ME 132 Fall 2020 Midterm 1

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ID # :

# 1	# 2	# 3	# 4	Subtotal
4	18	12	4	38

#5	# 6	# 7	# 8	Subtotal	TOTAL
8	10	14	6	38	76

Instructions:

1 Write your name and student ID number.

2 Read the questions carefully.

3 Write your solution clearly. **Please, please please ...**

4 This exam has 8 questions worth 76 points.

Problem # 1 ($1 \times 4 = 4$ points)

For each statement, state whether the claim is True or False.
Circle you answer. No explanation is necessary.
-1 **points for incorrect answers** so guessing is not advised.

1	**True** or **False**	First-order linear systems can never have an oscillatory free-response.
2	**True** or **False**	Proportional control can always stabilize a first-order LTI plant.
3	**True** or **False**	Increasing the proportional gain k_p, generally increases the time constant.
4	**True** or **False**	The steady-state response of a stable linear system due to a sinusoidal input depends on the initial conditions.

Problem # 2 ($2\times 9 = 18$ points)

Circle the **most appropriate** answer. No explanation is necessary.
Each correct answer gets 2 points. Incorrect answers get 0 points.
Since there is no penalty for wrong answers, you might as well guess if you are not sure.

(a) The DC gain of the LTI differential equation model $\ddot{y}(t) + 1.7\dot{y}(t) + 0.72y(t) = u(t)$ is:

1. 1
2. $\frac{1}{0.72}$
3. 0.72
4. 50
5. undefined

(b) The DC gain of the LTI differential equation model $\ddot{y}(t) - 1.7\dot{y}(t) + 0.72y(t) = u(t)$ is:

1. 1
2. $\frac{1}{0.72}$
3. 0.72
4. 50
5. undefined not stable!

(c) The magnitude of the complex number $\dfrac{e^j}{e^{-j}}$ is:

1. -1
2. 1
3. e^{2j}
4. $\cos(1)$
5. $\cos(2)$

(d) Consider the ordinary differential equation: $\dot{y}(t) = t$, $y(0) = 4$. The solution is:

1. 4
2. $2t^2$
3. $0.5t^2 + 4$
4. $0.5t^2 + e^{-t}$
5. None of the above.

(e) The differential equation $\dfrac{d^3y(t)}{dt^3} + 3t^2\dfrac{d^2y(t)}{dt^2} + 2\dfrac{dy(t)}{dt} = 6\dfrac{d^2u(t)}{dt^2} - 7u(t-2)$ is:

1. Not causal.
2. Linear and Time-invariant
3. Time-invariant
4. Nonlinear
5. Linear and Time-varying

(f) Which of the following is NOT a benefit of closed loop control over open loop control?

1. Improved disturbance rejection.
2. Decreased sensitivity to plant model inaccuracies.
3. Improved reference tracking.
4. Improved noise rejection.
5. None of the above.

(g) For which systems do transfer functions exist?

1. Linear
2. Linear and Time-invariant
3. Linear and Time-varying
4. Nonlinear and Time-invariant
5. Nonlinear and Time-varying

(h) Consider the unit step response of the under-damped second order system $H(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$. Decreasing the damping ξ results in

1. Larger oscillations.
2. Higher peak overshoot.
3. Longer settling time.
4. No change in DC gain.
5. All of the above.

(i) Which of the following is a FALSE statement?

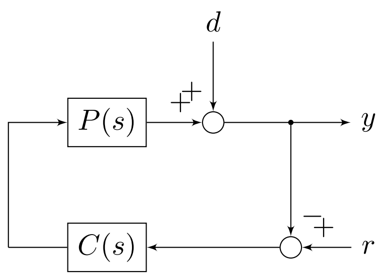
1. Disturbances are typically unknown.
2. Very accurate plant models are needed for simulation.
3. Feedback can destabilize a stable plant.
4. Very accurate plant models are needed for controller design.
5. Open loop control relies on calibration.

