Consider the following continuous-time system:

$$\frac{d}{dt}\vec{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

Unfortunately, we can only measure $x_1(t)$, so we can only perform state feedback on $x_1(t)$, i.e., $u(t) = kx_1(t)$ for some $k \in \mathbb{R}$.

We want the system to be stable and overdamped. Which of the following is a possible value for k?

- (selected) $k = -2\sqrt{2}$
- $k = \sqrt{3}$
- Not possible to find an appropriate value for k.
- $k = -\sqrt{3}$
- $k = 2\sqrt{2}$

Question 2

1 / 1 pts

Which of the following statements are true?

I. An ideal capacitor acts as an open circuit at very high frequencies.

II. An ideal inductor acts as a short circuit at DC (zero frequency).

III. The voltage across a current source remains constant irrespective of the output current.

IV. Resistance can be determined from the device's current-voltage characteristic.

- (selected) II and IV only.

- II, III, and IV only.
- II and III only.
- I and II only.
- I, III, and IV only.

Question 3

1 / 1 pts

Which of the following could be the first singular value σ_1, the first left singular vector $\vec{u}_1$, and the first right singular vector $\vec{v}_1$ of an SVD of the 3×3 identity matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$?

- $\sigma_1 = 1 \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \vec{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$
- $\sigma_1 = 1 \quad \vec{u}_1 = \begin{bmatrix} -\frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix} \quad \vec{v}_1 = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}$
- $\sigma_1 = -1 \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \vec{v}_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$
- (selected) $\sigma_1 = 1 \quad \vec{u}_1 = \begin{bmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \end{bmatrix} \quad \vec{v}_1 = \begin{bmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}$

- [ ] $\sigma_1 = -1$ $\vec{u}_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$ $\vec{v}_1 = \begin{bmatrix} -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$

Question 4

1 / 1 pts

Suppose that A is a 3×4 matrix and has rank 2. Which of the following statements are true about the SVD A = $U\Sigma V^\top$, where Σ is a 3×4 matrix?

I. $\|A\vec{v}_1\| \geq \|A\vec{v}_2\|$, where $\vec{v}_1$ is the first column and $\vec{v}_2$ is the second column of V.

II. The third column of V is in the null space of A.

III. The third column of U is in the column space of A.

IV. The columns of V are eigenvectors of $A^T A$.

- [ ] I and III only.
- [ ] I and II only.
- [ ] I, III, and IV only.
- [ ] II, III, and IV only.
- [x] I, II, and IV only.

Question 5

1 / 1 pts

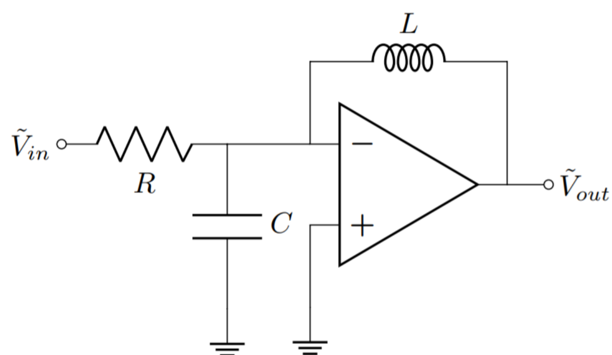
Maruf discretized a continuous-time linear system with eigenvalues $1, -1, -2$. One of the eigenvalues of the discretized system was 2. What are the other two eigenvalues?

- 0.5 and 4.
- 1 and -1.
- e and e^{-1}.
- 0.5 and 0.25.
- Cannot be determined without the sampling period specified.

Question 6

1 / 1 pts

Consider the following circuit with an ideal op-amp:



Find the transfer function $H(j\omega) = \frac{\tilde{V}_{out}}{\tilde{V}_{in}}$.

- $H(j\omega) = \frac{1}{j\omega RC}$
- $H(j\omega) = -\frac{L}{j\omega R}$

- $H(j\omega) = \frac{R}{R+j(\omega L - \frac{1}{RC})}$
- (selected) $H(j\omega) = -\frac{j\omega L}{R}$
- $H(j\omega) = -\frac{\omega^2 L}{C}$

Question 7

1 / 1 pts

Given the transfer function $H(j\omega) = \frac{j\omega RC}{1+j\omega RC}$, which of the following is an **incorrect** statement?

