PHYSICS 7B

UC BERKELEY

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7B Lectures 2 & 3 Final Solutions

Problem 1

Consider the conducting sphere of radius R and charge Q_0 shown in the figure below. The conducting sphere is located in the interior of a spherical shell of internal radius R_1 and external radius $R_2, R_2 > R_1$. The inside of the spherical shell has a charge $Q = -Q_0/2$, whose charge distribution obeys the following rule $\rho(r) = Ar$.

Find:

(a) (5 pts.) The value of the constant A.

Solution: To find the constant A, we need to use the self-consistency equation

$$Q_{\text{shell}} = \int \mathrm{d}\mathbf{r}^3 \rho(\mathbf{r}) \qquad \text{(2pts.)} \tag{1}$$

Now we plug in $Q_{\text{shell}} = -Q_0/2$ and $\rho(\mathbf{r}) = Ar$,

$$-\frac{1}{2}Q_0 = 4\pi \int_{R_1}^{R_2} Arr^2 \mathrm{d}r = \pi A \left(R_2^4 - R_1^4\right) \qquad \text{(2pts.)} \tag{2}$$

Finally we can write A as

$$A = -\frac{Q_0}{2\pi \left(R_2^4 - R_1^4\right)} \qquad \text{(1pt.)} \tag{3}$$

(b) (5 pts.) The electric field as a function of distance from the center of the conducting sphere.

Solution: We can use Gauss's law to find the electric field everywhere,

$$\int \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q_{\text{env}}}{\epsilon_0} \quad \text{(1pt.)} \tag{4}$$

where we assumed spherical symmetry in the second equality. Inside the conducting sphere, there is no charge,

$$\mathbf{E}_{r<R} = 0 \qquad \text{(1pt.)} \tag{5}$$

Between the sphere and the shell, we have charge Q_0 on the surface of the conducting sphere,

$$\mathbf{E}_{R<r<R_1} = \frac{Q_0}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}} \qquad \text{(0.5pt.)} \tag{6}$$

Inside the shell, we have both charge Q_0 on the sphere and those $\rho(r) = Ar$ inside the shell,

$$\mathbf{E}_{R_1<r<R_2} = \frac{1}{4\pi\epsilon_0 r^2}\left(Q_0 + 4\pi \int_0^r Ar' r'^2 \mathrm{d}r'\right)\hat{\mathbf{r}} = \frac{1}{4\pi\epsilon_0 r^2}\left(Q_0 + \pi A(r^4 - R_1^4)\right)\hat{\mathbf{r}} \tag{7}$$

$$= \frac{Q_0}{4\pi\epsilon_0 r^2}\left(1 - \frac{1}{2}\frac{r^4 - R_1^4}{R_2^4 - R_1^4}\right)\hat{\mathbf{r}} \qquad \text{(2pt.)} \tag{8}$$

Finally, outside the shell, the total charge inside is just $Q_0/2$,

$$\mathbf{E}_{r>R_2} = \frac{Q_0}{8\pi\epsilon_0 r^2}\hat{\mathbf{r}} \qquad \text{(0.5pt.)} \tag{9}$$

Missing the direction will result in 1 pt. off.

(c) (5 pts.) Consider a point like particle of charge q, mass m located at a distance $R_3 > R_2$ from the center of the sphere. If the particle is moving with initial velocity v_0, directed toward the center of the sphere, find the minimum value of v_0 that will allow the particle to reach the center of the sphere.

Solution: The minimum value of v_0 is determined by energy conservation $KE = PE$,

$$\frac{1}{2}mv_0^2 = q\Delta V \qquad \text{(1pt.)} \tag{10}$$

where $\Delta U = q\Delta V$ is the potential energy difference the particle has to overcome. Now we integrate from R_3 all the way down to 0 to find the potential difference ΔV,

$$\Delta V = \int_0^{R_3} \mathbf{E}\cdot \mathrm{d}\mathbf{r} = \Delta V_{r<R} + \Delta V_{R<r<R_1} + \Delta V_{R_1<r<R_2} + \Delta V_{R_2<r<R_3} \qquad \text{(0.5pt.)} \tag{11}$$

