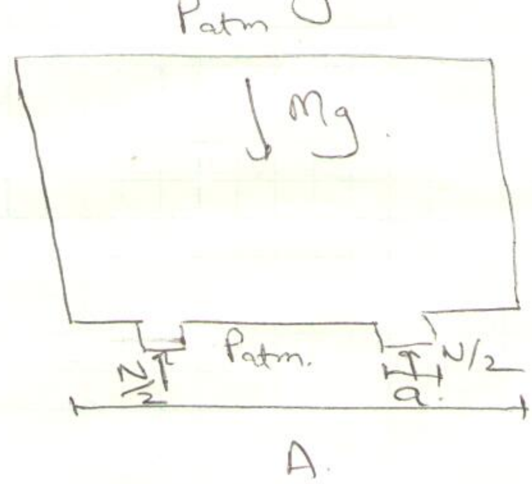
MIDTERM 1 SOLUTION FALL 2020.

Q1 ~

Consider the system.



The cuboid body with feet.

Here $N/2$ is the force acting on the body from the weighing machine.
Lets find this normal force first.
In equilibrium ~

$$\underbrace{-P_{atm} A \hat{z}}_{\text{Top surface}} + \underbrace{P_{atm}(A-2a)\hat{z}}_{\text{Bottom Surface}} - Mg\hat{z} + N\hat{z} = 0.$$

$$N \cancel{\hat{z}} = P_{atm}\, 2a + Mg$$

The total force acting on the weighing machine is.

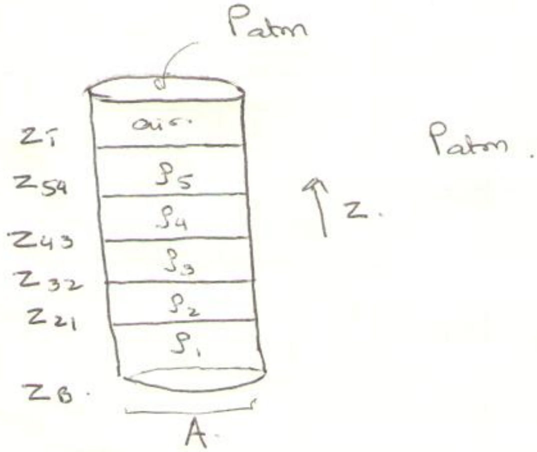
~~$\vec{W}$~~

$$|W| = \underbrace{Mg + P_{atm} 2a}_{\text{force by body}} + \underbrace{P_{atm}(A - 2a)}_{\text{force due to air pressure.}}$$

$$|W| = Mg + P_{atm} A.$$

$|W|$ is independent of a.

Q2 –



Writing down expression for gage pressure –

$$P(z) = \begin{cases} -\rho_5 g(z - z_T) & z_T > z \geq z_{54} \\ -\rho_5 g(z_{54} - z_T) - \rho_4 g(z - z_{54}) & z_{54} > z \geq z_{43} \\ -\rho_5 g(z_{54} - z_T) - \rho_4 g(z_{43} - z_{54}) - \rho_3 g(z - z_{43}) & z_{43} > z \geq z_{32} \end{cases}$$

$\cancel{-\rho_5 g - \rho_4 g}$

$$-\rho_5 g(z_{54} - z_T) - \rho_4 g(z_{43} - z_{54}) - \rho_3 g(z_{32} - z_{43}) - \rho_2 g(z - z_{32}) \qquad z_{32} > z \geq z_{21}$$

$$-\rho_5 g(z_{54} - z_T) - \rho_4 g(z_{43} - z_{54}) - \rho_3 g(z_{32} - z_{43}) - \rho_2 g(z_{21} - z_{32}) - \rho_1 g(z - z_{21})$$

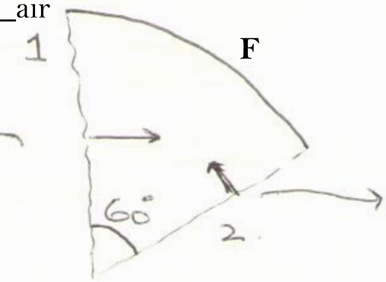
$$z_{21} > z \geq z_B$$

Lets consider the cylinder section :-

Our control volume considered here is the fictitious boundary 1 and 2, and the metal plate. We are considering the metal plate here because air pressure is acting on both sides. The total pressure inside the plate is P_total = P_glue + P_air