- $|H(j\omega)| = \frac{1}{\sqrt{5}}$ at $\omega = \frac{1}{2RC}$.
- (selected) The phase of $H(j\omega)$ at $\omega = \infty$ is $90°$.
- $|H(j\omega)| = \frac{1}{\sqrt{2}}$ at $\omega = \frac{1}{RC}$.
- $|H(j\omega)| = 1$ at $\omega = \infty$.
- The phase of $H(j\omega)$ at $\omega = \frac{1}{RC}$ is $45°$.

Question 8

1 / 1 pts

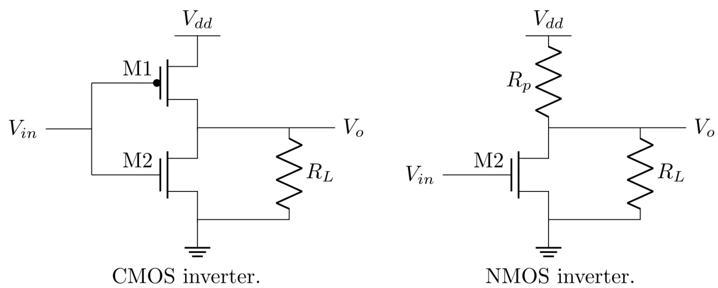
Which of the following are advantages of using CMOS over NMOS for an inverter that is loaded with a resistor as shown in the diagram below?

I. The minimum output voltage is lower.

II. The maximum output voltage is higher.

III. Power consumption when no switching is occurring is lower.

IV. Power consumption when switching is occurring is lower.



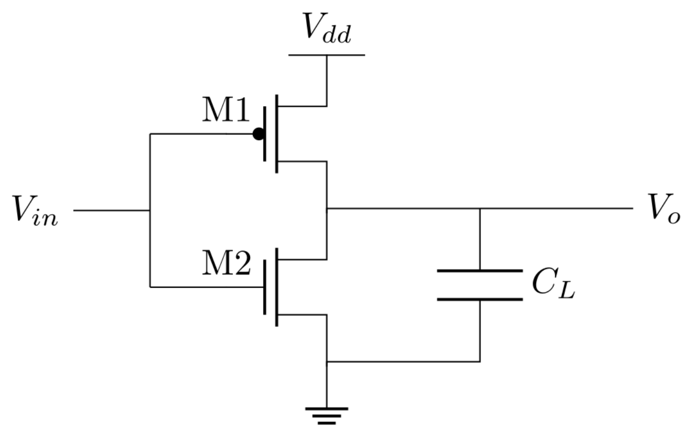
- II and IV only.
- I, II, III, and IV.
- II and III only. (selected)
- II, III, and IV only.
- I and III only.

Question 9

1 / 1 pts

You are given the inverter circuit shown below. You would like to reduce the power supplied by the power supply by a factor of 2.

You are given that $C_L = 1\text{ pF}$, $f_s = 1\text{ GHz}$ (the clock rate), $R_{on,p} = R_{on,n} = 10\Omega$, and $V_{dd} = 1\text{ V}$. You are only allowed to adjust the circuit as described below. Which of the following could you do?



- [ ] Add another PMOS transistor in series with M1 and another NMOS transistor in series with M2.
- [x] None of the other choices.
- [ ] Double C_L.
- [ ] Double V_{dd}.
- [ ] Add another PMOS transistor in parallel with M1 and another NMOS transistor in parallel with M2.

Question 10

1 / 1 pts

Consider the following discrete-time system:

$$\vec{x}(t+1) = A\vec{x}(t), \vec{x}(t) \in \mathbb{R}^3$$

We know that the eigenvalues of A are $0, -0.5, -2$. Which of the following statements are true?

