
Midterm 2

Last Name	First Name	SID
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Rules.

- **Unless otherwise stated, all your answers need to be justified and your work must be shown. Answers without sufficient justification will get no credit.**
- You have 70 minutes to complete the exam and 10 minutes exclusively for submitting your exam to Gradescope. (DSP students with $X\%$ time accomodation should spend $70 \cdot X\%$ time on the exam and 10 minutes to submit).
- Collaboration with others is strictly prohibited.
- You may reference your notes, the textbook, and any material that can be found through the course website. You may use Google to search up general knowledge. However, searching up a question is not allowed.
- You may not use online solvers or graphing tools (ex. WolframAlpha, Desmos, Python). Simple functions (ex. combinations, multiplication) are fine.
- For any clarifications you have, please create a private Piazza post. We will have a Google Doc that shows our official clarifications.

Problem	points earned	out of
Problem 1		12
Problem 2		14
Problem 3		12
Problem 4		10
Problem 5		12
Problem 6		8
Problem 7		20
Total		88

1 Imposter [4+4+4 points]

The EECS 126 course staff are playing a game of Among Us 2! In this game, everyone is stranded aboard a spaceship. Nikita and Sean are imposters and are trying to kill as many crewmates as possible; there are infinitely many crewmates on the ship. Kills can be discovered by crewmates, but some kills will never be discovered. Nikita kills crewmates according to a Poisson process of rate λ_1 , and each of her kills will be discovered at some point later in the game with probability p_1 . Sean kills crewmates according to an independent Poisson process of rate λ_2 , and each of his kills will be discovered at some point later in the game with probability p_2 . There are no interruptions to this process, i.e. there are no stops/pauses/meetings.

(a) Sean has just killed a crewmate. Starting from this point, what is the expected amount of time until Nikita kills a crewmate?

(b) What is the expected amount of time until the first kill that will later be discovered is made?

(c) What is the probability that the kill in part (b) was caused by Nikita?

2 Is Anything Really Continuous? [4+5+5 points]

You observe the arrivals of a Poisson process with unknown parameter λ , which we will seek to estimate. However, your stopwatch doesn't track milliseconds, so you can't see the exact times and can only see them rounded up to the next second: instead of interarrival times t_1, t_2, \dots , you can only observe $T_1 = \lceil t_1 \rceil, T_2 = \lceil t_2 \rceil, \dots$.

1. Find the MLE of λ if we knew the *true* interarrival times t_1, \dots, t_n .
2. Show that the rounded-up interarrival times T_i are distributed according to $\text{Geometric}(1 - e^{-\lambda})$.
3. Find the MLE of λ given rounded-up interarrival times T_1, \dots, T_n .

3 Times Up! [5+7 points]

Consider a sequence of independent random variables X_1, X_2, \dots where $X_n \sim \text{Exponential}(10 \ln(n))$.

1. Does the sequence converge in probability?

2. Does the sequence converge almost surely? [*Hint*: Borel-Cantelli could be of use here]

4 A Hypothetical Scenario [10 points]

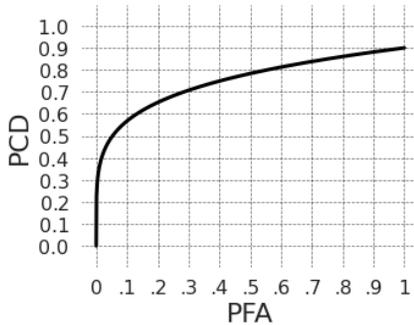
You are polling people to find their favorite sport between Table Tennis and Badminton in your town. Whichever wins will become the official sport of your town! Right now, badminton is ahead of table tennis, and you are worried because you want table tennis to win (it is your favorite sport). Votes are tallied by districts, and there is only one district left to count: if table tennis wins a plurality of the votes in this district, table tennis will win overall in the entire town.

This district has n people. You know each person votes for table tennis, badminton, or baseball with probability 0.6, 0.3, and 0.1, respectively, and each vote is independent. Assume baseball won't win (i.e., the votes for baseball are ignored because the people counting the votes are so sure it won't win). You would be relaxed if you knew that table tennis would win with 95% certainty. Using the CLT, for which n would table tennis win with 95% probability? *Hint*: $\Phi^{-1}(0.05) \approx -1.65$, where Φ is the standard Gaussian CDF.

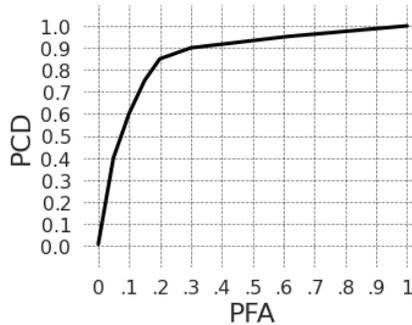
- d) You want to use your Huffman encoding to save space. You decide to encode all the other characters naively (i.e. in a manner similar to part (c) but for 32 characters), but use your Huffman scheme from part (a) for the vowels a , i , o , and u . How can you do this in a prefix free manner? How many bits do you expect to use if you use this strategy? Give your answer in terms of n and B .

6 More Hypothetical Scenarios [8 points]

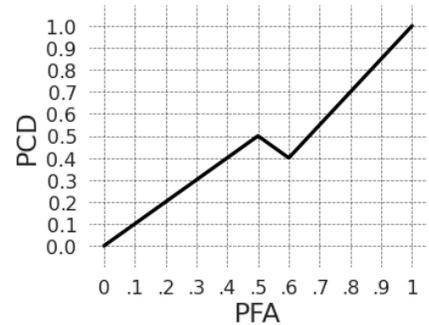
Consider the following graphs. Which of the following corresponds to a valid Receiver Operating Characteristic (ROC) curve (only one is valid)? Find the PCD of the corresponding Neyman-Pearson decision rule based on a constraint that the PFA is less than or equal to 0.4. *Note: You can approximately read the numbers off of the graph; answers with reasonable decimal values will receive full credit. Remember to justify your answers.*



Graph A



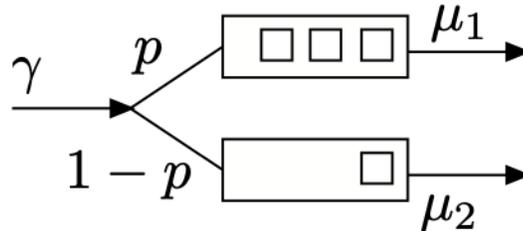
Graph B



Graph C

7 Two-Server System [5 + 6 + 3 + 3 + 3 points]

Suppose now that jobs come in at rate γ and each job gets routed to server one with probability p , and server two with probability $1 - p$ (see below diagram). If we say there are n jobs in the queue this includes the job getting serviced at that moment.



Assume that $\mu_1 > \gamma p$ and $\mu_2 > \gamma(1 - p)$. For all subparts, consider the queue to have been running for an infinitely long time (i.e., in stationarity).

- (a) Draw the state transition diagram for the associated continuous time Markov chain of this queue. Clearly describe what each state means (there are multiple possible correct answers).

- (b) Solve for the stationary distribution of the CTMC.

(c) What is the probability that there are 4 jobs in queue two given that there are currently 3 jobs in queue one?

(d) What is the expected number of jobs in queue one at any given time?

(e) What is the expected number of jobs in queue two at any given time?