Thus P_gage = P_glue

$\hat{n}_1 = \hat{x}$



$\hat{n}_2 = -\sin 30^\circ \hat{x} + \cos 30^\circ \hat{y}$

This section is surrounded by fluid. In equilibrium, we can write the force balance as :-

$$\underline{F}_1 + \underline{F}_2 + \underline{F} = 0.$$

Where $\underline{F}$ is the force acting on the curved surface.

$$\underline{F}_1 = \int_A P(z)\, \hat{n}_1\, dA.$$

$$= R\, \hat{x} \int_{z_B}^{z_T} P(z)\, dz$$

$$\underline{F}_2 = \int_A P(z)\, \hat{n}_2\, dA.$$

$$= R\left(-\sin 30^\circ \hat{x} + \cos 30^\circ \hat{y}\right) \int_{z_B}^{z_T} P(z)\, dz$$

$$\underline{F} = -\underline{F}_1 - \underline{F}_2$$

$$= \left[-R\hat{x} - R\left(-\sin 30^\circ \hat{x} + \cos 30^\circ \hat{y}\right)\right] \int_{z_B}^{z_T} P(z)\,dz$$

$$= \left[-\frac{1}{2} R\hat{x} - R\frac{\sqrt{3}}{2}\hat{y}\right] \int_{z_B}^{z_T} P(z)\,dz \quad \text{—} \; 1$$

$$\int_{z_B}^{z_T} P(z)\,dz = \int_{z_{54}}^{z_T} P(z)\,dz + \int_{z_{43}}^{z_{54}} P(z)\,dz + \int_{z_{32}}^{z_{43}} P(z)\,dz$$

$$+ \int_{z_{21}}^{z_{32}} P(z)\,dz + \int_{z_B}^{z_{21}} P(z)\,dz \quad \text{—} \; 2$$

Notice that we use gage pressure since air is exerting pressure on both sides of the cylinder.

$$\int_{z_{54}}^{z_T} P(z)\,dz = \int_{z_{54}}^{z_T} \left(-\rho_5 g z + \rho_5 g z_T\right) dz$$

$$= -\rho_5 g \frac{z^2}{2} + \rho_5 g z_T z \Big|_{z_{54}}^{z_T}$$

$$= -\rho_5 g \frac{z_T^2}{2} + \rho_5 g z_T^2 + \rho_5 g \frac{z_{54}^2}{2} - \rho_5 g z_{54} z_T$$

$$= +\rho_5 g \frac{z_T^2}{2} + \rho_5 g \frac{z_{54}^2}{2} - \rho_5 g z_{54} z_T$$

$$= \rho_5 g \frac{(z_T - z_{54})^2}{2}$$

$$\int_{z_{43}}^{z_{54}} P(z)\,dz = \int_{z_{43}}^{z_{54}} -\rho_5 g(z_{54} - z_T) - \rho_4 g z + \rho_4 g z_{54} \quad dz$$

$$= -\rho_5 g z_{54} z + \rho_5 g z_T z - \rho_4 g \frac{z^2}{2} + \rho_4 g z_{54} z \Big|_{z_{43}}^{z_{54}}$$

$$= -\rho_5 g z_{54}^2 + \rho_5 g z_T z_{54} - \rho_4 g \frac{z_{54}^2}{2} + \rho_4 g z_{54}^2$$
$$+ \rho_5 g z_{54} z_{43} - \rho_5 g z_T z_{43} + \rho_4 g \frac{z_{43}^2}{2} - \rho_4 g z_{54} z_{43}$$

$$= \cancel{-\rho_5 g z_{54}^2 + \rho_5 g}$$

$$= \rho_5 g \left(z_T z_{54} + z_{54} z_{43} - z_T z_{43} - z_{54}^2 \right)$$
$$+ \rho_4 g \frac{(z_{54} - z_{43})^2}{2}$$

$$\int_{z_{32}}^{z_{43}} P(z)\,dz = \int_{z_{32}}^{z_{43}} \left[-\rho_5 g (z_{54} - z_T) - \rho_4 g (z_{43} - z_{54}) - \rho_3 g (z - z_{43}) \right] dz$$

$$= -\rho_5 g (z_{54} - z_T)(z_{43} - z_{32}) - \rho_4 g (z_{43} - z_{54})(z_{43} - z_{32})$$
$$+ \rho_3 g \frac{(z_{43} - z_{32})^2}{2}$$