where

$$\Delta V_{r<R} = \int_0^R E_{r<R}\mathrm{d}r = 0 \qquad \text{(0.5pt.)} \tag{12}$$

$$\Delta V_{R<r<R_1} = \int_R^{R_1} E_{R<r<R_1}\mathrm{d}r = \frac{Q_0}{4\pi\epsilon_0}\int_R^{R_1}\frac{\mathrm{d}r}{r^2} = \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{R}-\frac{1}{R_1}\right) \qquad \text{(0.5pt.)} \tag{13}$$

$$\Delta V_{R_2<r<R_3} = \int_{R_2}^{R_3} E_{r>R_2}\mathrm{d}r = \frac{Q_0}{8\pi\epsilon_0}\int_{R_2}^{R_3}\frac{\mathrm{d}r}{r^2} = \frac{Q_0}{8\pi\epsilon_0}\left(\frac{1}{R_2}-\frac{1}{R_3}\right) \qquad \text{(0.5pt.)} \tag{14}$$

$$\Delta V_{R_1<r<R_2} = \int_{R_1}^{R_2} E_{R_1<r<R_2}\mathrm{d}r = \frac{Q_0}{4\pi\epsilon_0}\int_{R_1}^{R_2}\left(\frac{1}{r^2} - \frac{1}{2r^2}\frac{r^4-R_1^4}{R_2^4-R_1^4}\right)\mathrm{d}r \tag{15}$$

$$= \frac{Q_0}{4\pi\epsilon_0}\left(-\frac{R_1+R_2}{3\left(R_1^2+R_2^2\right)} - \frac{1}{3(R_1+R_2)} + \frac{1}{R_1} - \frac{1}{2R_2}\right) \qquad \text{(1.5pts.)} \tag{16}$$

It is ok to not simplify the formula and write

$$\Delta V_{R_1<r<R_2} = \frac{Q_0}{4\pi\epsilon_0}\left(\frac{R_1^4\left(\frac{1}{R_1}-\frac{1}{R_2}\right) - \frac{1}{3}\left(R_2^3-R_1^3\right)}{R_2^4-R_1^4} + \frac{1}{R_1} - \frac{1}{R_2}\right) \tag{17}$$

Now we can put everything together and find

$$\Delta V = \frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{R_1+R_2}{3\left(R_1^2+R_2^2\right)} - \frac{1}{3(R_1+R_2)} - \frac{1}{2R_3}\right) \tag{18}$$

$$\implies v_0 = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2q}{m}}\sqrt{\frac{Q_0}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{R_1+R_2}{3\left(R_1^2+R_2^2\right)} - \frac{1}{3(R_1+R_2)} - \frac{1}{2R_3}\right)} \qquad \text{(0.5pt.)} \tag{19}$$

Problem 2:

a. (4 pts) Since the shaded region has uniform charge density, the charge density is simply

$$\rho = \frac{Q}{V}, \qquad \text{(1 point)}$$

where Q is the total charge given by Gauss's law

$$\int \mathbf{E} \cdot \mathbf{da} = \phi = \frac{Q}{\epsilon_0} \rightarrow Q = \epsilon_0 \phi, \qquad \text{(1 point)}$$

and the volume is given by the volume of the large sphere minus the volume of the two cavities

$$V = \frac{4\pi}{3}\left[R^3 - 2R_1^3\right]. \qquad \text{(1 point)}$$

Putting this together,

$$\rho = \frac{3\epsilon_0 \phi}{4\pi\left[R^3 - 2R_1^3\right]}. \qquad \text{(1 point)}$$

b. (6 pts) To obtain the electric field, along the z axis, we must use the principle of superposition to calculate the electric field due to the uniform sphere of radius R and charge density ρ and the two cavities of radius R_1 and charge density $-\rho$. Mathematically,

$$\mathbf{E}_{\text{tot}} = \mathbf{E}_{\text{s}} + \mathbf{E}_{\text{cl}} + \mathbf{E}_{\text{cr}}, \qquad \text{(2 point)}$$

where we must remember that these are vector quantities.

