Physics 7B, Lecture 2 (Speliotopoulos)
First Midterm, Spring 2021
Berkeley, CA

Rules: *You may work on this exam from 8-10pm, and you must submit your solutions on Gradescope and mark where your answers are by 10:15. You may use our textbook (vol. 2 of Giancoli's Physics for Scientists and Engineers, custom edition for the University of California, Berkeley),* ***any notes taken by yourself*** *during our lectures, and the supplemental lecture notes on kinetic theory posted on our bCourses site.* ***You are not allowed to use any other resources, including but not limited to discussion worksheets, solutions, and the internet.*** *You are not allowed to communicate with any other students or discuss the content of the exam with anyone besides the Physics 7B teaching staff. Any violations of this policy will be considered a breach of academic integrity. All questions during the exam should be directed to your proctor via Zoom direct messages.*

Exam format: *This exam consists of five problems, each worth 20 points.*

Your starting point for each problem must be an equation listed at the end of the exam. You may NOT use any equation from the homework or any other source as the starting point. In addition, you must provide a reason based on physical or mathematical principles in your answers in order to receive credit. Simply stating an answer is NOT sufficient even if it is correct.

We will give partial credit on this midterm*, so if you are not sure how to do a problem, or if you do not have time to complete a problem, try to demonstrate your understanding of the physics involved by drawing a clear diagram and explaining (in terms of physics) why you believe your answer to be incorrect, or how you would do the problem if you had time. Please be aware, however, that writing irrelevant or incorrect information will result in less partial credit.*

Submission format: *Please start a new page for each new problem. You may write on paper, or on a tablet/computer, but final submissions must be uploaded to Gradescope and each part of each problem must be linked to a page(s) of your submission to receive credit.*

Honors Statement: *It is expected that during this examination, as with any examination that they undertake at the university, students adhere to the usual standards of academic integrity at the University of California at Berkeley as outlined on by the Center for Teaching and Learning.* ***We therefore ask that you write the following statement on the last page of your submission along with the date and your signature:***

"I swear on my honor that I have neither given nor received aid on this exam."

1. When we talked about thermal expansion, we chose to reference the change in the length of a rod by its length L_0 at its initial temperature T_0:

$$\Delta L = L_0 \alpha \Delta T,$$

from which we found that the length of the rod at temperature T is

$$L = L_0(1 + \alpha \Delta T). \tag{1}$$

Here, $\Delta L = L - L_0$, $\Delta T = T - T_0$, and α is the linear coefficient of expansion.

a. Suppose we chose to reference the change in the length of the rod by its length L at temperature T instead of L_0 so that

$$\Delta L = L\tilde{\alpha}\Delta T,$$

and $\tilde{\alpha}$ is the linear coefficient of expansion with this choice. What is L in terms of L_0, ΔT, and $\tilde{\alpha}$ with this choice?

b. What is $\tilde{\alpha}$ in terms of α and ΔT?

c. The expression obtained in part a for L should be different from Eq. (1) above. Given the typical size of α, are these differences significant? Justify your answer in detail. (You might find the approximation $(1+u)^p \approx 1 + pu$ for $u \ll 1$ useful.)

2. A bottle contains water with mass m_w at a temperature T_w. A piece of ice with mass m_{ice} at a temperature T_{ice} is placed into the water. The bottle is then thermally isolated, and you wait until the system returns to equilibrium.

   a. If no ice melts, and no water freezes, what is the temperature T (in Celsius) of the system? Justify your answer in detail.

   b. If once again no ice melts, and no water freezes, what is the ratio m_w/m_{ice}? You may express your answer in terms of any or all of the following quantities: T; T_{ice}; T_w; the specific heats, c_w and c_{ice}, of water and ice, respectively; the latent heat of fusion, L_{ice}; and $T_{abs} = -273.15$ °C.

3. Consider a monatomic gas in a container. As we are interested only in motion along the x-direction, take the probability of finding a particle with velocity between v_x and $v_x + dv_x$ to be

$$d\mathcal{P} = N\left(\frac{m}{2\pi k_B T}\right)^{\frac{1}{2}} e^{-\frac{1mv_x^2}{2k_BT}} dv_x,$$

and the average of any quantity $B(v_x)$ per particle to be

$$\langle B(v_x)\rangle = \frac{1}{N}\int_{-\infty}^{\infty} B(v_x)\,d\mathcal{P}.$$

Here, m is the mass of a particle that makes up the gas.

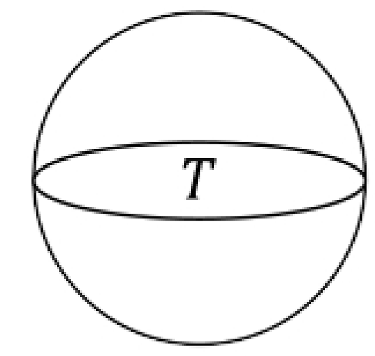
a. We can represent the velocity of a particle along the positive x-axis to be:

$$v_x^+ \equiv \begin{cases} v_x & \text{if } v_x > 0, \\ 0 & \text{if } v_x \leq 0. \end{cases}$$

What is $\langle v_x^+ \rangle$? Express it in terms of m, T, and k_B. (You might find the integral table at the back of the exam useful.)

b. What is $\langle (v_x^+)^2 \rangle$? Express it in terms of m, T, and k_B.

