Physics 7B Summer 2020 - Midterm Exam Formula Sheet

$$PV = nRT = Nk_BT$$

$$\Delta E_{int} = Q - W$$

$$dE_{int} = dQ - PdV$$

$$\Delta S = \int \frac{dQ}{T}$$

$$\eta = \frac{W}{Q_h}$$

$$PV^\gamma = \text{const.}$$

$$d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$$
(Cartesian Coordinates)

$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z}$$
(Cylindrical Coordinates)

$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi}$$
(Spherical Coordinates)

$$\int \frac{dx}{(x^2+y^2)^{1/2}} = \log(\sqrt{x^2+y^2} + x) + c$$

$$\int \frac{dx}{(x^2+y^2)} = \frac{\tan^{-1}(x/y)}{y} + c$$

$$\int \frac{dx}{(x^2+y^2)^{3/2}} = \frac{x}{y^2\sqrt{x^2+y^2}} + c$$

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$
solves $\frac{dy}{dt} = -Ay + B$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$
solves $\frac{d^2y}{dt^2} = -Ay$

$$\log(2) = 0.69$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$k_B = 1.4 \times 10^{-23} J/K$$

$$R = 8.3 J/(\text{mol} \cdot K)$$

$$\epsilon_0 = 8.9 \times 10^{-12} C^2/(N \cdot m^2)$$

Midterm Exam

- You **may** use your notes, textbook, lectures, and other physical and online materials while taking this exam.
- You **may not** use another person as a resource either online or in person. No collaboration of any kind!
- Please **start each problem on a new page.**
- Put a **box around your final answer** and cross out any work you wish the grader to disregard.
- Try to be **neat** and **organized**.

Problems are weighted as indicated. Remember to look over your work. Good luck!

When you are finished with the exam….

1) **On a new page, you must write and sign the following honor statement:**

   "I swear on my honor that I have not given aid to another person on this exam, have not received aid from another person on this exam, will not discuss this exam with another student who has not yet taken this exam, and have not discussed this exam with another student who has already taken this exam."

   Make this the first page of your exam.

2) **Scan your exam with each problem on a new page, create a PDF, and upload at www.gradescope.com.**

1) (20 pts) A very long, solid metal cylinder of radius r_a is centered on the long axis of a very long, cylindrical metal shell of inner radius r_b and outer radius r_c. There exists a positive area charge density σ on the surface of the inner cylinder and the metal shell is neutral overall.

   a) (6 pts) Find an expression for the magnitude and direction of the electric field in the region $r_a < r < r_b$

   b) (8 pts) Find an expression for the magnitude and direction of the electric field in the region $r_b < r < r_c$

   c) (6 pts) Find an expression for the magnitude and direction of the electric field in the region $r > r_c$

2) (20 pts) A hollow insulating spherical shell of inner radius a and outer radius b has a volume charge density given by $\rho(r) = \alpha/r$ where α is a positive constant.

a) (6 pts) Find an expression for the magnitude of the electric field at a distance r from the center of the shell where $a < r < b$ in terms of a, r, α, and constants.

b) (6 pts) Now a point charge q is placed at the center of the hollow space at $r = 0$ in an effort to make the E field constant from $a < r < b$. What must the sign and magnitude of the charge q be, in terms of α, a, and constants? What will the resulting constant E field be, in terms of α and constants?

c) (8 pts) Find an expression for the electric potential outside the shell ($r > b$), in terms of a, b, r, q, α, and constants.

3) (15 pts) A balloon containing $2.10 \times 10^3\ m^3$ of helium gas at 1.00 atm and a temperature of 16.0°C rises rapidly from the ground to a point in the air where the atmospheric pressure is 0.899 atm. You may assume that the helium is monatomic, behaves like an ideal gas, and that the balloon ascended too quickly to allow significant heat exchange with the surrounding air.

a) (10 pts) Calculate the volume and temperature of the gas at the higher altitude.

b) (5 pts) Find the change in internal energy of the helium between ground level and the higher altitude.

4) (20 pts) Oxygen (O_2) has a molar mass of 32.0 g/mol.

a) (2 pts) Find the average translational kinetic energy of an oxygen molecule at 300 K.

b) (3 pts) Find the average value of the square of its speed.

c) (2 pts) Find the rms speed.

d) (2 pts) Find the momentum (p=mv) of an oxygen molecule traveling at the rms speed.

e) (3 pts) If an oxygen molecule traveling at the rms speed bounces back and forth between opposite sides of a cubical vessel of 0.10 m on a side, what is the average force the molecule exerts on one of the walls of the container? Assume the molecule's velocity is perpendicular to the walls it hits.

f) (2 pts) Find the average force per unit area.

g) (2 pts) How many oxygen molecules traveling at the rms speed are necessary to produce an average pressure of 1 atm?

h) (2 pts) Calculate the number of oxygen molecules that are actually in a container of this size at 300 K and 1 atm.

i) (2 pts) There should be a discrepancy between your answers for part (h) and part (g). Explain why this difference exists and why they differ by the factor they do.

5) (12.5 pts) A standard Carnot engine is setup to operate between 2 heat reservoirs at T_H and T_C. A clever individual decides to attempt to increase the efficiency by running one engine between T_H and some intermediate temperature T' and a second engine between T' and T_C using heat expelled by the first engine as input for the second engine. Compute the efficiency of this new system in terms of T_C and T_H and compare it to the original standard Carnot engine.

6) (12.5 pts) You decide to make a "refrigerator" for your dorm room consisting of a box made of Styrofoam containing a block of ice. The initial mass of the ice is 25.0 kg, and the box has dimensions of 0.600 m x 0.800 m x 0.500 m. Water from the melting ice collects inside the box and stays in thermal equilibrium with the ice. The ice block is at 0.00 °C, and the temperature outside the box is 22.0 °C. If you keep the box closed and you want the inside of the box to stay at 8.00 °C for one week until all the ice melts, how thick should the Styrofoam be?

$C_{water} = 4186$ J/kg·K $\qquad L_{f,water} = 333$ kJ/kg $\qquad L_{v,water} = 2260$ kJ/kg

$\kappa_{styrofoam} = 0.033$ W/m·K