$$\int_{z_{21}}^{z_{32}} P(z)\,dz = -\rho_5 g(z_{54}-z_T)(z_{32}-z_{21}) - \rho_4 g(z_{43}-z_{54})(z_{32}-z_{21}) - \rho_3 g(z_{32}-z_{43})(z_{32}-z_{21}) + \rho_2 g \frac{(z_{32}-z_{21})^2}{2}$$

$$\int_{z_B}^{z_{21}} P(z)\,dz = -\rho_5 g(z_{54}-z_T)(z_{21}-z_B) - \rho_4 g(z_{43}-z_{54})(z_{21}-z_B) - \rho_3 g(z_{32}-z_{43})(z_{21}-z_B) - \rho_2 g(z_{21}-z_{32})(z_{21}-z_B) + \rho_1 g \frac{(z_{21}-z_B)^2}{2}$$

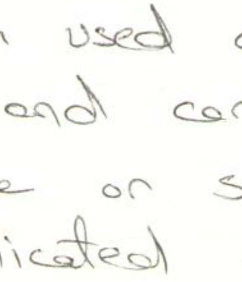
Plugging these into ① & ② gives us the force $\underline{F}$ supplied by the glue.

Solution algebra on next page

Note:-

The force can also be calculated using the unit vector on the curved surface. However, the approach used above is much more geral and can be used to find force on surface which are much more complicated & their normal vectors are difficult to find.

Such as.



we can find force on this curved surface using similar approach

$$\underline{E} = \left[-\tfrac{1}{2} R \hat{x} - R \tfrac{\sqrt{3}}{2} \hat{y} \right] \int_{z_B}^{z_T} \rho(z)\, dz$$

Plugging into equations -

$$\cancel{\int_{z_B}^{z_T} \rho(z)\, dz = \rho_5 g \frac{(z_T - z_{54})^2}{2} + \ldots}$$

$$\int_{z_B}^{z_T} \rho(z)\, dz = \rho_5 g \frac{z_T^2}{2} + \rho_5 g \frac{z_{54}^2}{2} - \cancel{\rho_5 g z_T z_{54}} + \cancel{\rho_5 g z_T z_{54}}$$

$$+ \cancel{\rho_5 g z_{54} z_{43}} - \cancel{\rho_5 g z_T z_{43}} - \rho_5 g z_{54}^2$$

$$+ \rho_4 g \frac{z_{54}^2}{2} + \rho_4 g \frac{z_{43}^2}{2} - \cancel{\rho_4 g z_{54} z_{43}} - \cancel{\rho g z_{54} z_{43}}$$

$$+ \cancel{\rho_5 g z_{54} z_{32}} + \cancel{\rho_5 g z_{54}} + \cancel{\rho_5 g z_T z_{43}}$$

$$- \cancel{\rho_5 g z_T z_{32}} - \rho_4 g z_{43}^2 + \cancel{\rho_4 g z_{43} z_{32}} + \cancel{\rho_4 g z_{54} z_{43}}$$

$$- \cancel{\rho_4 g z_{54} z_{32}} + \rho_3 g \frac{z_{43}^2}{2} + \rho_3 g \frac{z_{32}^2}{2} - \cancel{\rho_3 g z_{43} z_{32}}$$

$$- \cancel{\rho_5 g z_{54} z_{32}} + \cancel{\rho_5 g z_{54} z_{21}} + \cancel{\rho_5 g z_T z_{32}} - \cancel{\rho_5 g z_T z_{21}}$$

$$- \cancel{\rho_4 g z_{43} z_{32}} + \cancel{\rho_4 g z_{43} z_{21}} + \cancel{\rho_4 g z_{54} z_{32}} - \cancel{\rho_4 g z_{54} z_{21}}$$

$$- \rho_3 g z_{32}^2 + \cancel{\rho_3 g z_{32} z_{21}} + \cancel{\rho_3 g z_{43} z_{32}} - \cancel{\rho_3 g z_{43} z_{21}}$$

$$+ \rho_2 g \frac{z_{32}^2}{2} + \rho_2 g \frac{z_{21}^2}{2} - \cancel{\rho_2 g z_{32} z_{21}}$$