4. Empty space is not empty; it is filled with radiation (the cosmic microwave background), which can be thought of as having a temperature of T_{CMB}. A sphere of iron with mass m, surface area A, and an initial temperature of $T_0 = 2^7\, T_{CMB}$ is suspended in empty space (see figure to right). (You can assume that the sphere has a uniform temperature at all times.)

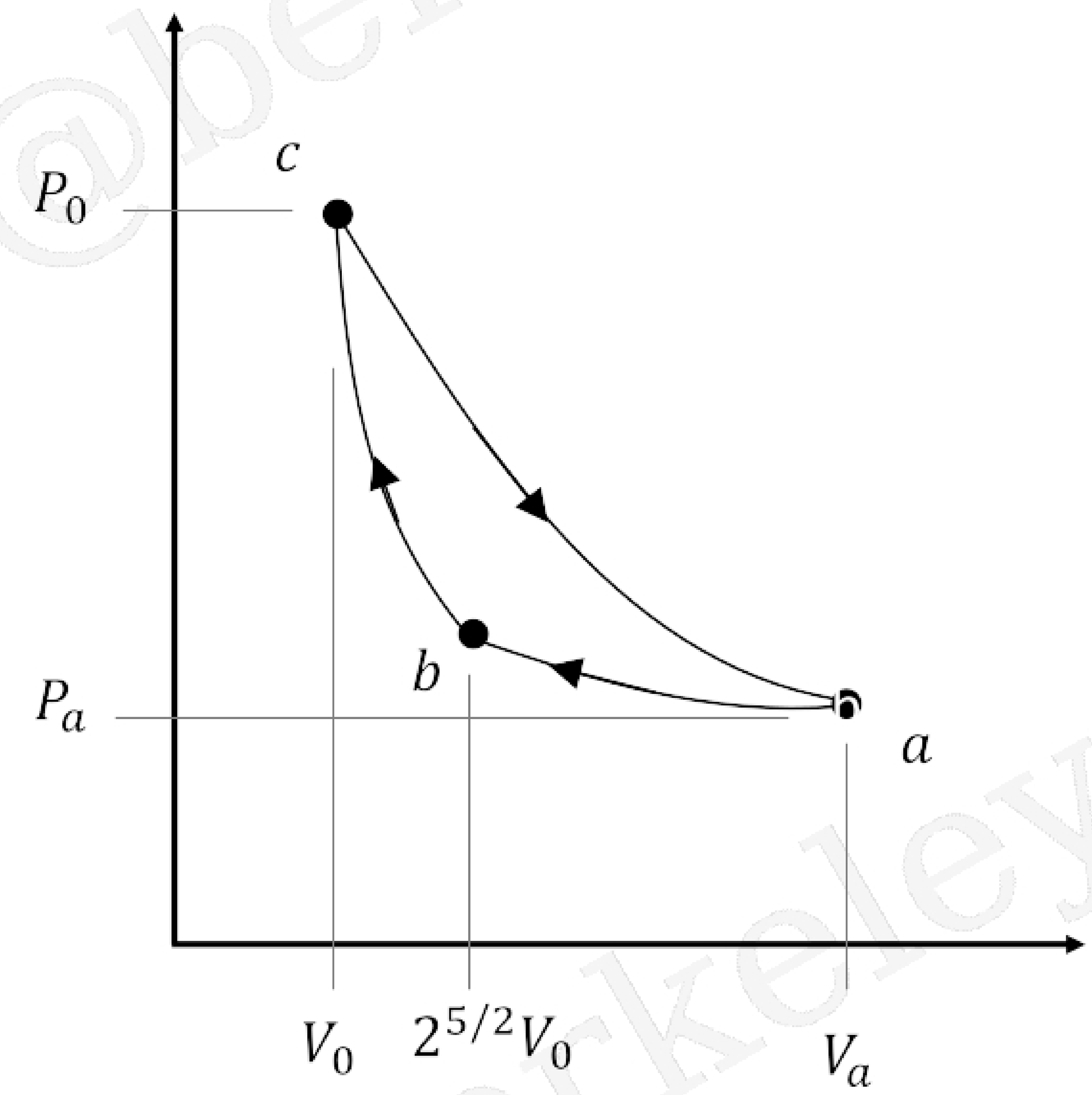


a. While the temperature of the iron will slowly decrease, will it ever be less than T_{CMB}? Explain in detail way.