I. The system is stable.

II. For some non-zero initial conditions, at least one of the state variables will grow exponentially unbounded.

III. For some non-zero initial conditions, $\vec{x}(t)$ will converge to $\vec{0}$ within one time step.

IV. For some non-zero initial conditions, $\vec{x}(t)$ will remain bounded for all $t = 0, 1, 2, \ldots$.

V. A possible system response is $\vec{x}(t) = \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$ for all $t = 0, 1, 2, \ldots$.

- I, IV, and V only.
- II and III only.
- II, III, IV, and V only.
- (selected) II, III, and IV only.
- I, III, and IV only.

Question 11

1 / 1 pts

Given the following system, $\vec{x}(t+1) = \begin{bmatrix} a & 2 \\ 1 & b \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t)$, which of the following values of a and b result in an **uncontrollable** system?

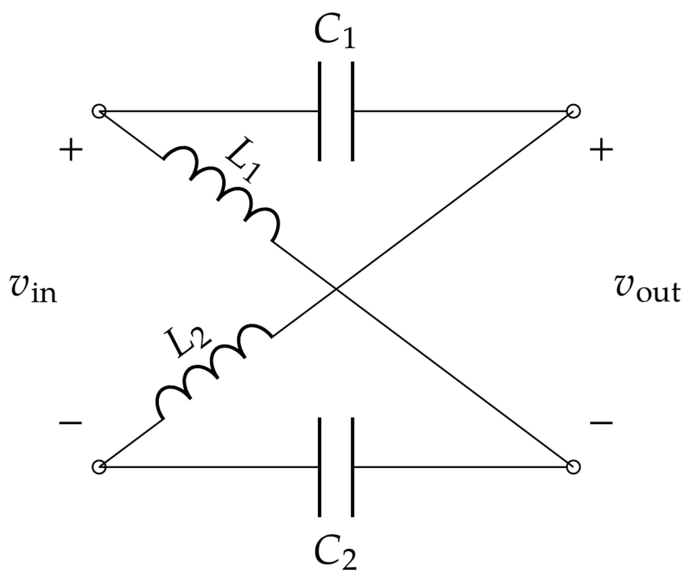
- $a = 0, b = 1$
- None of the other choices.
- $a = 0, b = 0$

- $a = 1, b = 0$
- $a = 1, b = 1$

Question 12

1 / 1 pts

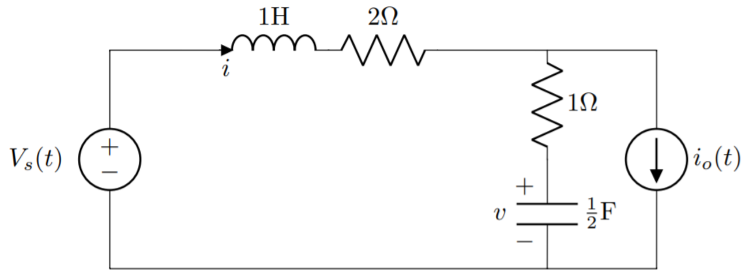
What are the values of $H(j\omega)$ at $\omega = 0$ and $\omega \to \infty$, respectively, where $H(j\omega) = \frac{\tilde{V}_{\text{out}}}{\tilde{V}_{\text{in}}}$?



- 1 and 1
- 0 and 0
- -1 and 1
- -1 and 0
- 0 and 1

Question 13

1 / 1 pts



MB3. Consider the circuit above with the inductor current and capacitor voltage as state variables.

Suppose that one finds a state-space model in standard form. With an invertible transformation of variables, the A matrix can be transformed to which of the following?

- ○ $A = \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}$
- ◉ $A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$
- ○ $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
- ○ None of the other choices.
- ○ $A = \begin{bmatrix} -2 & -1 \\ -2 & -1 \end{bmatrix}$

Question 14

1 / 1 pts

Consider the discrete-time system $\vec{x}(t+1) = A\vec{x}(t) + Bu(t)$, where $A = \begin{bmatrix} 0.5 & 0 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, with the state feedback $u(t) = k_1 x_1(t) + k_2 x_2(t)$.

Which of the following statements are true about the resulting closed-loop system?