$$- \cancel{\rho_5 g z_{54} z_{21}} + \rho_5 g z_{54} z_B + \cancel{\rho_5 g z_T z_{21}} - \rho_5 g z_T z_B$$

$$- \cancel{\rho_4 g z_{43} z_{21}} + \rho_4 g z_{43} z_B + \cancel{\rho_4 g z_{54} z_{21}} - \rho_4 g z_{54} z_B$$

$$- \cancel{\rho_3 g z_{32} z_{21}} + \rho_3 g z_{32} z_B + \cancel{\rho_3 g z_{43} z_{21}} - \rho_3 g z_{43} z_B$$

$$- \rho_2 g z_{21}^2 + \rho_2 g z_{21} z_B + \cancel{\rho_2 g z_{32} z_{21}} - \rho_2 g z_{32} z_B$$

$$+ \rho_1 g \frac{z_{21}^2}{2} + \rho_1 g \frac{z_B^2}{2} - \rho_1 g z_{21} z_B$$

Using (2) -

$$\underline{F_1} = \underline{B_1} - \underline{F_{L1}} - \underline{F_{B1}}$$

$$= \left[\rho_4 g L H^2 \ \frac{3\sqrt{3}}{32} \ \hat{z}\right] - \left[\frac{3}{4} H P_{atm} L + \rho_4 g z_4 L H \frac{3}{4}\right.$$

$$- \cancel{\rho_4 g \frac{z_1^2 L}{2}} - \frac{9}{\cancel{16}\,32} H^2 \rho_4 g L + \frac{3}{4}\rho_4 g z_1 H L + \left.\cancel{\rho_4 g \frac{z_1^2 L}{2}}\right] \hat{x}$$

$$- \left[\frac{\sqrt{3}}{4} H L P_{atm} - \rho_4 g \frac{\sqrt{3}}{4} H z_1 L + \frac{\sqrt{3}}{4}\rho_4 g z_4 H L\right] \hat{z}$$

Using (3) -

$$\underline{F_2} = \left[\frac{\sqrt{3}}{32} \ \rho_2 g H^2 L\right] \hat{z} + \left[\frac{\sqrt{3}}{4} H L P_{atm} - \rho_4 g z_1 \frac{\sqrt{3}}{4} H L\right.$$

$$+ \left.\rho_4 g z_4 \frac{\sqrt{3}}{4} H L\right] \hat{z} - \left[\frac{H}{4} L P_{atm} - \rho_4 g z_1 \frac{H}{4} L + \rho_4 g z_4 \frac{H}{4} L\right.$$

$$+ \rho_2 g z_1 \frac{H}{4} L - \cancel{\rho_2 g \frac{z_1^2}{2} L} + \cancel{\rho_2 g \frac{z_1^2 L}{2}} + \frac{H^2}{\cancel{16}\,32} \rho_2 g L + \left.L \rho_2 g \frac{H}{\cancel{8}\,4} z_1\right] \hat{x}$$

Using (4) -

$$\underline{F_3} = \left[\frac{3\sqrt{3}}{32} \ \rho_3 g H^2 L \ \cancel{\tfrac{x}{4}}\right] \hat{z} + \left[\frac{\sqrt{3}}{4} H L P_{atm} + \rho_3 g z_1 \frac{\sqrt{3}}{4} H L\right.$$

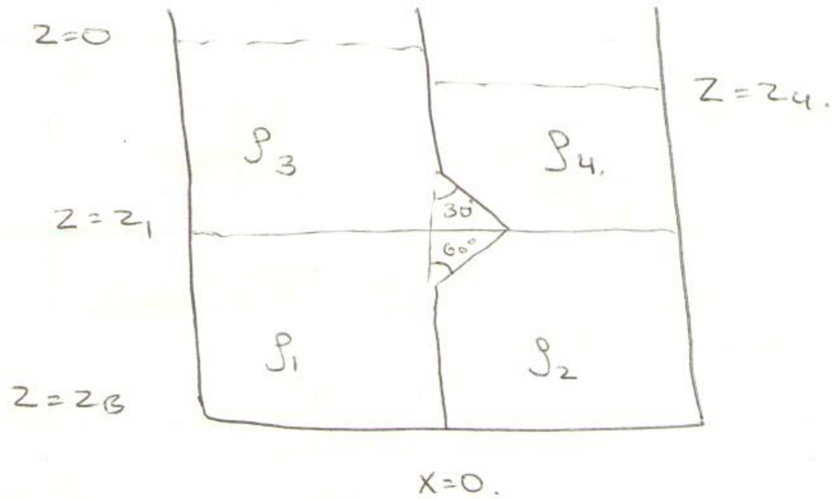
$$- \rho_3 g H^2 L \ \cancel{\tfrac{3\sqrt{3}}{16}} \ \left.\frac{3\sqrt{3}}{16}\right] \hat{z} + \left[\frac{3}{4} H L P_{atm} - \rho_3 g H^2 \cancel{\tfrac{9}{16}} \frac{9}{32}\right.$$