T_{CMB}

b. What is τ, the time it takes the sphere to cool to $2T_{CMB}$? Express your answer in terms of any or all of the following constants: $T_{CMB}, A, m,$ the emissivity of iron e_{Fe}, the specific heat of iron c_{Fe}, and Stefan-Boltzmann's constant σ_{SB}. (As you will have to do an integral, you may find the integral table at the end of the exam useful.)

5. In the thermal cycle shown on the right, the process from $a \longrightarrow b$ is isothermal, while the process from $b \longrightarrow c$ is adiabatic. The working gas is a ***diatomic molecule*** operating around 100 ℃, and can be treated as ideal.



a. Can the process from $c \longrightarrow a$ be isothermal? Justify your answer in detail.

b. Along process from $c \longrightarrow a$

$$P = P_0 \left(\frac{V}{V_0}\right)^{-\frac{6}{5}}.$$

What is the pressure P_a and the volume V_a of the gas at a? Express your answer in terms of P_0 and/or V_0.

c. What is the net heat flow Q_{cycle} for this cycle? Express it in terms of P_0 and V_0. (You may find the integral table at the back of the exam useful.)

d. What is the efficiency e of this cycle?

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MT1 Equation Sheet

$$PV = Nk_BT,\ PV = nRT$$

$$P = \frac{nRT}{V - nb} - a\frac{n^2}{V^2}$$

$$\langle B(\vec{v})\rangle = N\left(\frac{m}{2\pi k_BT}\right)^{3/2}\iiint B(\vec{v})e^{-\frac{m\vec{v}^2}{2k_BT}}d^3\vec{v}$$

$$\langle E_p\rangle = \frac{3}{2}k_BT,\ v_{mp} = \sqrt{\frac{2k_BT}{m}},$$

$$v_{RMs} = \sqrt{\frac{3k_BT}{m}},\ \langle v\rangle = \sqrt{\frac{8k_BT}{\pi m}}$$

$$B_{RMS} = \sqrt{\langle B(\vec{v})^2\rangle - \langle B(\vec{v})\rangle^2}$$

$$\Delta L = L_0\alpha\Delta T,\ \Delta V = V_0\beta\Delta T$$

$$Q = mc\Delta T, \qquad Q = \pm mL$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$\frac{dQ}{dt} = \pm e\sigma_{SB}AT^4$$

$$\Delta E = Q - W,\ \ dE = dQ - PdV$$

$$W = \int_{path} PdV,\ \ dW = PdV$$

$$E = \frac{D}{2}Nk_BT = \frac{D}{2}PV$$

$$dS = \frac{dQ}{T}$$

$$(\boldsymbol{\Delta}E)_{cycle} = 0,\ W_{cycle} = Q_{cycle}$$

$$e = 1 - \frac{Q_L}{Q_H}, \qquad e = 1 - \frac{T_L}{T_H}$$

$$C.O.P. = \frac{Q_L}{Q_H - Q_L}, \qquad C.O.P. = \frac{Q_H}{Q_H - Q_L}$$

Isobaric Process:

$$V \propto T, \quad W = P\Delta V, \quad \Delta E = \frac{D}{2}P\Delta V$$

$$Q = \left(\frac{D}{2} + 1\right)P\Delta V,\ Q = C_Pn\Delta T,$$

$$C_P = \left(\frac{D}{2} + 1\right)R$$

Isochoric Process:

$$P \propto T, \quad W = 0, \quad \Delta E = \frac{D}{2}V\Delta P,$$

$$Q = \Delta E, \quad Q = C_Vn\Delta T, \quad C_V = \frac{D}{2}R$$

Isothermal Process:

$$PV = constant, \quad W = PV\ ln(V_2/V_1)$$

$$\Delta E = 0,\ \ Q = W$$

Adiabatic Process:

$$PV^{\gamma} = constant, \quad TV^{\gamma-1} = constant$$

$$W = -\Delta E,\ \ \Delta E = \frac{D}{2}(P_2V_2 - P_1V_1)$$

$$Q = 0,\ \ \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{D}$$

Integral Table

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}, n \neq -1$$

$$\int \frac{dx}{x} = \ln x$$

$$\int_0^{\infty} u^{2n}e^{-u^2}\,du = \frac{\sqrt{\pi}}{2^{n+1}}\frac{(2n-1)!}{n!}$$

$$\int_0^{\infty} u^{2n+1}e^{-u^2}du = \frac{1}{2}n!$$

$$\int \frac{dx}{x^2 - 1} = \frac{1}{2}\log\left(\frac{1-x}{1+x}\right).$$

$$\int \frac{dx}{x^4 - 1} = \frac{1}{4}\left[\log\left(\frac{1-x}{1+x}\right) - 2\tan^{-1}x\right].$$

$$\int \frac{xdx}{x^2 + 1} = \frac{1}{2}\log(x^2 + 1)$$

$$\int \frac{x^4dx}{x^4 - 1} = \frac{1}{4}\left[4x + \log\left(\frac{1-x}{1+x}\right) - 2\tan^{-1}x\right]$$