For $a > 1$, from application of Gauss's law we obtain

$$\begin{aligned}
\mathbf{E}_{\text{s}}(z) &= \frac{\rho}{3\epsilon_0}\left[\frac{R^3}{z^2}\right]\hat{z} \\
\mathbf{E}_{\text{cl}}(r_l) &= -\frac{\rho}{3\epsilon_0}\left[\frac{R_1^3}{r_l^2}\right]\hat{r_l} \\
\mathbf{E}_{\text{cr}}(r_r) &= -\frac{\rho}{3\epsilon_0}\left[\frac{R_1^3}{r_r^2}\right]\hat{r_r}, \qquad \text{(1 point)}
\end{aligned}$$

where $\mathbf{r_l}$ and $\mathbf{r_r}$ refer to the radial vectors centered on the left and right cavities respectively and ending on the point $z = aR$. They both have magnitude $r = \sqrt{z^2 + d^2}$. Adding these vectors and noting that by symmetry only components along $\hat{z}$ don't cancel,

$$\mathbf{E}_{\text{cl}} + \mathbf{E}_{\text{cr}} = 2E_{\text{cl}} \cos\theta \hat{z} = 2E_{\text{cl}} \frac{z}{r_l} \hat{z} \qquad \text{(1 point)}$$

$$\begin{aligned} \rightarrow \mathbf{E}_{\text{tot}}^{a>1} &= \left[E_s + 2E_{cl} \frac{z}{r_l} \right] \hat{z} \\ &= \frac{\rho}{3\epsilon_0} \left[\frac{R^3}{z^2} - \frac{2R_1^3 z}{r^3} \right] \hat{z} \\ &= \frac{\rho}{3\epsilon_0} \left[\frac{R}{a^2} - \frac{2aR_1^3 R}{((aR)^2 + d^2)^{3/2}} \right] \hat{z} \\ &= \frac{\phi R}{4\pi \left[R^3 - 2R_1^3 \right]} \left[\frac{1}{a^2} - \frac{2aR_1^3}{((aR)^2 + d^2)^{3/2}} \right] \hat{z}. \qquad \text{(0.5 point)} \end{aligned}$$

For $a < 1$, $\mathbf{E}_{\text{cl}}$ and $\mathbf{E}_{\text{cr}}$ are the same as above since all point along the z axis lie outside the cavities, however now Gauss's law yields

$$\mathbf{E}_{\text{s}}(z) = \frac{\rho}{3\epsilon_0} z \hat{z}. \qquad \text{(1 point)}$$

With this replacement, the total field is given by

$$\begin{aligned} \mathbf{E}_{\text{tot}}^{a<1} &= \frac{\rho}{3\epsilon_0} \left[z - \frac{2R_1^3 z}{r^3} \right] \hat{z} \\ &= \frac{\phi a R}{4\pi \left[R^3 - 2R_1^3 \right]} \left[1 - \frac{2R_1^3}{((aR)^2 + d^2)^{3/2}} \right] \hat{z}. \qquad \text{(0.5 point)} \end{aligned}$$

Alternatively, this problem can be solved by direct integration using the Coulomb law, for which credit will be given as follows:

$$\begin{aligned} \mathbf{E}_{\text{tot}} &= \mathbf{E}_{\text{s}} + \mathbf{E}_{\text{cl}} + \mathbf{E}_{\text{cr}}, \qquad \text{(2 point)} \\ \mathbf{E}_{\text{tot}}^{a>1} &= \frac{1}{4\pi\epsilon_0} \int_0^R dr \rho 4\pi r^2 \frac{1}{(aR)^2} - \frac{2}{4\pi\epsilon_0} \frac{\frac{4}{3}\pi R_1^3 \rho a R}{((aR)^2 + d^2)^{3/2}} \qquad \text{(2 point)} \\ \mathbf{E}_{\text{tot}}^{a<1} &= \frac{1}{4\pi\epsilon_0} \int_0^{aR} dr \rho 4\pi r^2 \frac{1}{(aR)^2} - \frac{2}{4\pi\epsilon_0} \frac{\frac{4}{3}\pi R_1^3 \rho a R}{((aR)^2 + d^2)^{3/2}} \qquad \text{(2 point)} \end{aligned}$$