$$+ \left.\rho_3 g z_1 H \frac{3}{4}\right] \hat{x}$$

Q3 ~

Fluid 3 with density 3 is from z=0 to z=z_1

Fluid 1 with density 2 is from z=z_1 to z=z_B



Fluid 4 with density 4 is from z=z_4 to z=z_1

Fluid 2 with density 2 is from z=z_1 to z=z_B

On RHS ~

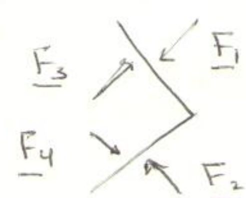
$$P_R(z) = \begin{cases} P_{atm} - \rho_4 g(z - z_4) & z_4 \geq z \geq z_1 \\ P_{atm} - \rho_4 g(z_1 - z_4) - \rho_2 g(z - z_1). & z_1 \geq z \geq z_B. \end{cases}$$

On LHS ~

$$P_L(z) = \begin{cases} P_{atm} - \rho_3 g z & 0 \geq z > z_1 \\ P_{atm} - \rho_3 g z_1 - \rho_1 g(z - z_1) & z_1 \geq z \geq z_B \end{cases}$$

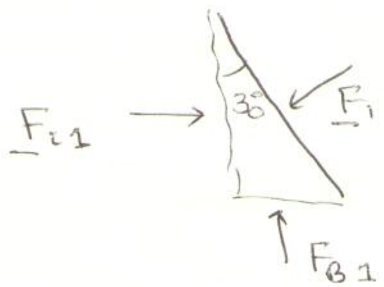
The sum of forces equal 0 ..

$$\underline{F} + \underline{F}_1 + \underline{F}_2 + \underline{F}_3 + \underline{F}_4 = 0. \quad (1).$$



Where $\underline{F}$ is the force that must be supplied by the hinges to keep plates in equilibrium.

Consider upper plate ~ Here we do a thought experiment that our triangular control volume is surrounded in fluid 4



Byoyancy balance can be writen as ~

$$\underline{B}_1 = \underline{F}_1 + \underline{F}_{L1} + \underline{F}_{B1}$$

$$\underline{B}_1 = \frac{1}{2}\left(\frac{\sqrt{3}}{2} H \cos 30^\circ\right)\left(\frac{\sqrt{3}}{2} H \sin 30^\circ\right) L \rho_4 g \, \hat{z}$$

$$\underline{F}_{L1} = L\hat{x} \int_{-z_1}^{-z_1 + \frac{\sqrt{3}}{2} H \cos 30^\circ} P_{atm} - \rho_4 g (z - z_4) \quad dz.$$

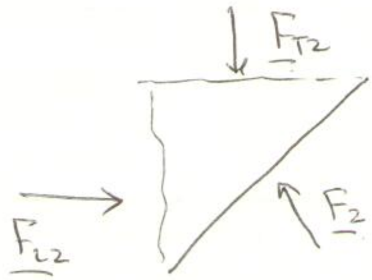
$$= L\hat{x}\left[\left(P_{atm} + \rho_4 g z_4\right)\left(\frac{\sqrt{3}}{2} H \cos 30^\circ\right) - \rho_4 g \frac{\left(-z_1 + \frac{\sqrt{3}}{2} H \cos 30^\circ\right)^2}{2} + \rho_4 g \frac{z_1^2}{2}\right].$$

$$\underline{F}_{B1} = \frac{\sqrt{3}}{2} H \sin 30^\circ \, L \left(P_{atm} - \rho_4 g (z_1 - z_4)\right) \text{ z\_hat}$$

$$\underline{F}_1 = \underline{B}_1 - \underline{F}_{L1} - \underline{F}_{B1} \quad (2)$$