I. The system is unstable with $k_1 = k_2 = 0$.

II. We can find some k_1 and k_2, such that the system is stable.

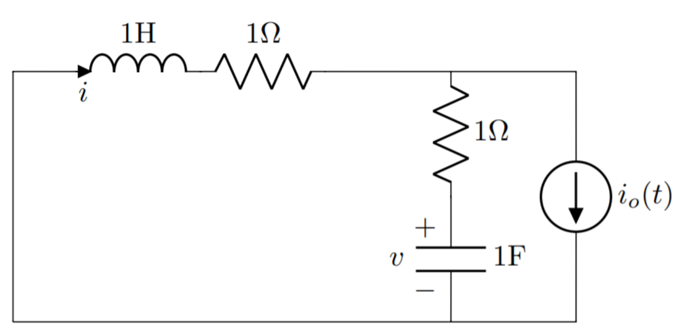
III. If we restrict k_2 to zero, we can find some k_1, such that the system is stable.

IV. If we restrict k_1 to zero, we can find some k_2, such that the system is stable.

- ○ II, III, and IV only.
- ◉ I, II, and IV only.
- ○ I, II, and III only.
- ○ I and II only.
- ○ I only.

Question 15

1 / 1 pts



Consider the circuit above with the inductor current i and capacitor voltage v as state variables and with the independent current source i_o as input.

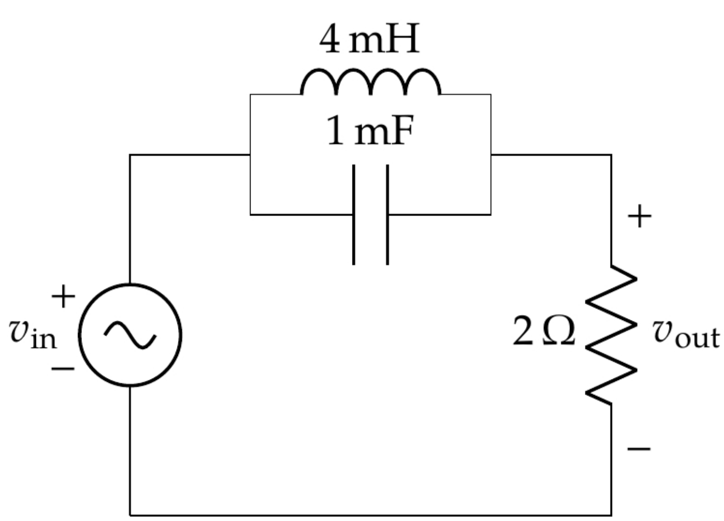
Suppose that the circuit is at equilibrium corresponding to $i_o^* = 0\text{ A}$ at $t = 0$. The states can be driven to the final state $(0\text{ V}, 1\text{ A})$ in time:

- Only on the interval $[0, 10]$.
- Any finite time.
- Only on the interval $[0, 1]$.
- Only as $t \to \infty$.
- (selected) None of the other choices.

Question 16

1 / 1 pts

The following circuit is a notch filter having $H(j\omega_c) = 0$, where $H(j\omega) = \frac{\tilde{V}_{\text{out}}}{\tilde{V}_{\text{in}}}$.



What is ω_c?

- ○ $0.5\,\frac{\text{rad}}{\text{s}}$
- ◉ $500\,\frac{\text{rad}}{\text{s}}$
- ○ $20\,\frac{\text{rad}}{\text{s}}$
- ○ $22\,\frac{\text{rad}}{\text{s}}$
- ○ $400\,\frac{\text{rad}}{\text{s}}$

Question 17

1 / 1 pts

Consider a matrix A and its SVD $A = U\Sigma V^\top$. Assume that the ith singular value of A is σ and that the ith right singular vector of A is $\vec{v}$.

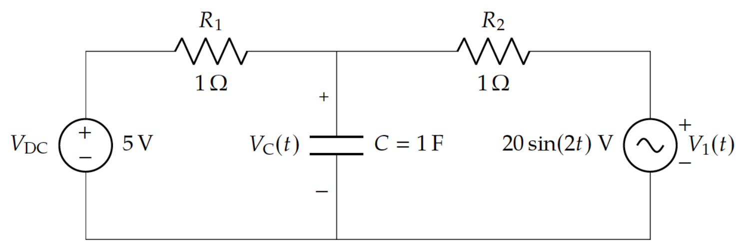
Now consider the matrix $B = \begin{bmatrix} A & \sqrt{3}A \end{bmatrix}$. Which of the following could be the corresponding ith singular value σ_i and ith right singular vector $\vec{v}_i$ of B?