c. (5 pts) Taking $V = 0$ at $z = \infty$, the electric potential at the surface of the sphere is given by integrating over the electric field for $a > 1$ with magnitude $E(z)$

$$\begin{aligned} V &= -\int_{z=\infty}^{z=R} \mathbf{E} \cdot \mathbf{dl} \qquad \text{(2 point)} \\ &= -\int_{\infty}^{R} E(z) dz \qquad \text{(1 point)} \\ &= -\frac{\rho}{3\epsilon_0} \int_{\infty}^{R} \left[\frac{R^3}{z^2} - \frac{2R_1^3 z}{(z^2 + d^2)^{3/2}} \right] dz \qquad \text{(1 point)} \\ &= \frac{\phi}{4\pi \left[R^3 - 2R_1^3 \right]} \left[R^2 - \frac{2R_1^3}{\sqrt{R^2 + d^2}} \right] \qquad \text{(1 point)} \end{aligned}$$

Problem 3

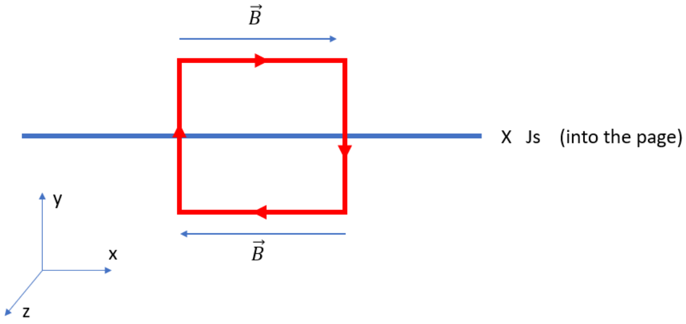
1 Solution

Part a) Find the magnetic field generated by the infinite sheet.

Use Ampere's law:

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \tag{1}$$

By symmetry, B cannot depend on x or z. Draw square of side l around the sheet. Also by the symmetry of the problem only the top and bottom terms of the path integral contribute. By the right hand rule, the direction of the magnetic field due to a current going into the page is clockwise.



$$\int_{top} \vec{B} \cdot d\vec{l} + \int_{bottom} \vec{B} \cdot d\vec{l} + \int_{sides} \vec{B} \cdot d\vec{l} = Bl + Bl + 0 = 2Bl = \mu_0 I_{enc} \tag{2}$$

The current enclosed is:

$$I_{enc} = J_s l \tag{3}$$

So the magnetic field is:

$$B = \frac{\mu_0 J_s l}{2l} = \frac{\mu_0 J_s}{2} \tag{4}$$

It points to the right above the sheet and to the left below the sheet.

Part b Find the force per unit length exerted on the wire.

The force on the wire is the magnetic force due to the magnetic field produced by the sheet of current.

$$\vec{F} = i_f \vec{l} \times \vec{B} = \frac{i_f l \mu_0 J_s}{2}(-\hat{y}) \tag{5}$$

So the force per length is:

$$\frac{\vec{F}}{l} = \frac{\mu_0 i_f J_s}{2}(-\hat{y}) \tag{6}$$

The direction of the force was found using the right hand rule for the force exerted by the magnetic field on top of the sheet.

Part c) Find the distance y above the infinite sheet where the total magnetic field is zero.

The magnetic field is zero when the magnetic field generated by the current in the wire cancels out the magnetic field produced by the current density in the sheet.