For lower plate

Here we do a thought experiment that our triangular control volume is surrounded in fluid 2



$$\underline{B_2} = \underline{F_2} + \underline{F_{T2}} + \underline{F_{L2}}$$

$$\underline{B_2} = \rho_2 g \frac{1}{2}\left(\frac{H}{2}\cos 60^\circ\right)\left(H/_2 \sin 60^\circ\right) L\hat{z}$$

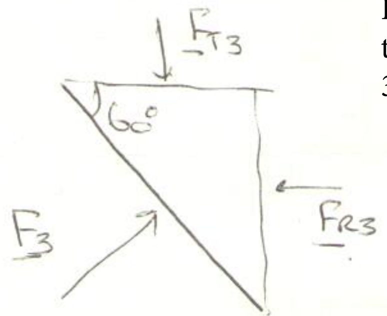
$$\underline{F_{T2}} = -L\hat{z}\left(\frac{H}{2}\sin 60^\circ\right)\left(P_{atm} - \rho_4 g(z_1 - z_4)\right)$$

$$\underline{F_{L2}} = L\hat{x}\int_{-z_1 - \frac{H}{2}\cos 60^\circ}^{-z_1} P_{atm} - \rho_4 g(z_1 - z_4) + \rho_2 g z_1 - \rho_2 g z \; dz$$

$$= L\hat{x}\left[\left(P_{atm} - \rho_4 g(z_1 - z_4) + \rho_2 g z_1\right)\left(\frac{H}{2}\cos 60^\circ\right) - \rho_2 g \frac{z_1^2}{2} + \rho_2 g \frac{\left(-z_1 - \frac{H}{2}\cos 60^\circ\right)^2}{2}\right]$$

$$\underline{F_2} = \underline{B_2} - \underline{F_{T2}} - \underline{F_{L2}} \quad (3)$$

Consider the upper plate again, but this time with LHS fluid.



Here we do a thought experiment that our triangular control volume is surrounded in fluid 3

$$\underline{B}_3 = \underline{F}_3 + \underline{F}_{T3} + \underline{F}_{R3}$$

$$\underline{B}_3 = \frac{1}{2}\left(\frac{\sqrt{3}}{2}H\cos 60^\circ\right)\left(\frac{\sqrt{3}}{2}H\sin 60^\circ\right)L\rho_3 g\,\hat{z}$$

$$\underline{F}_{T3} = -L\hat{z}\left(\frac{\sqrt{3}}{2}H\cos 60^\circ\right)\left(P_{atm} - \rho_3 g\left(-z_1 + \frac{\sqrt{3}}{2}H\cos 30^\circ\right)\right)$$

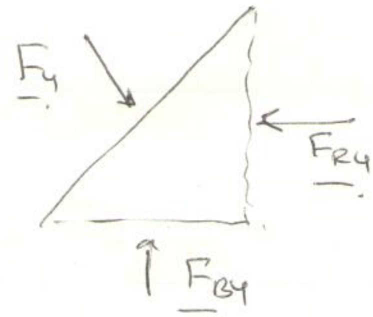
$$\underline{F}_{R3} = -L\hat{x}\int_{-z_1}^{-z_1+\frac{\sqrt{3}}{2}H\cos 30^\circ}\left(P_{atm} - \rho_3 g z\right)dz.$$

$$= -L\hat{x}\left[P_{atm}\left(\frac{\sqrt{3}}{2}H\cos 30^\circ\right) - \rho_3 g\frac{\left(-z_1 + \frac{\sqrt{3}}{2}H\cos 30^\circ\right)^2}{2} + \rho_3 g\frac{z_1^2}{2}\right]$$

$$\underline{F}_3 = \underline{B}_3 - \underline{F}_{T3} - \underline{F}_{R3} \qquad \text{—(4)}$$

Now finally consider lower plate with LHS fluid -

Here we do a thought experiment that our triangular control volume is surrounded in fluid 1



$$\underline{B_4} = \underline{F_4} + \underline{F_{R4}} + \underline{F_{B4}}$$

$$\underline{F_{B4}} = \left[P_{atm} - \rho_3 g z_1 - \rho_1 g \left(-z_1 - \frac{H}{2} \cancel{\cos 30} \cos 60^\circ - z_1 \right) \right] \left(H/_2 \cos 30^\circ \right) L \hat{z}.$$

$$\underline{B_4} = \tfrac{1}{2} \left(H/_2 \cos 30^\circ \right) \left(\frac{H}{2} \sin 30^\circ \right) L \rho_1 g \hat{z}$$