- ○ $\sigma_i = (1+\sqrt{3})\sigma \qquad \vec{v}_i^\top = \begin{bmatrix} \frac{\sqrt{2}}{2}\vec{v}^\top & \frac{\sqrt{2}}{2}\vec{v}^\top \end{bmatrix}$
- ○ $\sigma_i = \sigma \qquad \vec{v}_i^\top = \begin{bmatrix} \vec{v}^\top & \sqrt{3}\vec{v}^\top \end{bmatrix}$
- ○ $\sigma_i = \sigma \qquad \vec{v}_i^\top = \begin{bmatrix} \frac{1}{2}\vec{v}^\top & \frac{\sqrt{3}}{2}\vec{v}^\top \end{bmatrix}$
- ◉ $\sigma_i = 2\sigma \qquad \vec{v}_i^\top = \begin{bmatrix} \frac{1}{2}\vec{v}^\top & \frac{\sqrt{3}}{2}\vec{v}^\top \end{bmatrix}$
- ○ $\sigma_i = \sqrt{3}\sigma \qquad \vec{v}_i^\top = \begin{bmatrix} \frac{\sqrt{2}}{2}\vec{v}^\top & \frac{\sqrt{2}}{2}\vec{v}^\top \end{bmatrix}$

Question 18

1 / 1 pts

In the following circuit, $V_{\text{DC}} = 5$ Volts and $V_1(t) = 20\sin(2t)$ Volts. Here, $R_1 = R_2 = 1\ \Omega$, and $C = 1$ F. Find the steady-state response of $V_{\text{C}}(t)$.



- $V_{\text{C}}(t) = 2.5 - 5\sqrt{2}\sin(2t + \frac{\pi}{4})$ V
- (selected) $V_{\text{C}}(t) = 2.5 + 5\sqrt{2}\sin(2t - \frac{\pi}{4})$ V
- $V_{\text{C}}(t) = 5\sqrt{2}\sin(2t + \frac{\pi}{4})$ V
- $V_{\text{C}}(t) = 2.5$ V
- $V_{\text{C}}(t) = 2.5 + 5\sqrt{2}\cos(2t - \frac{\pi}{4})$ V

Question 19

1 / 1 pts

Which of the following statements are true concerning the phasor analysis method used in 16B to solve circuit differential equations?

I. Phasor analysis yields the particular solution only.

II. The homogeneous solution contains the same frequency component as the input.

III. Differentiation is equivalent to rotation and scaling in the phasor domain.

IV. The input to the circuit has a constant frequency.

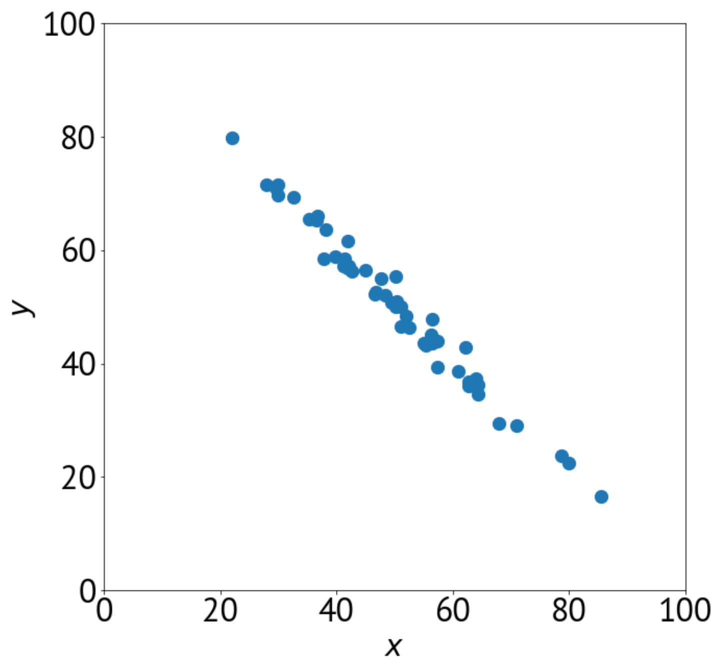
- ○ I and IV only.
- ○ I, II, and IV only.
- ○ I, II, and III only.
- ○ III and IV only.
- ◉ I, III, and IV only.