The magnetic field generated by the wire is:

$$B_w = \frac{i_f \mu_0}{2\pi r} \tag{7}$$

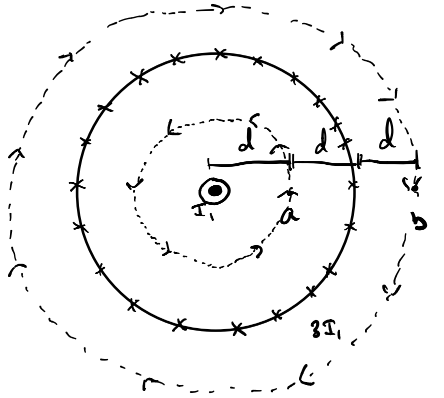
Where r is the distance from the wire.

So the distance y can be found by equating the two fields:

$$B = B_w \rightarrow \frac{i_f \mu_0}{2\pi y} = \frac{\mu_0 J_s}{2} \rightarrow y = \frac{i_f}{\pi J_s} \tag{8}$$

Lecture 2 & 3 Final Exam Problem 4 Solution + Rubric

Solution:



Method: Use Ampere's Law $\mu_0 I_{enc} = \oint \vec{B}\cdot d\vec{l}$

Field at a: $I_{enc} = I_1$, $\oint \vec{B}\cdot d\vec{l} = B\cdot(2\pi d) = 2\pi d B$

$$\Rightarrow \mu_0 I_1 = 2\pi d B \Rightarrow \boxed{B = \frac{\mu_0 I_1}{2\pi d}}$$

Since the current is out of the page, RHR tells us the field circulates counter-clockwise around the wire, so the magnetic field points up at a.

Field at b: $|I_{enc}| = I_2 - I_1 = 3I_1 - I_1 = 2I_1$, $\oint \vec{B}\cdot d\vec{l} = B\cdot(2\pi\cdot 3d) = 6\pi d B$

$$\Rightarrow 2\mu_0 I = 6\pi d B \Rightarrow \boxed{B = \frac{\mu_0 I}{3\pi d}}$$

Now the net current enclosed is into the page, so the RHR tells us that the field at b points down.

Rubric

(i) Correctly state/use Ampere's Law $\mu_0 I_{enc} = \oint \vec{B}\cdot d\vec{l}$ (4 points)

(ii) Correctly find I_{enc} for a (1 point)

(iii) Correctly find $\oint \vec{B}\cdot d\vec{l}$ for a (1 point)

(iv) Find the correct direction for a (1 point)

(v) Correctly find I_{enc} for b (1 point)

(vi) Correctly find $\oint \vec{B}\cdot d\vec{l}$ for b (1 point)

(vii) Find the correct direction for b (1 point)

Problem 5 - Solution

A metallic rod of length L can slide right and left on two conducting tracks of a circuit as shown in the figure below. The rod moves with velocity $\mathbf{v}$ in the presence of a magnetic field $\mathbf{B}$. The field $\mathbf{B}$ is generated by an infinitely long vertical wire placed at a distance a from the circuit. A constant current I passes through the wire.

The rod and the tracks of the circuit have a resistance R. Neglect the friction between the rod and the track and the inductance of the circuit. At $t = 0$ the rod is located on the left side of the circuit at a distance a from the infinitely long wire. The rod moves with constant velocity $\mathbf{v}$.

a) (6pts)

Find the direction and magnitude of the current in the rod at time T.

Let $\hat{\mathbf{z}}$ be the unit vector pointing out of the page and $\hat{\mathbf{x}}$ be the unit vector pointing to the right, the direction of motion of the rod. The magnetic field produced by the wire at a position $r\hat{\mathbf{x}}$ is:

$$\mathbf{B} = -\frac{\mu_0 I}{2\pi r}\hat{\mathbf{z}} \tag{1}$$

That is, pointing into the page. For our purposes, $r = a + x(t)$ where $x(t) = vt$ is the distance of the rod from its starting position. At a time t, then, the flux through the loop is given by:

$$\Phi = -L\int_0^{x(t)} \frac{\mu_0 I}{2\pi(a+x')}dx' = \frac{\mu_0 I}{2\pi}\ln\frac{a}{a+x(t)} \tag{2}$$

where $x(t) = vt$ as before. Faraday's law then gives the induced EMF as:

$$\mathcal{E} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{\mu_0 I L v}{2\pi(a+vt)} \tag{3}$$

