Physics 7B Fall 2020 Lectures 2 & 3 Solutions

Problem 1

1. (a) The ice needs heat $m_i c_i (T_m - T_i)$ to raise its temperature to the melting temperature T_m and heat $m_i L_f$ to melt. The water's temperature can decrease until it reaches T_m, at which point it will begin to freeze itself. Therefore, the limiting case is

$$m_i L_f + m_i c_i (T_m - T_i) + m_w c_w (T_m - T_w) = 0 \rightarrow m_i = \frac{m_w c_w (T_w - T_m)}{L_f + c_i (T_m - T_i)}. \tag{1}$$

Any m_i larger than this will cause the water to freeze, so

$$m_i \leq \frac{m_w c_w (T_w - T_m)}{L_f + c_i (T_m - T_i)}. \tag{2}$$

(b) Assume the ice cube has volume V. Since $2/3$ of the cube is submerged, the upward force due to buoyancy is

$$F_b = \frac{2}{3} V \rho_w g, \tag{3}$$

and the downward force due to gravity is

$$F_g = V \rho_i g. \tag{4}$$

While floating, these two forces are balanced. Therefore,

$$\frac{2}{3} V \rho_w g = V \rho_i g \rightarrow \rho_i = \frac{2}{3} \rho_w. \tag{5}$$

Because these two densities are different, the volume of the ice cube will change after melting. Let V' be the volume occupied by the ice cube after melting. The total mass of the ice cube will be conserved, so we have

$$m_i = \rho_i V = \frac{2}{3} \rho_w V = \rho_w V' \rightarrow V' = \frac{2}{3} V. \tag{6}$$

Since V' is the same as the volume of water displaced by the ice cube, the glass will not overflow.

(c) Begin by assuming the system is thermally isolated:

$$\Delta Q = \Delta Q_w + \Delta Q_i = 0 \tag{7}$$

After the ice melts, its specific heat will be the same as that of water, so we have

$$\Delta Q = m_w c_w (T_f - T_w) + m_i c_w (T_f - T_m) + m_i L_f + m_i c_i (T_m - T_i) = 0 \tag{8}$$

Next, we use $m_i = m_w/2$ to simplify and then solve for T_f.

$$m_w c_w (T_f - T_w) + \frac{m_w}{2} c_w (T_f - T_m) + \frac{m_w}{2} L_f + \frac{m_w}{2} c_i (T_m - T_i) = 0 \tag{9}$$

$$T_f = \frac{1}{3} \left(2T_w + \frac{c_i}{c_w} T_i + \left(1 - \frac{c_i}{c_w} \right) T_m - \frac{L_f}{c_w} \right) \tag{10}$$

Problem 2

One mole of an ideal monoatomic gas undergoes the cycle shown in the figure below. AB is a reversible isotherm at temperature T_A. BC is a reversible isobar at pressure P_B. CA is an irreversible isovolumetric process that brings the system back to the temperature T_A through an exchange of heat. If $V_B = 2V_C$ and $P_A = 2P_B$ find the work done by the gas during the cycle and the amount that it exchanges with the environment during the three transformations.

Solution: Let's first find the temperature at all points T_A, T_B, T_C. T_A is given and since AB is isothermal $T_B = T_A$.

Since $P_A = 2P_B$ and $V_B = 2V_C$, we can use the ideal gas law to solve for T_C

$$P_A V_A = nRT_A \to_{V_C=V_A} P_A V_c = nRT_A$$
$$P_C V_C = nRT_C \to_{P_C=P_B} P_B V_C = nRT_C$$
$$\implies \frac{V_C}{nR} = \frac{P_B}{T_C} = \frac{P_A}{T_A} \to_{P_A=2P_B} T_C = \frac{1}{2}T_A$$