$\cancel{\underline{F_4}}$

$$\underline{F_{R4}} = -L \hat{x} \int_{-z_1 - \frac{H}{2} \cos 60^\circ}^{-z_1} \left[P_{atm} - \rho_3 g z_1 - \rho_1 g (z - z_1) \right] dz.$$

$$= -L \hat{x} \left[\left(P_{atm} - \rho_3 g z_1 + \rho_1 g z_1 \right) \left(\frac{H}{2} \cos 60^\circ \right) - \rho_1 g \frac{z_1^2}{2} + \rho_1 g \frac{\left(-z_1 - H/_2 \cos 60^\circ \right)^2}{2} \right]$$

$$\underline{F_y} = \underline{B_y} - \underline{F_{Ry}} - \underline{F_{By}} \quad — (5).$$

Plugging values into (1), (2), (3), (4) & (5), we get the required force from the hinges.

Solution algebra is on next page

Note -

The important thing to note here is that the CV is dipped into the liquid we are considering at the time.

$$\int_{z_B}^{z_T} P(z)\,dz = \rho_5 g \frac{z_T^2}{2} - \rho_5 g \frac{z_{54}^2}{2} + \rho_4 g \frac{z_{54}^2}{2} - \rho_4 g \frac{z_{43}^2}{2}$$

$$+ \rho_3 g \frac{z_{43}^2}{2} + \cancel{\rho_3 g \frac{z_{43}^2}{2}} - \rho_3 g \frac{z_{32}^2}{2} + \rho_2 g \frac{z_{32}^2}{2}$$

$$- \rho_2 g \frac{z_{21}^2}{2} + \rho_1 g \frac{z_{21}^2}{2} + \rho_1 g \frac{z_B^2}{2}$$

$$+ \rho_5 g z_{54} z_B - \rho_5 g z_T z_B + \rho_4 g z_{43} z_B - \rho_4 g z_{54} z_B$$

$$+ \rho_3 g z_{32} z_B - \rho_3 g z_{43} z_B + \rho_2 g z_{21} z_B$$

$$- \rho_2 g z_{32} z_B - \rho_1 g z_{21} z_B$$

$$= \rho_5 g \left(\frac{z_T^2}{2} - \frac{z_{54}^2}{2} + z_{54} z_B + z_T z_B \right) +$$

$$\rho_4 g \left(\frac{z_{54}^2}{2} - \frac{z_{43}^2}{2} + z_{43} z_B - z_{54} z_B \right) +$$

$$\rho_3 g \left(\frac{z_{43}^2}{2} - \frac{z_{32}^2}{2} + z_{32} z_B - z_{43} z_B \right) +$$

$$\rho_2 g \left(-\frac{z_{21}^2}{2} + \frac{z_{32}^2}{2} + z_{21} z_B - z_{32} z_B \right) +$$

$$\rho_1 g \left(\frac{z_{21}^2}{2} + \frac{z_B^2}{2} - z_{21} z_B \right).$$

Thus ~

$$\underline{F} = \left[-\tfrac{1}{2} R \hat{x} - \frac{\sqrt{3}}{2} R \hat{y} \right] \left[\rho_5 g \left(\frac{z_T^2}{2} - \frac{z_{54}^2}{2} + z_{54} z_B + z_T z_B \right) \right.$$

$$+ \rho_4 g \left(\frac{z_{54}^2}{2} - \frac{z_{43}^2}{2} + z_{43} z_B - z_{54} z_B \right) + \rho_3 g \left(\frac{z_{43}^2}{2} - \frac{z_{32}^2}{2} + z_{32} z_B - z_{43} z_B \right)$$

$$\left. + \rho_2 g \left(\frac{z_{32}^2}{2} - \frac{z_{21}^2}{2} + z_{21} z_B - z_{32} z_B \right) + \rho_1 g \left(\frac{z_{21} - z_B}{2} \right)^2 \right]$$

