Name: ______________________________

SID: ______________________________

Physics 7B Final - Fall 2020
Professor R. Birgeneau

Total Points: 105 (7 Problems + bonus)
This exam is out of 100 points with 5 bonus points. Show all your work and take particular care to explain your steps. Partial credit will be given. Use symbols defined in problems and define any new symbols you introduce. ***If a problem requires that you obtain a numerical result, first write a symbolic answer and then plug in numbers.*** *Label any drawings you make.* **Good luck!**

Problem 1 (15 pts.) A mole of a monatomic ideal gas is taken from an initial state of pressure P and volume V to a final state of pressure $2P$ and volume $2V$ by two different processes: (I) It expands isothermally until its volume is double, and then its pressure is increased at constant volume to the final state. (II) It is compressed isothermally until its pressure is doubled, and then its volume is increased at constant pressure to the final state. In terms of P and V,

(a) (3 pts.) draw and label the path of each process on a $p-V$ diagram.

(b) (4 pts.) calculate the heat absorbed by the gas in each part of the process

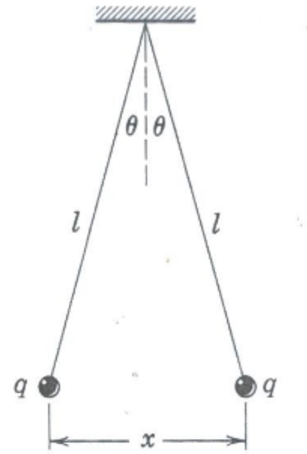
(c) (3 pts.) calculate the change in internal energy of the gas

(d) (4 pts.) calculate the change in entropy of the gas

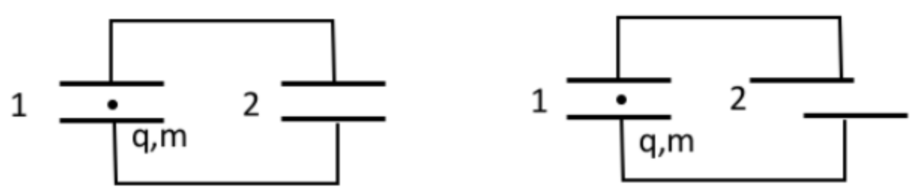
Problem 2 (15 pts.)
Consider the diagram below, which consists of identical balls of mass m and charge q that are hung from wires of length L.

(a) (8 pts.) Assume that the angle θ is very small. Find an expression for the distance between the balls, x, in terms of given quantities and fundamental constants.

(b) (7 pts.) Without assuming that theta is small, find an expression from which theta can be found



Problem 3 (10 pts.) Two identical capacitors with capacitance C have been charged to charge Q by a battery and connected as shown in the left figure below. A particle with charge q and mass m stays stable in the middle of capacitor 1. Assume that there is a gravitational field g that points downward. After disconnecting the battery, the two plates of capacitor 2 are staggered horizontally to make the overlapping area to be half of the original area (figure on the right below). Find the acceleration of the charged particle in the capacitor 1.

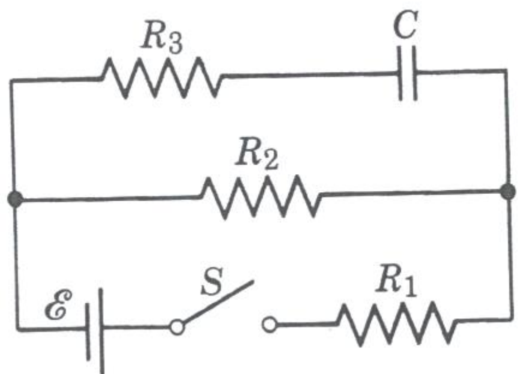


Problem 4 (15 pts.) Consider the circuit below. The battery has an emf $\mathcal{E} = 1200$ V and the capacitor has capacitance $C = 6.5\mu$f. The resistors have resistances $R_1 = R_2 = R_3 = 7.3 \times 10^5\Omega$. With C uncharged, the switch S is suddenly closed at $t = 0$.

(a) (5 pts.) Find, for $t = 0$ and $t \to \infty$, the current through each resistor

(b) (5 pts.) Draw a graph of the potential drop V_2 across R_2 from $t = 0$ to $t \to \infty$

(c) (5 pts.) What are the numerical values of V_2 at $t = 0$ and $t \to \infty$



Problem 5 (15 pts.) Consider a wire with radius R and a non-uniform current density

$$J = \begin{cases} J_0\left(\frac{r}{3R} - \frac{r^2}{2R^2}\right) & r \leq R \\ 0 & R < r \end{cases}$$

where J_0 and R are constants.

(a) (6 pts.) Determine the magnetic field inside the wire

(b) (6 pts.) Determine the magnetic field for $r > R$

(c) (3 pts.) Determine the location r where the magnetic field is a maximum.

Problem 6 (15 pts.)

(a) (5 pts.) Consider a wire that has a uniform current density such that it carries a total current I. Find the magnetic energy per unit length **inside the wire** in terms of the current I and fundamental constants.

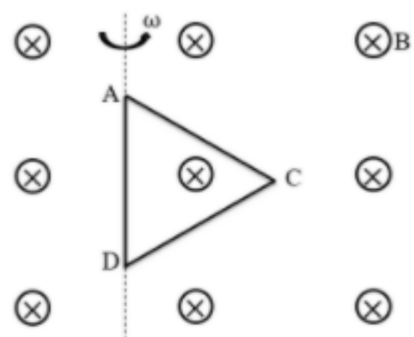
(b) (5 pts.) For the wire above, calculate the magnetic energy per unit length **outside the wire**. Does this result make sense physically?

(c) (5 pts.) If we consider a length L of the above wire, find the self-inductance of the wire in terms of given variables and fundamental constants.

Problem 7 (15 pts.) An equilateral triangle coil with side length L and electric resistance R is rotating with constant angular velocity ω with respect to axis AD. The uniform magnetic field B is perpendicular to axis AD, as indicated below. At time $t = 0$, B is exactly perpendicular to the triangle coil. At any given time t,

(a) (8 pts.)Calculate the induced emf of the coil

(b) (7 pts.) Calculate the electric potential between A and C.



Bonus(5 pts.) Maxwell augmented Ampere's Law with a new term which is called the displacement current so that in differential form Ampere's Law reads:

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{d\vec{E}}{dt} \tag{1}$$

Show that the addition of the displacement current makes Ampere's Law consistent with the charge continuity equation:

$$\nabla \cdot \vec{j} + \frac{d\rho}{dt} = 0 \tag{2}$$