Problem # 3 (6+6 = 12 points)

You must provide explanations or show your work for partial credit.

Consider the feedback system with the first-order plant shown below.
The plant and controller are

$$P(s) = \frac{1}{s+1}, \quad C(s) = \frac{b}{s+a}$$



$$P(s) = \frac{1}{s+1}$$

$$C(s) = \frac{b}{s+a}$$

Design the controller parameters a, b such that

- the closed-loop system rejects constant disturbances d
- the closed loop system is stable and has damping $\xi = 0.5$

$$y = d + PC(r-y)$$

$$\Rightarrow \quad y = \left[\frac{1}{1+PC}\right] d + \left[\frac{PC}{1+PC}\right] r$$

$$= \left[\frac{s^2+(1+a)s+a}{s^2+(1+a)s+a+b}\right] d + \left[\frac{b}{s^2+(1+a)s+(a+b)}\right] d$$

$Q(0) \to 0$

$\Rightarrow a = 0$

$\xi = 0.5$

$\Downarrow$

$\omega_n = 1$

$\Downarrow$

$2\xi\omega_n = 1+a = 1 \qquad \omega_n^2 = b \qquad b = 1$

$a =$ 0	$b =$ 1

Problem 4 (4 points)

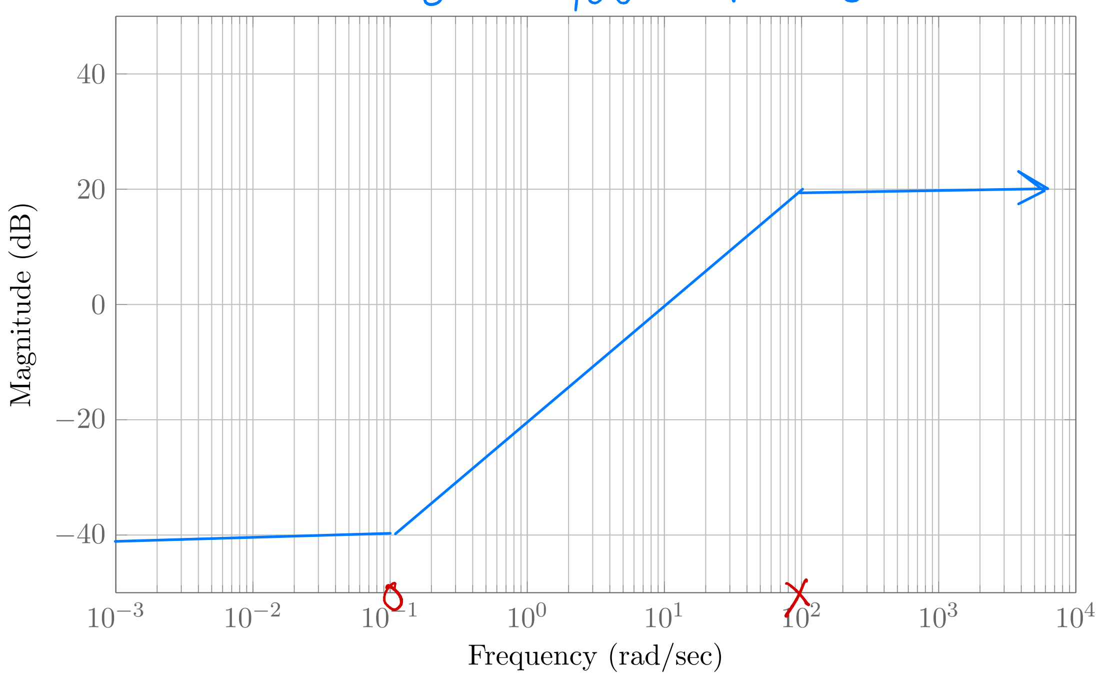
Consider the first-order LTI system

$$C(s) = \frac{10s - 1}{s + 100}$$

zero @ 0.1 pole @ -100

Sketch the magnitude frequency response plot of $C(s)$ on the graph paper below. Use the straight-line approximations covered in lecture.
No explanations are needed.

