University of California, Berkeley Mechanical Engineering ME104 Engineering Mechanics Test 3.Q1 S20 Prof S. Morris

1. (125) To maintain balance, the cyclist travelling at constant speed v on the horizontal bend leans into the bend through a certain angle θ , as illustrated in the vertical section (ii). The bend is a quarter circle having radius b; the total mass of the rider and bicycle is m, and the coefficient of static friction between road and tyre is μ_s .



(a) Draw the free–body diagram showing the forces acting on the bicycle and rider in the vertical section (ii).

(b) Derive the equation relating the angle θ to v and other quantities shown in the figure.

(c) What is the maximum allowable value of θ ? Explain your answer.

(d) Determine the corresponding maximum speed allowable if the cyclist to follow the bend without slipping or tipping.

University of California, Berkeley Mechanical Engineering ME104 Engineering Mechanics Test 3.Q2 S20 Prof S. Morris

2. (125) The measuring cup can be modelled as a uniform cylindrical shell of mass m_1 , radius b and height 2b, with the handle being a uniform plate of mass m_2 , length 2b, and negligibly small thickness. The cup is initially at rest in the kitchen drawer; the coefficient of static friction between the drawer and the bottom of the cup is μ_s . (For the purpose of this test, the mass of the bottom of the cup is assumed to be negligibly small.) The drawer is then pulled, imposing a uniform acceleration $\mathbf{a} = a_x \mathbf{e}_x$ on the drawer. The cup handle is aligned parallel to this acceleration. As a result, the cup can slip, tip, or travel with the drawer.



(a) Assuming the cup to translate without tipping, and a_x to be positive, determine, as a function of b, g, m_1 , m_2 and μ_s , the maximum value of a_x allowable if the cup is not to slip.

(b) Assuming the cup does not slip, determine the maximum (positive) value of a_x allowable if the cup is not to tip.

(c) By comparing the results of parts (a) and (b), determine whether slipping or tipping limits a_x for the configuration shown.

(d) Now suppose that $a_x < 0$, so that the handle now points in the direction opposite to **a**. By repeating your analysis above determine whether slipping or tipping limits the magnitude of a_x in this case. (This cup in our kitchen drawer irritated me, until I became conscious of the annoyance, and pointed the handle in the appropriate direction.)

University of California, Berkeley Mechanical Engineering ME104 Engineering Mechanics Test 3.Q3 S20 Prof S. Morris

3. (125) The non–uniform, rectangular plate in figure (i) is initially at rest. An impulsive force is then applied at point P and after the impulse, point Q is observed to have acquired velocity \boldsymbol{v} . No other forces or moments act on the plate. Because the plate is not uniform, however, \boldsymbol{v} need not be parallel to the impulse.)



Assuming now that the same impulse is applied to point Q (rather than P) to the same plate in the same initial configuation, determine the velocity at point P. As in the first case, no other forces or moments act on the plate. To receive any credit you must show the appropriate equations and working.

University of California, Berkeley Mechanical Engineering ME104 Engineering Mechanics Test 3.Q4 S20 Prof S. Morris

4. (125) The circular ring of radius a rolls without slipping around the cylinder of radius b. Initially, point P on the ring is in contact with the cylinder, as shown in figure (i). When the contact point has made one complete turn about the cylinder, a different point Q of the hoop is now in contact with the cylinder, as shown in figure (iii). In between those times, the ring has touched the cylinder at all points of the circumference of the cylinder; for example, figure (ii) shows the configuration when the contact point has made a quarter turn.



(a) Determine the angle through which the ring has revolved when the contact point has completed one complete turn around the cylinder, as shown in figure (iii).

(b) Assuming gravity to be negligible, draw the free-body diagram for the ring.

(c) If the contact point takes time T to make one complete turn from the initial state shown in figure (i), to its final state shown in figure (iii), determine the normal force exerted by the cylinder on the ring as a function of ring mass m and radius a, cylinder radius b and the period T. (Assume that the ring rotates with uniform speed.)