Using (5) ~

$$\underline{F_4} = \left(\frac{\sqrt{3}}{32}\rho_1 g H^2 L\right)\hat{z} - \left[\frac{\sqrt{3}}{4} HLP_{atm} - \rho_3 g z_1 H \frac{\sqrt{3}}{4} L + \rho_1 g z_1 HL \frac{\sqrt{3}}{2} + \rho_1 g H^2 L \frac{\sqrt{3}}{16}\right]\hat{z} + \left[\frac{H}{4} L P_{atm} - \rho_3 g z_1 \frac{H}{4} L + \rho_1 g z_1 \frac{H}{4} L + \rho_1 g \frac{H^2}{32} L + \rho_1 g z_1 \frac{H}{4} L\right]\hat{x}$$

Using (1) ~

$$\underline{F} = -\underline{F_1} - \underline{F_2} - \underline{F_3} - \underline{F_4}$$

$$= \Big[-\rho_4 g L H^2 \frac{3\sqrt{3}}{32} + \cancel{\frac{\sqrt{3}}{4} HLP_{atm}} - \rho_4 g \frac{\sqrt{3}}{4} H z_1 L + \frac{\sqrt{3}}{4}\rho_4 g z_4 HL$$

$$- \frac{\sqrt{3}}{32}\rho_2 g H^2 L - \cancel{\frac{\sqrt{3}}{4} HLP_{atm}} + \rho_4 g z_1 \frac{\sqrt{3}}{4} HL - \rho_4 g z_4 HL \frac{\sqrt{3}}{4}$$

$$- \rho_3 g H^2 L \frac{3\sqrt{3}}{32} - \cancel{\frac{\sqrt{3}}{4} HLP_{atm}} - \rho_3 g z_1 \frac{\sqrt{3}}{4} HL$$

$$+ \rho_3 g H^2 L \frac{3\sqrt{3}}{16} - \frac{\sqrt{3}}{32}\rho_1 g H^2 L + \cancel{\frac{\sqrt{3}}{4} HLP_{atm}}$$

$$- \rho_3 g z_1 H \frac{\sqrt{3}}{4} L + \rho_1 g z_1 HL \frac{\sqrt{3}}{2} + \rho_1 g H^2 L \frac{\sqrt{3}}{16}\Big]\hat{z}$$

$$+ \Big[\cancel{\frac{3}{4} HLP_{atm}} + \frac{3}{4}\rho_4 g z_4 LH - \frac{9}{32} H^2 \rho_4 g L + \frac{3}{4}\rho_4 g z_1 HL$$

$$+ \cancel{\frac{H}{4} LP_{atm}} - \rho_4 g z_1 \frac{HL}{4} + \rho_4 g z_4 \frac{HL}{4} + \rho_2 g z_1 \frac{HL}{4} + H^2 \frac{\rho_2 g L}{32}$$

$$+ \frac{\rho_2 g H z_1 L}{4} - \cancel{\frac{3}{4} HLP_{atm}} + \frac{9}{32}\rho_3 g H^2 L - \rho_3 g z_1 HL \frac{3}{4}$$

$$- \cancel{\frac{H}{4} LP_{atm}} + \rho_3 g z_1 L \frac{H}{4} - \rho_1 g z_1 \frac{HL}{4} - \rho_1 g \frac{H^2 L}{32} - \rho_1 g z_1 \frac{HL}{4}\Big]\hat{x}$$

$$\underline{F} = \Big[-\rho_4 g L H^2 \frac{3\sqrt{3}}{32} - \rho_3 g z_1 H L \frac{\sqrt{3}}{4} - \rho_3 g H^2 L \frac{3\sqrt{3}}{32}$$

$$+ \rho_3 g H^2 L \frac{3\sqrt{3}}{16} - \rho_3 g z_1 H L \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{32} \rho_2 g H^2 L$$

$$- \frac{\sqrt{3}}{32} \rho_1 g H^2 L + \rho_1 g z_1 H L \frac{\sqrt{3}}{2} + \rho_1 g H^2 L \frac{\sqrt{3}}{16} \Big] \hat{z}$$

$$+ \Big[\rho_4 g z_4 H L - \frac{9}{32} \rho_4 g L H^2 + \frac{1}{2} \rho_4 g z_1 L H$$

$$+ \frac{9}{32} \rho_3 g H^2 L - \frac{1}{2} \rho_3 g z_1 L H + \frac{1}{2} \rho_2 g z_1 H L$$

$$- \rho_1 g z_1 H L \tfrac{1}{2} - \frac{1}{32} \rho_1 g H^2 L \Big] \hat{x}$$