Econ 101A – Midterm 1

Th 25 February 2020.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. After you are finished, scan and upload the exam on Gradescope by 12.40 the latest. There is only 1 problem in today's exam. Show your work!

Problem 1. Consumption and Leisure. (67 points) A consumer makes a choice about consumption c and leisure l. The utility function is

$$u(c, l) = ln\left(c\right) + 30l$$

1. Plot 3 indifference curves $u(c, l) - \bar{u} = 0$ for different values of $\bar{u}$. Have the leisure l as the y axis and consumption c as the x axis. Do you notice a special feature of these preferences? (Hint: calculating the slope of the indifference curve, the MRS, might help you with the drawing) (5 points)

   - To obtain the indifference curves, solve out $ln\left(c\right) + 30l - \bar{u} = 0$ or $l = \bar{u}/30 - ln\left(c\right)/30$. Thus the MRS is $dl/dc = -\frac{1}{30c}$. The important aspect to notice is that the MRS is independent of the value of l. This is because these preferences have the special feature of being quasi-linear, that is, the utility function is linear in the second variable, as in your PSet 2. Because of this, when we plot them, the indifference curves are all parallel, upward shifts of one another, due to the quasi-linear feature.

2. In the next 4 sub-questions, I ask for features of the preferences represented by the utility function above. Write the definition if you can recall it and explain to the best you can why you think the preferences have, or do not have, those features. (12 points)

   (a) Are the preferences complete?

      - Preferences are complete if for all x, y, either $x \succeq y$ or $y \succeq x$ or both. Note that x, y are vectors, where $x = (c_x, l_x)$ and $y = (c_y, l_y)$. In this case, the property is easily satisfied because these preferences are represented by a utility function and clearly $u(x) \geq u(y)$, or $u(y) \geq u(x)$,or both. As we stated in lecture, when preferences can be represented by a utility function, they are always complete and transitive.

   (b) Are the preferences transitive?

      - Preferences are transitive if $x \succeq y$ and $y \succeq z$ implies $x \succeq z$. This easily follows because if $u(x) \geq u(y) \geq u(z)$,clearly $u(x) \geq u(z)$. Again, when preferences can be represented by a utility function, they are always complete and transitive.

   (c) Are the preferences monotonic?

      - Preferences are monotonic if $x \geq y$ implies $x \succeq y$. Recall that in vector terms $x \geq y$ means $c_x \geq c_y$ and $l_x \geq l_y$. In this case, the utility function is increasing in both c and l and thus this holds.

   (d) Are the preferences convex?

      - Preferences are convex is for any y the set of x such that $x \succeq y$ is a convex set (that is, includes all the convex combinations of elements of the set). We can see that from the plot above that the indifference curves are convex and thus the property is satisfied.

3. We now consider the budget constraint. The individual has income M and in addition can earn from working at wage w, where hours worked h satisfy $h + l = 24$. The price of c is p. Thus, $pc \leq M + wh$. Transform the budget constraint into a constraint written over c and l, and the constants M, p and w. (4 points)

   - In $pc \leq M + wh$ substitute the expresison for h to get $pc \leq M + w(24 - l)$ and thus $pc + wl \leq M + 24w$.

4. Why is w the price of leisure l in the budget constraint? (3 points)

   - Leisure does not have a direct cost, and yet w acts as price of leisure because it is the opportunity cost of leisure: by taking an hour of leisure, the person forgoes an hour of work, which would have earned wage w.

5. The consumer maximizes utility subject to budget constraint. Assume that the budget constraint is satisfied with equality (it is) and write the Lagrangean function. Derive the set of first order conditions. (4 points).

