the given circuit. The switch S has been in position 1 as shown for a very long time. The battery and inductors have negligible resistance.

1   2

S

75.0 V

125.0 Ω

15.0 mH

5.0 mH

25.0 μF

35.0 μF
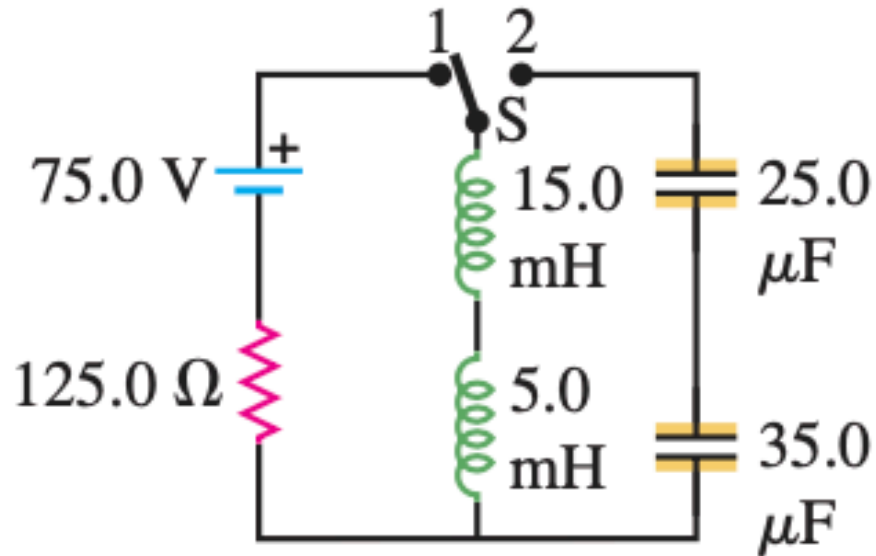
a) (5 pts) Find the current in the circuit.

b) (5 pts) Now the switch is quickly flipped to position 2. What is the maximum charge acquired by each capacitor?

c) (5 pts) How much time after the switch is flipped will it take for the capacitors to reach the charge found in part b?