The current induced in the wire is then given by $|\mathcal{E}|/R$ by Ohm's law:

$$\boxed{I_{\text{induced}}(T) = \frac{\mu_0 I L v}{2\pi R(a+vT)}} \tag{4}$$

The direction of this current can be found by Lenz's law – the induced current should create a magnetic field opposing the change in flux. The flux increases in the $-\hat{\mathbf{z}}$ direction as the rod moves to the right, so the induced field should be in the $+\hat{\mathbf{z}}$ direction. Using the right hand rule, we conclude that the induced current will be **upward in the rod, parallel to the current in the infinite wire, counter-clockwise around the circuit formed by the rod and the tracks.**

b) (4pts)

Find the total energy dissipated in the resistance of the circuit at time T.

The power (rate of energy change) dissipated by a resistor of resistance R with a current I flowing through it is $P = I^2R$. In our case, then:

$$P(t) = \frac{\mathrm{d}E}{\mathrm{d}t} = I_{\text{induced}}(t)^2 R = \frac{\mu_0^2 I^2 L^2 v^2}{4\pi^2 R(a+vt)^2} \tag{5}$$

The total energy dissipated is then the integral of this:

$$E(T) = \int_0^T \frac{\mu_0^2 I^2 L^2 v^2}{4\pi^2 R(a+vt)^2} dt \tag{6}$$

$$\implies \boxed{E(T) = \frac{\mu_0^2 I^2 L^2 v^2}{4\pi^2 Ra(a+vT)} T} \tag{7}$$

c) (5pts)

Find an expression for the force required to maintain the uniform motion.

The magnetic field from the infinite wire will exert a force on the induced current:

$$\mathbf{F}(t) = L\mathbf{I}_{\text{induced}}(t) \times \mathbf{B}(vt) = (-\hat{\mathbf{x}}) \frac{\mu_0^2 I^2 L^2 v}{4\pi^2 R(a+vt)^2} \tag{8}$$

We need an equal and opposite force $\mathbf{F}_u$ to maintain uniform motion:

$$\boxed{\mathbf{F}_u(t) = \hat{\mathbf{x}} \frac{\mu_0^2 I^2 L^2 v}{4\pi^2 R(a+vt)^2}} \tag{9}$$

The energy dissipated by the resistor will be replaced by this force, so that the kinetic energy of the rod stays constant.

Problem 6

a. By Faraday's Law

$$i = -\frac{1}{R}\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

1

$$= \frac{\mu_0 n I(0) \pi a^2 \cos\theta}{R\tau}$$

3

in the positive $\hat{\boldsymbol{\phi}}$ direction **1**

b.

$$\mathbf{B}_{tot} = \frac{\mu_0 n I(0) t}{\tau}\hat{\mathbf{z}} - \frac{\mu_0 i}{2a}\cos\theta\hat{\mathbf{z}} - \frac{\mu_0 i}{2a}\sin\theta\hat{\mathbf{x}}$$

2+2+2

c.

$$\mathbf{A} = \pi a^2(\cos\theta\hat{\mathbf{z}} + \sin\theta\hat{\mathbf{x}})$$

$$\boldsymbol{\mu} = i\mathbf{A}$$

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}_{tot}$$

1

$$= \frac{\mu_0^2 n^2 I(0)^2 \pi^2 a^4 \cos\theta\sin\theta}{R\tau^2}\hat{\mathbf{y}}$$

3

Problem 7 Solution

(a) We first note that the pressure balances the elastic force:

$$PA = k\Delta x \Rightarrow \boxed{P = \frac{k\Delta x}{A}.}$$

The temperature is then given by the ideal gas law,

$$PV = nRT \Rightarrow T = \frac{PV}{nR}.$$

The volume is $\frac{V}{2}$, $n = 1$, and we plug in our answer for the pressure to get

$$\boxed{T = \frac{k\Delta x V}{2AR}.}$$

(b) After a long time, the two sides of the box will have equal temperature and number density and therefore equal pressure. Thus, the spring must be in its equilibrium position and exert no force. We then note that the total change in energy (the change in internal energy of the gas plus the change in elastic energy of the spring) must be zero because the container is adiabatic and does no work as a whole:

$$\Delta E_{\text{int}} + \left(\frac{1}{2}k\,(0)^2 - \frac{1}{2}k\Delta x^2\right) = 0 \Rightarrow \boxed{\Delta E_{\text{int}} = \frac{1}{2}k\Delta x^2.}$$