- A→ B

  As AB is an isotherm, $\Delta T = 0$ and $\Delta E_{int} = \frac{3}{2}Nk_B\Delta T = 0$ and $Q = U + W_{by} = W_{by}$ where

  $$W_{AB} = \int_{V_C}^{V_B} P\,\mathrm{d}V \;= nRT_A \int_{V_C}^{V_B} \frac{\mathrm{d}V}{V} = nRT_A \ln\frac{V_B}{V_C} = nRT_A \ln 2$$

  where we plugged in $V_B = 2V_C$. Thus, for $n = 1$,

  $$\boxed{W_{AB} = RT_A \ln 2} \tag{1}$$

- B→C For the isobaric process, $U, Q, W \neq 0$. Then, since P_B constant,

  $$W_{BC} = \int_{V_B}^{V_C} P_B\,\mathrm{d}V \;= P_B(V_B - V_A) = -P_B V_C$$

  where in the last equality we used the fact that $V_B = 2V_C$. Since $P_B V_C = nRT_C = nRT_A/2$, $W_{BC} = nRT_A/2$ Thus, for $n = 1$

  $$\boxed{W_{BC} = \frac{1}{2}RT_A} \tag{2}$$

- C→A For the isovolumetric process, $\Delta V = 0$ so,

  $$W_{CA} = \int_{V_C}^{V_A} P\,\mathrm{d}V \;= 0$$

  Thus,

  $$\boxed{W_{CA} = 0} \tag{3}$$

  During the entire cycle, since $P_B V_C = nRT_C = nRT_A/2$,

  $$W_{\text{total}} = W_{AB} + W_{BC} + W_{CA} = nRT_A \ln 2 - P_B V_C = nRT_A \ln 2 - nRT_A/2 = \left(\ln 2 - \frac{1}{2}\right) nRT_A$$

  Thus, for $n = 1$

  $$\boxed{W_{\text{total}} = \left(\ln 2 - \frac{1}{2}\right) RT_A} \tag{4}$$

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Grading Rubric

Out of 20 points:

- + 6 AB: Correct Answer for $W_{AB} = RT_A \ln 2$
  - + 2 Writing Expression $W = \int P \,\mathrm{d}V$
  - + 2 Writing Expression $W = nRT_A \int \frac{\mathrm{d}V}{V}$
  - + 1 Correct Integration $W = nRT_A \ln \frac{V_B}{V_C}$
  - + 1 Substitution for $V_B = 2V_C$
- + 6 BC: Correct Expression $W = \frac{1}{2}RT_A = -P_B V_C = -\frac{1}{2}P_A V_C = -\frac{1}{4}P_A V_B$
  - + 2 Writing Expression $W = \int P \,\mathrm{d}V$
  - + 2 Writing Expression $\Delta P = 0$
  - + 2 Correct Answer
- + 6 CA: Correct Answer for $W_{CA} = 0$
  - + 2 Writing Expression $W = \int P \,\mathrm{d}V$
  - + 2 Writing Expression $\Delta V = 0$
  - + 2 Correct Answer
- + 2 Cycle: Correct Expression for $W_{tot} = \left(\ln 2 - \frac{1}{2}\right) RT_A$
  - + 1 Writing Expression $W_{tot} = W_{AB} + W_{BC} + W_{CA}$
  - + 1 Correct Answer

-1 Final answer in terms of T_C, P_C, V_A
-1 Off by a sign

Midterm 1, Problem 3 Rubric:

Part #	Point Total	5/5	4/5	3/5	2/5	1/5	0/5
Part A	5	Completely Correct	1 minor mistake, overall conceptual understanding is there.	More than 1 mistake.	Largely incorrect, but partial understanding displayed.	Complete lack of conceptual understanding.	No answer.
Part B	5	Completely Correct	1 minor mistake, overall conceptual understanding is there.	More than 1 mistake.	Largely incorrect, but partial understanding displayed.	Complete lack of conceptual understanding.	No answer.
Part C	5	Completely Correct	1 minor mistake, overall conceptual understanding is there.	More than 1 mistake.	Largely incorrect, but partial understanding displayed.	Complete lack of conceptual understanding.	No answer.

<u>Solution:</u>

3a). Using the ideal gas law, PV = NKT:

$$\mathrm{N} = \frac{PV}{KT} = \frac{(1.013*10^5)(4/3)\pi*(0.15)^3}{1.381*10^{-23}(273.15+20)} = 3.54*10^{23}\text{atoms}$$

3b). There are 3 degrees of freedom, thus the average kinetic energy is:

$$\left(\frac{3}{2}\right)*\mathrm{kT} = 6.07*10^{-21}\text{ Joules}$$

3c). The rms speed is related to the kinetic energy by:

$$E = \frac{3}{2}kT = \frac{1}{2}m < v^2 > => < v^2 > = \frac{2E}{m}$$

$$v_{rms} = \sqrt{(< v^2 >)} = \sqrt{2\frac{<E>}{m}} = \sqrt{\frac{2*6.07*10\text{^}-21}{4*1.66*10\text{^}-27}} = 1352 \text{ m/s}$$

Problem 4

4. (a) (15 points) Because there is vacuum above the piston, the only forces on it are the downward force due to gravity F_g, and the upward force due to the pressure of the gas F_P. These are equal in magnitude, so we can determine the pressure P.

$$P = \frac{F_P}{A} = \frac{F_g}{A} \tag{1}$$

$$= \frac{Mg}{A} \qquad \text{(1 pt)} \tag{2}$$

This is true throughout the process, i.e. this is an isobaric expansion (1 pt). Now, to find Q_1, we can use either the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, or the molar specific heat at constant volume, $Q = nC_p\Delta T$. In each case, we need to find the change in temperature, so we start with that. To find the final temperature T_2, we use the ideal gas law $PV = nRT$ (1 pt) and the fact that this is an isobaric process. Now,

$$\frac{T_2}{V_2} = \frac{P}{nR} = \frac{T_1}{V_1} \tag{3}$$

$$T_2 = T_1\left(\frac{V_2}{V_1}\right) \qquad \text{(1 pt)} \tag{4}$$

$$\Delta T = T_2 - T_1 \tag{5}$$

$$= T_1\left(\frac{V_2}{V_1}\right) - T_1 \tag{6}$$

$$= T_1\left(\frac{V_2}{V_1} - 1\right) \tag{7}$$

$$= \frac{T_1}{V_1}(V_2 - V_1) \tag{8}$$

$$= \frac{T_1}{V_1}\Delta V \tag{9}$$

where $\Delta V = V_2 - V_1$. We will also need the number of degrees of freedom d. This is a diatomic gas, and we assume that it is at a moderate temperature. This means that the temperature is high enough for the rotational degrees of freedom to be active while still cold enough that the vibrational degrees of freedom are frozen. In this case, $d = 5$ (1 pt).

i. If we use the first law, we need ΔE and W.

$$\Delta E = \frac{d}{2}nR\Delta T \qquad \text{(1 pt)} \tag{10}$$

$$= \frac{d}{2}nR\frac{T_1}{V_1}\Delta V \tag{11}$$

$$= \frac{d}{2}P\Delta V \qquad \left(\text{because } P = \frac{nRT_1}{V_1}\right) \tag{12}$$

$$= \frac{5}{2}\frac{Mg}{A}(V_2 - V_1) \qquad \text{(1 pt)} \tag{13}$$

$$W = P\Delta V \qquad \text{(1 pt)} \tag{14}$$

$$= \frac{Mg}{A}(V_2 - V_1) \qquad \text{(1 pt)} \tag{15}$$

$$Q_1 = \Delta E + W \qquad \text{(2 pts)} \tag{16}$$

$$= \frac{7}{2}\frac{Mg}{A}(V_2 - V_1) \tag{17}$$

ii. In order to use the molar specific heat, we need to recall the following.

$$C_v = \frac{d}{2}R \qquad \text{(1 pt)} \tag{18}$$

$$C_p = C_v + R \qquad \text{(1 pt)} \tag{19}$$

$$= \frac{d}{2}R + R \tag{20}$$

$$= \frac{7}{2}R \qquad \text{(1 pt)} \tag{21}$$

Now, we can use this to find Q_1.

$$Q_1 = nC_p\Delta T \qquad \text{(3 pts)} \tag{22}$$

$$= n\frac{d+2}{2}R\frac{T_1}{V_1}\Delta V \tag{23}$$

$$= \frac{d+2}{2}P\Delta V \qquad \text{(because } P = \frac{nRT_1}{V_1}\text{)} \tag{24}$$

$$= \frac{7}{2}\frac{Mg}{A}(V_2 - V_1) \tag{25}$$

(Students can only get points from one approach or the other.) With either approach, we find that

$$Q_1 = \frac{7}{2}\frac{Mg}{A}(V_2 - V_1) \qquad \text{(3 pts).} \tag{26}$$

Note that the number of moles n does not appear in the answer. If the algebra is done differently, it may be necessary to find n.

$$n = \frac{PV_1}{RT_1} \tag{27}$$

$$= \frac{Mg}{A}\frac{V_1}{RT_1} \qquad \text{(1 pt)} \tag{28}$$

(b) (5 points) The paddlewheel does work W on the gas, which must add to the change in internal energy of the gas. Let ΔE_1 and W_1 be the change in internal energy and work done by the gas in part a, respectively (which we previously called ΔE and W). Similarly, let ΔE_2 and W_2 be the change in internal energy and work done by the gas on the piston in part b, respectively. Since the final volume is the same, the final temperature is the same, and therefore the change in internal energy is the same. The final volume being the same also means that the work done by the gas on the piston will be the same.

$$\Delta E_1 = \Delta E_2 \qquad \text{(.5 pts)} \tag{29}$$

$$W_1 = W_2 \qquad \text{(.5 pts)} \tag{30}$$

$$\Delta E_1 = Q_1 - W_1 \tag{31}$$

$$\Delta E_2 = Q_2 - W_2 + W \qquad \text{(1 pt)} \tag{32}$$

$$Q_2 = Q_1 - W \qquad \text{(1 pt)} \tag{33}$$

$$= \frac{7}{2}\frac{Mg}{A}(V_2 - V_1) - W \qquad \text{(2 pts)} \tag{34}$$

Problem 5 solution

October 2020

(a) (5 points)

$$Q = -MC(T_1 - 273) - 1/2ML \qquad \text{(2.5 pts for each term)} \tag{1}$$

(b) (8 points)

$$dQ = -MCdT - LdM \qquad \text{(3 pts)} \tag{2}$$

$$\Delta S_c = \int \frac{dQ}{T} \qquad \text{(2 pts)} \tag{3}$$

$$= -\int_{273\text{K}}^{T_1} MCdT - \int_0^{M/2} LdM \qquad \text{(2 pts)} \tag{4}$$

$$= -MCln\left(\frac{T_1}{273\text{K}}\right) - \frac{ML}{546\text{K}} \qquad \text{(1 pt)} \tag{5}$$

(c) (4 points)
The second law of thermodynamics states that any physical process must increase the total entropy of the universe.
The entropy computed in (b) is negative. However, that just says that the entropy of the system decreases. As long as the entropy of the surroundings increases more, the net entropy of the universe will increase. [If a student got a wrong answer for part (a), I won't take any further points here and will evaluate this part based on their result from (a).]

(d) (8 points) The refrigerator is ideal, and so the process must be reversible. The total entropy change of the setup must be zero:

$$\Delta S_H = MCln\left(\frac{T_1}{273\text{K}}\right) + \frac{ML}{546\text{K}} \qquad \text{(3 pts)} \tag{6}$$

$$= \frac{Q_H}{T_2} \qquad \text{(2 pt)} \tag{7}$$

So,

$$Q_H = T_2\left(ln\left(\frac{T_1}{273\text{K}}\right) + \frac{ML}{546\text{K}}\right) \tag{8}$$

$$W = Q_H - Q_C \qquad \text{(2 pts)} \tag{9}$$

$$= T_2\left(ln\left(\frac{T_1}{273\text{K}}\right) + \frac{ML}{546\text{K}}\right) - MC(T_1 - 273\text{K}) - 1/2ML \qquad \text{(2 pts)} \tag{10}$$