DC gain $= \frac{-1}{100}$, $|\text{DC gain}| = 0.01 = -40\,\text{db}$



Problem # 5 (4 +4 = 8 points)

You must provide explanations or show your work for partial credit.

(a) Consider the plant with transfer function

$$H(s) = \frac{s^2 + cs + d}{s^2 + 4s + 4}$$

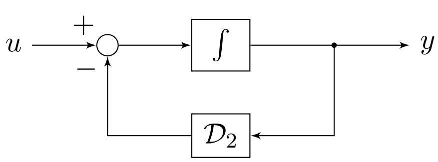
We apply the input $u(t) = \sin(2t)$. The steady-state response is zero. Find c and d.

$$H(j2) = 0 \Rightarrow \frac{-4 + 2cj + d}{*} = 0$$

$$\Rightarrow c = 0, \; d = 4$$

$c =$ 0	$d =$ 4

(b) Let $\mathcal{D}_2$ denote the 2-second delay operator. Consider the feedback system shown below



Find the transfer function from u to y.

$$y = \frac{1}{s}(u - D_2 y)$$

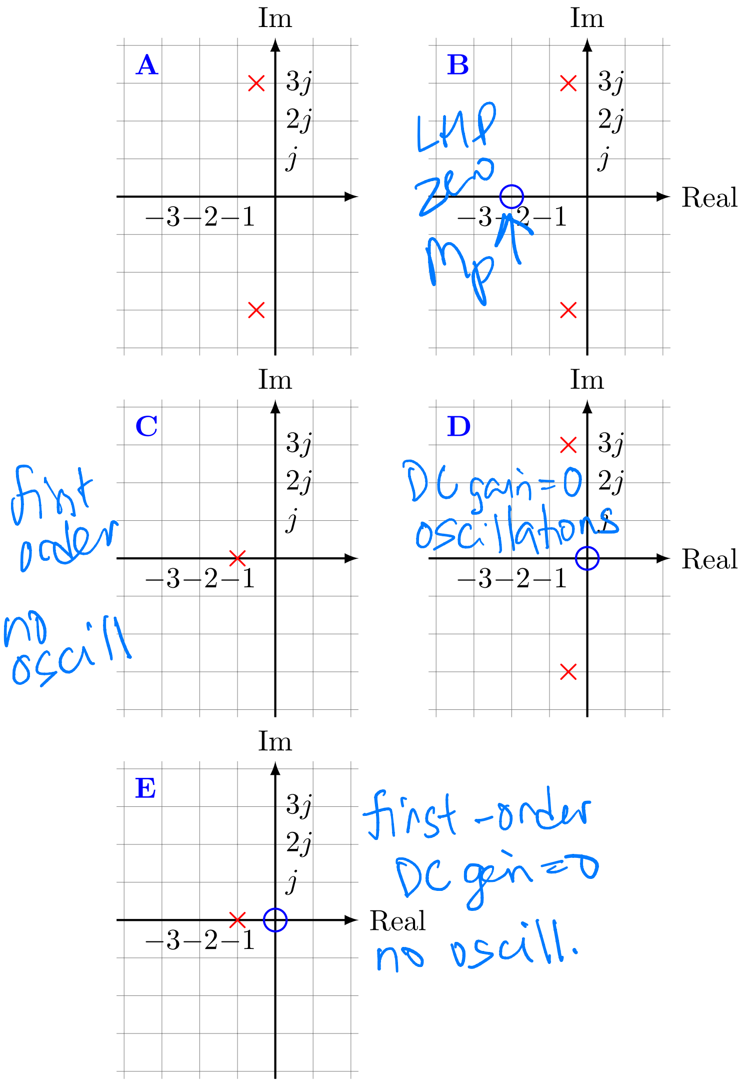
$$sy = u - e^{-2s} y$$

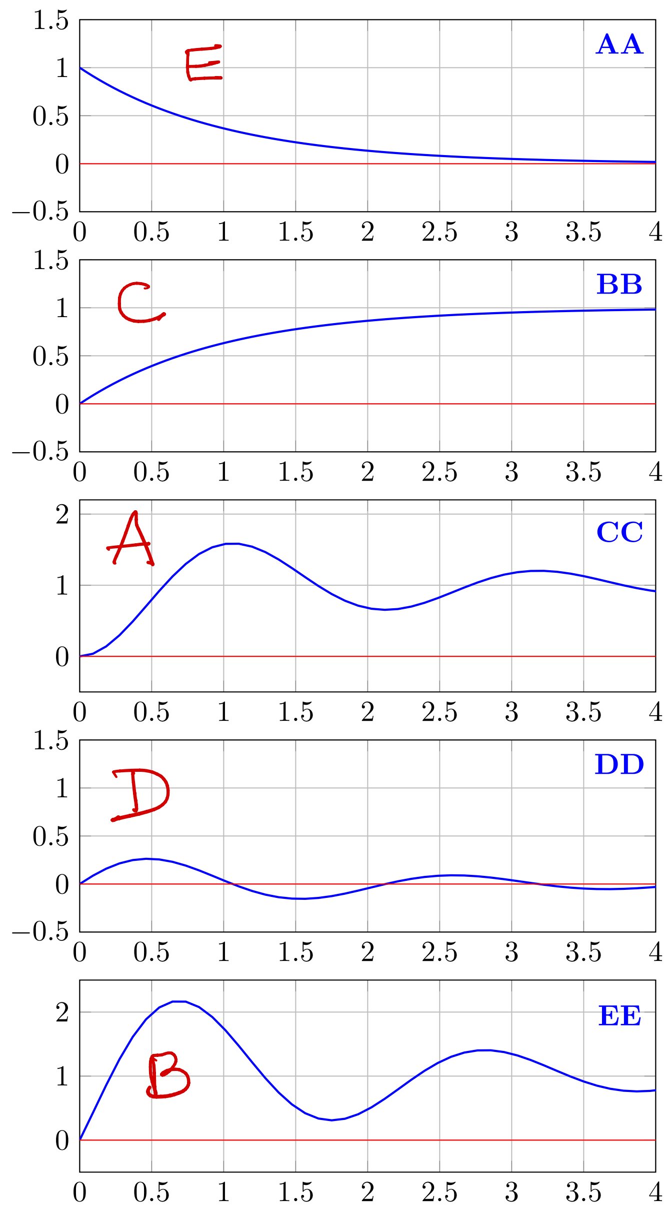
$$y = \left[\frac{1}{s + e^{-2s}}\right] u$$

Answer: $\dfrac{1}{s + e^{-2s}}$

Problem # 6 (10 points)

Consider a linear time-invariant system $H(s)$. Shown below are several pole-zero diagrams for $H(s)$ together with several possible **unit step response** plots. Pair each pole-zero diagram (A-E) with the most appropriate step response (AA-EE).
No explanations are necessary.
Each correct answer gets 2 points, each incorrect answer gets -1 point.