(c) To find the final temperature, we use the change in internal energy:

$$\Delta E_{\text{int}} = \frac{3}{2}nR\Delta T \Rightarrow T = \frac{k\Delta x V}{2AR} + \frac{2}{3}\frac{\Delta E_{\text{int}}}{nR} = \boxed{\frac{k\Delta x}{R}\left(\frac{\Delta x}{3} + \frac{V}{2A}\right).}$$

The final pressure is simply given by the ideal gas law,

$$P = \frac{nRT}{V} = \boxed{k\Delta x\left(\frac{\Delta x}{3V} + \frac{1}{2A}\right).}$$

(d) For this problem, we note that entropy is a function of state. We then find a *reversible* process that leads us to the same final macrostate as the process in the problem and compute $\Delta S = \int \frac{dQ}{T}$ for that reversible process. In other words, we want the change in entropy for a reversible process with

$$\begin{aligned}
&\text{Volume: } \frac{V}{2} \to V \\
&\text{Pressure: } \frac{k\Delta x}{A} \to k\Delta x\left(\frac{\Delta x}{3V} + \frac{1}{2A}\right) \\
&\text{Temperature: } \frac{k\Delta x V}{2AR} \to \frac{k\Delta x}{R}\left(\frac{\Delta x}{3} + \frac{V}{2A}\right).
\end{aligned}$$

There are many ways to do this. We will break this into two processes: isothermal expansion and isovolumetric heating. The isothermal expansion is

$$\begin{aligned}
&\text{Volume: } \frac{V}{2} \to V \\
&\text{Pressure: } \frac{k\Delta x}{A} \to \frac{k\Delta x}{2A} \\
&\text{Temperature: } \frac{k\Delta x V}{2AR} \to \frac{k\Delta x V}{2AR}.
\end{aligned}$$

The isovolumetric heating is

$$\begin{aligned}&\text{Volume: } V \to V\\&\text{Pressure: } \frac{k\Delta x}{2A} \to k\Delta x\left(\frac{\Delta x}{3V}+\frac{1}{2A}\right)\\&\text{Temperature: } \frac{k\Delta x V}{2AR} \to \frac{k\Delta x}{R}\left(\frac{\Delta x}{3}+\frac{V}{2A}\right).\end{aligned}$$

We will now compute the entropy change for these processes. Starting with the isothermal expansion, we have $\Delta U = Q - W = 0 \Rightarrow Q = W$. Therefore,

$$\Delta S = \frac{Q}{T} = \frac{W}{T} = \frac{1}{T}\int P\left(V'\right)dV' = \frac{1}{T}\int \frac{nRT}{V'}dV' = R\int_{\frac{V}{2}}^{V}\frac{dV'}{V'} = R\ln 2.$$

For the isovolumetric heating, $dQ = \frac{3}{2}nRdT = \frac{3}{2}RdT$ for $n = 1$. Therefore,

$$\Delta S = \int\frac{dQ}{T} = \int\frac{3}{2}R\frac{dT}{T} = \frac{3}{2}R\int_{\frac{k\Delta x V}{2AR}}^{\frac{k\Delta x}{R}\left(\frac{\Delta x}{3}+\frac{V}{2A}\right)}\frac{dT}{T} = \frac{3}{2}R\ln\left(1+\frac{2A\Delta x}{3V}\right).$$

We then sum the two changes in entropy, obtaining

$$\boxed{\Delta S = R\left(\ln 2 + \frac{3}{2}\ln\left(1+\frac{2A\Delta x}{3V}\right)\right).}$$