Midterm 1, 10/1.

(1) The series

$$\sum_{n=1}^{\infty} a_n$$

is convergent.

(a) (4 pnts) True or False? The series

$$\sum_{n=1}^{\infty} (-1)^n a_n$$

converges.

F $\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges $\quad \sum_{n=1}^{\infty} \frac{1}{n}$ diverges

(b) (4 pnts) True or False? The series

$$\sum_{n=1}^{\infty} a_n^2$$

converges.

F $\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ conv., $\quad \sum_{n=1}^{\infty} \frac{1}{n}$ div.

(2)

$$f(x,y) = \ln\left((\frac{x+y}{x-y})^{\frac{1}{2}} \right)$$

$f = \frac{1}{2}\ln(x+y) - \frac{1}{2}\ln(x-y)$

(a) (4 pnt) True or False?

$$\frac{\partial f}{\partial x} = \frac{y}{x^2-y^2}$$

F

$\frac{\partial f}{\partial x} = \frac{1}{2}\frac{1}{x+y} - \frac{1}{2}\frac{1}{x-y} = -\frac{y}{x^2-y^2}$

(b) (4 pnt) True or False?

$$\frac{\partial f}{\partial y} = \frac{x}{x^2-y^2}$$

T

$\frac{\partial f}{\partial y} = \frac{1}{2}\frac{1}{x+y} + \frac{1}{2(x-y)} = \frac{x}{x^2-y^2}$

(3)

$$f(x,y) = \sin(x+y)e^{x-y} = \sum_{n,m=0}^{\infty} a_{n,m}x^n y^m$$

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(a) (4 pnt) True or False?

$$a_{1,1} = 0$$

T

(b) (4 pnt) True or False?

$$a_{2,1} = 0$$

F

(c) (4 pnt) True or False?

$$a_{3,0} = 0$$

F

(4) Consider the power series

$$\sum_{n=0}^{\infty} \frac{1}{n+5}\left(\frac{z+2i}{1+2i}\right)^n$$

(a) (9 pnts) Find the radius of convergence of this power series.

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(b) (2 pnts) True or False? This powers series can conditionally converge inside the disc of convergence. F

(c) (2 pnts) True or False? This power series can conditionally converge outside of the disc of convergence. F

Any power series conv. absolutely inside the disc of conv. and div. outside.

(3) $f = \sin(x+y)e^{x-y} = \left((x+y) - \frac{(x+y)^3}{6} + \dots\right)$

$\left(1 + x-y + \frac{(x-y)^2}{2} + \dots\right) = x+y + (x+y)(x-y) +$

$+ (x+y)\frac{(x-y)^2}{2} - \frac{(x+y)^3}{6} + \dots = x+y + x^2-y^2 +$

$+ \frac{(x^2-y^2)(x-y)}{2} - \frac{x^3+3x^2y+\dots}{6} = x+y+x^2-y^2+$

$+ \frac{x^3-x^2y}{2} - \frac{x^3+3x^2y}{6} + \dots = x+y+x^2-y^2+\frac{1}{3}x^3 - x^2y + \dots$

$a_{11} = 0, \quad a_{2,1} = \frac{1}{3} \neq 0, \quad a_{3,0} = -1 \neq 0$

(4) $\sum_{n=0}^{\infty} \underbrace{\frac{1}{(n+5)}\left(\frac{z+2i}{1+2i}\right)^n}_{a_n}$

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{n}{n+5}\left|\frac{z+2i}{1+2i}\right| = \frac{n}{n+5}\frac{|z+2i|}{\sqrt{5}} \to \frac{|z+2i|}{\sqrt{5}}$

By the Ratio Test: $\begin{cases} |z+2i| < \sqrt{5} & \text{the series conv. absolutely} \\ |z+2i| > \sqrt{5} & \text{diverges} \end{cases}$

$\Rightarrow R = \sqrt{5}$ and our series is centered at $-2i$

(5) (9 pnts) Find local minima of the function

$$f(x,y) = \frac{x^4}{4} + \frac{y^4}{4} - \frac{x^2}{2} + \frac{y^2}{2}$$

.

Hint: Find critical points. Use the second derivative test on critical points.

$f_x = x^3 - x$, $f_y = y^3 + y$

. critical points: $f_x = 0$, $x^3 - x = 0$ $\begin{cases} x = 0 \\ x = \pm 1 \end{cases}$

$f_y = 0$, $y^3 + y = 0$, $y = 0$

Thus: $(0,0)$, $(1,0)$, $(-1,0)$

Second derivatives: $f_{xx} = 3x^2 - 1$, $f_{yy} = 3y^2 + 1$, $f_{xy} = 0$

$(0,0)$, $f_{xx} = -1$, $f_{yy} = 1$, saddle pnt

$(1,0)$, $f_{xx} = 2$, $f_{yy} = 1$, $f_{xx} > 0$, $f_{yy} > 0$,
$f_{xx} f_{yy} > f_{xy}^2$
$\Rightarrow$ min

$(-1,0)$, $f_{xx} = 2$, $f_{yy} = 1$, same, min

local minima: $(1,0)$, $(-1,0)$. Note, they are also global minima $f(1,0) = f(-1,0)$