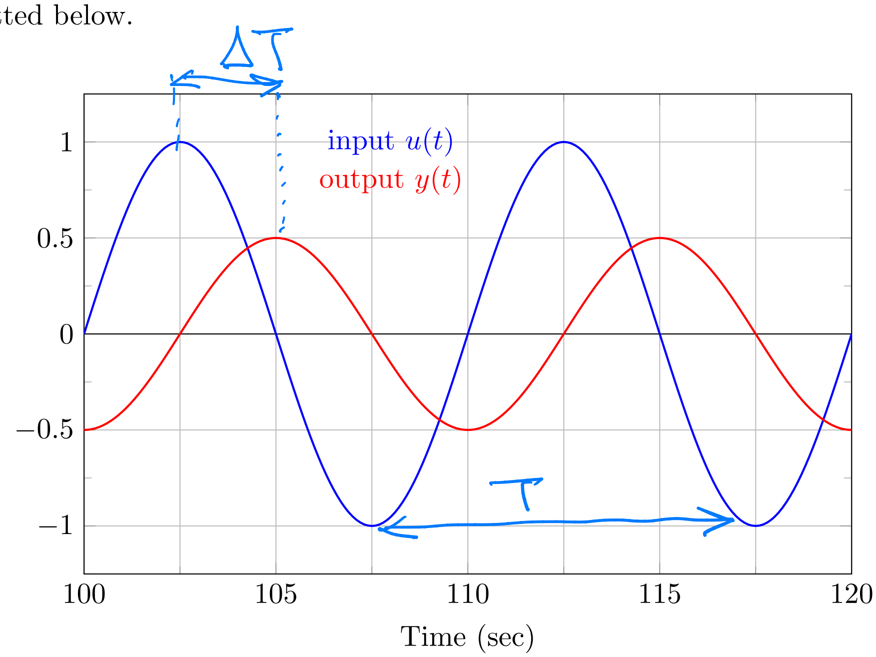




A = CC	B = EE	C = BB	D = DD	E = AA

Problem # 7 (2+2+2+4+4 = 14 points)

A second order system has transfer function $G(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$.
On applying the input $u(t) = \sin(\omega t)$, we observe the **steady state** output $y(t) = M\sin(\omega t + \phi)$.
These are plotted below.



Find ω, the amplification/attenuation factor M, and the phase shift ϕ.
Using this, determine ω_n and ξ.
You must provide explanations or show your work for partial credit.

$T = 10$ sec $\quad \omega = \dfrac{2\pi}{T} = \dfrac{\pi}{5}$ rad/sec

$\Delta T = -2.5$ sec $\quad \Delta\theta = \omega \Delta T = -\dfrac{\pi}{2}$ rad $= \phi$

$M = \dfrac{1}{2} \Rightarrow H(j\omega) = Me^{j\phi} = -\dfrac{j}{2} = \dfrac{1}{2j}$

$\Rightarrow \dfrac{\omega_n^2}{-\omega^2 + 2j\omega\omega_n\xi + \omega_n^2} \Rightarrow \omega = \omega_n, \; \xi = 1$

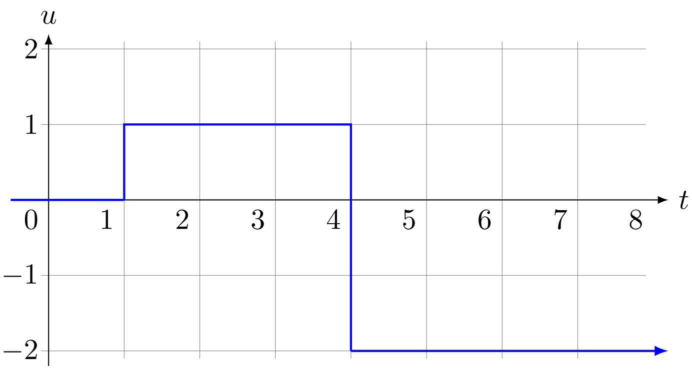
$\omega =$	$M =$	$\phi =$	$\omega_n =$	$\xi =$
$\pi/5$	$1/2$	$-90°$	$\pi/5$	1

Problem # 8 (6 points)

Sketch the response of the system with transfer function $\left[\frac{2e^{-2s}}{s+1}\right]$ to the input u shown below. Assume that the initial conditions are zero.

You do not have to show your work, find any numerical values, or label values on your plot.

No partial credit.



Delay = 2 sec.

DC gain = 2

Tc = 1 sec

Draw your answer here: