University of California, Berkeley Physics

7B Spring 2020, Lecture 2, Midterm Exam II

C. Bordel and F. Wang

Please read the following honor code and follow the instructions below:

Honor Code (adapted from the Stanford University Honor Code)

This Honor Code is an undertaking of the student:

- *that they will not give, receive, or seek to obtain unpermitted aid or resources during this examination*
- *that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of this Honor Code.*

The instructors on their part manifest their confidence in the honor of their students by refraining from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above.

To indicate your agreement to abide by the honor code above, please write "I agree to abide by the Honor Code printed on the examination" at the beginning of your solutions, followed by a signature and date.

A submission without an agreement to abide by the Honor Code may not be graded.

University of California, Berkeley Physics

7B Spring 2020, Lecture 2 regular and Lecture 1 alternate, Midterm Exam II

C. Bordel and F. Wang

Electrostatics and DC circuits

Maximum score: 100 points

Please clearly indicate on your solutions what problem/part you are working on. Make sure you show all your work and justify your answers in order to get full credit.

1. (20 points)

An infinitely long wire with charge per unit length λ passes through the center of a cube as shown. Assume the length of the edge of the cube is L.

(a) Sketch the electrical field lines from the charged wire.

(b) Find the flux coming out of each surface of the cube.

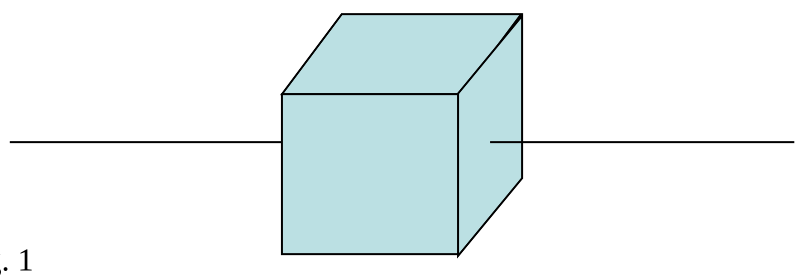

Fig. 1

2. (20 points) Three concentric conducting spherical shells have radii a, b, c such that $a < b < c$. Initially, the inner shell is uncharged, the middle shell has a positive charge $+Q$, and the outer shell has a negative charge $-Q$. The electrical potential at infinity is set at zero.

(a) Find the electric potential of the three conducting shells.

(b) The inner and outer shells are now connected by a thin conducting wire that is insulated as it passes through the middle shell. What is the final charge on each shell?

3. (20 points)

Consider a system shown in Fig. 2. It is composed of a sphere with a spherical cavity and a point charge Q. The sphere has a radius of R, a uniform charge density of ρ, and is centered at the position (0,0,0). The cavity has a radius of $0.75R$, zero charge density, and is centered at the position ($0.25R$, 0,0). The point charge is at position (L, 0, 0), and $L > R$.

(a) Find the electrical field along the x-axis between (R,0,0) and (L,0.0).

(b) What is the direction and magnitude of the force $\boldsymbol{F}$ acting on the sphere with the cavity by the point charge?

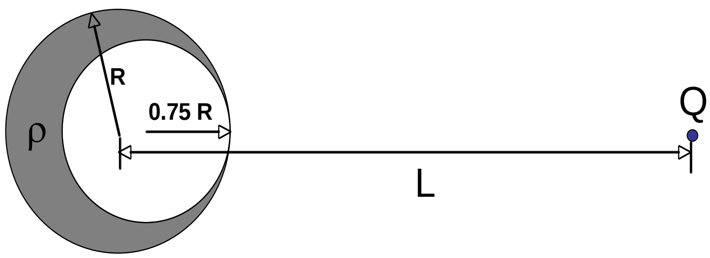

Fig. 2

4. (20 points)

A parallel plate capacitor with area A and separation d has charge Q and $-Q$ on the two plates. The capacitor is not connected to anything. Then a thick metal plate of thickness L is placed in between the plates, a distance x from the positively charged plate ($x + L < d$), as illustrated in Fig. 3. Assume that the capacitor's area is very large compared to the separation and that edge effects can be neglected.

(a) Find the electrical field in region I, II, and III in Fig. 3. What is the potential difference between the two capacitor plates?

(b) What is the energy stored in the system?

(c) Now if we want to pull the thick metal plate out of the capacitor to a position far away, how much work do we have to do?

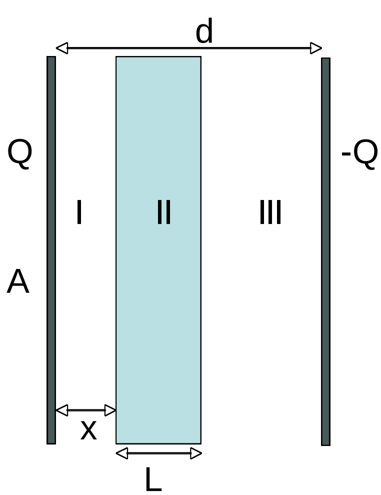

Fig. 3

5. (20 points) In the circuit shown in Fig. 4, all five resistors have the same value, 100 ohms, and each cell has an electromotive force of 1.5V.

(a) Find the potential difference between point *A* and *B*. This potential difference is known as open circuit voltage for the system.

(b) If we connect *A* and *B* by a metal wire with zero resistance, the voltage and current distribution in the circuit will change. What will be the direction and magnitude of the current flowing through the wire between *A* and *B*? This current is known as the short circuit current for the system.

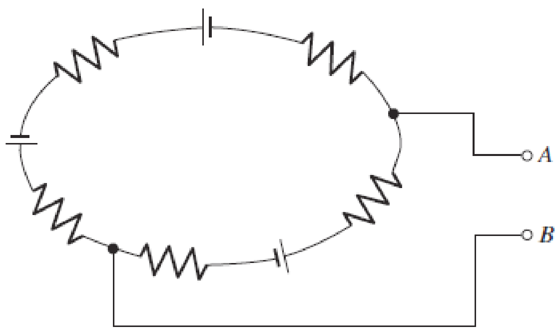

Fig. 4

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r} = \frac{kQ_1Q_2}{r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r} = \frac{k\,dQ}{r^2}\hat{r}$$

$$\rho = \frac{dQ}{dV}, \quad \sigma = \frac{dQ}{dA}, \quad \lambda = \frac{dQ}{dl}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p}\times\vec{E}, \quad V = -\vec{p}\cdot\vec{E}$$

$$\Phi_E = \int \vec{E}\cdot d\vec{A}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon}$$

$$U = QV$$

$$V = -\int \vec{E}\cdot d\vec{l} = \int \frac{dQ}{4\pi\epsilon_0 r} = \int \frac{k}{r}dQ$$

$$E = -\nabla V$$

$$Q = CV$$

$$C_{\text{eq}} = C_1 + C_2 \text{ (parallel)}$$

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (series)}$$

$$\epsilon = \kappa\epsilon_0$$

$$C = \kappa C_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \frac{\epsilon}{2}\int |\vec{E}|^2\,dV,$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$P = IV$$

$$R = \frac{\rho l}{A}$$

$$R_{\text{eq}} = R_1 + R_2 \text{ (series)}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ (parallel)}$$

$$\sum_{\text{junction}} I = 0$$

$$\sum_{\text{loop}} V = 0$$

$$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi \text{ (spherical)}$$

$$dV = \rho\, d\rho\, d\phi\, dz \text{ (cylindrical)}$$

$$dA = r\, dr\, d\theta \text{ (polar)}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$