   - The Lagrangean is $ln\,(c) + 30l - \lambda[pc + wl - M - 24w]$. The first order conditions are

$$f.o.c.c : \frac{1}{c} - \lambda p = 0,$$

$$f.o.c.l : 30 - \lambda w = 0,$$

$$f.o.c.\lambda : -[pc + wl - M - 24w] = 0.$$

6. Use the first-order conditions to solve for $l^*$and c^*. (5 points)

- Notice that in the ifrst order condition for l,l does not appear! This is because of the quasi-linear utility function. We can solve for $\lambda^* = 30/w$ from that first order condition (this is a case in which solving for the multiplier is easy) and plug it back in the first equation, which gives us

$$\frac{1}{c} - \frac{30}{w}p = 0$$

and thus

$$c = \frac{w}{30p}.$$

From the budget constraint, we solve for l as

$$l = -\frac{p}{w}(\frac{w}{30p}) + \frac{M}{w} + 24 = \frac{M}{w} + 24 - \frac{1}{30}$$

$l^* = \frac{M}{w} + 24 - \frac{1}{30}$

$c^* = \frac{w}{30p}$

7. How does the optimal choice of l^* and of c^* depend on income M? Are the two good normal goods? (4 points)

   - The optimal amount of leisure is increasing in income ($\partial l^*/\partial M = 1/w > 0$) and thus the good is a normal good. The optimal amount of consumption is independent of income ($\partial c^*/\partial M = 0$) and thus the consumption good is a neutral good, neither normal nor inferior. This is because of the quasi-linear nature of the preferences.

8. What is the special feature of the utility function above that accounts for the above result? (4 points)

   - This is because of the quasi-linear nature of the preferences so the optimal amount of consumption is independent of the level of income, and income affects just the amount of leisure.

9. Check now under what conditions $l^* \geq 0$ and $c^* \geq 0$ are satisfied. (4 points)

   - The condition $l^* = \frac{M}{w} + 24 - \frac{1}{30} \geq 0$ and $c^* = \frac{w}{30p} \geq 0$ are both clearly satisfied.

10. Consider now an individual with utility function

$$v(c, l) = ln\,(c) + 10l$$

which differs from the above because of the $10l$ term, instead of $30l$. An economist says – “You do not need to solve again for the optimal $l^*$and c^* for this case, as this utility function $v(c, l)$ is a monotonic transformation of the original utility function $u(c.l)$, thus it represents the same preferences and will have the same solutions”. Do you agree with this? Explain your thinking. (5 points)

   - The economist is wrong! The function $v(c, l) = u(c, l) - 20l$ and thus is not a monotonic transformation, since it subtracts a value that is a function of l. Indeed, if you solve with this utility function you will get a different solution.

11. Consider now an individual with utility function

$$w(c, l) = ln\,(c) + ln(l).$$

Write the Lagrangean, take first order conditions and solve for $l^*$and c^* for this case. (8 points)

- Notice that this utility function is a transformation of a Cobb-Douglas $c^{1/2}l^{1/2}$obtained taking logs (and multiplying by 2). The Lagrangean is $ln\,(c) + ln\,(l) - \lambda[pc + wl - M - 24w]$. The first order conditions are

$$f.o.c.c : \frac{1}{c} - \lambda p = 0,$$

$$f.o.c.l : \frac{1}{l} - \lambda w = 0,$$

$$f.o.c.\lambda : -[pc + wl - M - 24w] = 0.$$

  We move the lambda terms to the right-hand side and divide though in the first two conditions to get

$$\frac{l}{c} = \frac{p}{w},$$

  then can combine with the budget constraint to get

$$pc + w(\frac{p}{w}c) = M + 24w$$

  or

$$c^* = \frac{1}{2p}[M + 24w]$$

  and

$$l^* = \frac{p}{w}c^* = \frac{p}{w}\frac{1}{2p}[M + 24w] = \frac{1}{2w}[M + 24w].$$

  This is as excpted given that it is a Cobb-Douglas function.

12. For this utility function $w(c, l)$, how does the optimal choice of l^* and of c^* depend on income M? Are the two good normal goods? (4 points)

    - Both l^* and of c^* are increasing in M and thus both goods are normal goods. Formally, $\partial l^*/\partial M = 1/2w > 0$ and $\partial c^*/\partial M = 1/2p > 0$.

13. Compare the answers to question (12) to the answer for question (7). Relate the difference to the difference in utility functions. (5 points)

    - The key difference is that in question (7) we were maximizing a quasi-linear utility function in which the income effect is only reflected in consumption of the good that is linear in the utility function, in this case l. In contrast, for (12) we are dealing with a transformation of a Cobb-Douglas function, in which both goods are normal goods, that is, increase when income increases.