Question 20

1 / 1 pts

A tall matrix $A \in \mathbb{R}^{50\times 2}$, i.e., $A = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_{50} & y_{50} \end{bmatrix}$, is shown as a scatter plot below.

Each point (x_i, y_i) corresponds to a row $i = 1, 2, 3, \ldots, 50$ with x_i being the horizontal component and y_i the vertical component. Note that both columns have a mean of 50.



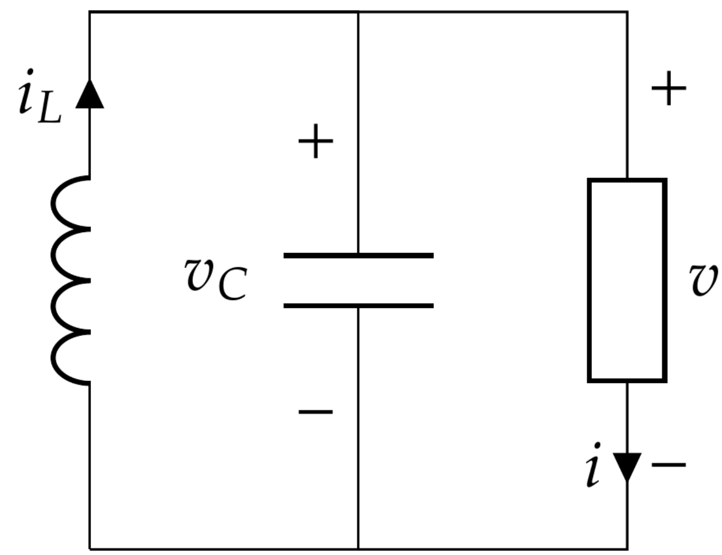
Suppose the matrix $\tilde{A}$ is obtained from A by subtracting the mean of the data from all entries of A. The right singular vectors of $\tilde{A}$ are given by which of the following?

- None of the other choices.
- $\vec{v}_1^\top = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ $\quad$ $\vec{v}_2^\top = \begin{bmatrix} 0 & 0 \end{bmatrix}$
- $\vec{v}_1^\top = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ $\quad$ $\vec{v}_2^\top = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$
- $\vec{v}_1^\top = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ $\quad$ $\vec{v}_2^\top = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$
- $\vec{v}_1^\top = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ $\quad$ $\vec{v}_2^\top = \begin{bmatrix} 0 & 0 \end{bmatrix}$

Question 21

1 / 1 pts

The following circuit contains a nonlinear resistor with the current-voltage characteristic $i = I_0 e^{v/V_0}$.



Which of the following is an equilibrium point?

- $v_C = V_0, i_L = 0$
- $v_C = 0, i_L = 0$
- $v_C = 0, i_L = I_0$
- $v_C = V_0, i_L = I_0$
- $v_C = 0, i_L = -I_0$

Question 22

1 / 1 pts

In the singular value decomposition, we can write any matrix A as the product of three matrices: $A = U\Sigma V^\top$, where U and V are both square matrices.

Which of the following statements are true?

I. U and V are both orthonormal matrices.

II. Σ has the same dimensions as A.

III. Left-multiplying a vector by $V^\top$ does not change its length.

IV. Left-multiplying a vector by U does not change its length.

- ○ I only.
- ○ II and III only.
- ○ I, III, and IV only.
- ◉ I, II, III, and IV.
- ○ II only.

Question 23

1 / 1 pts

A **discrete-time** system is given by $x(t+1) = 2x(t) - 2x(t)^2$.

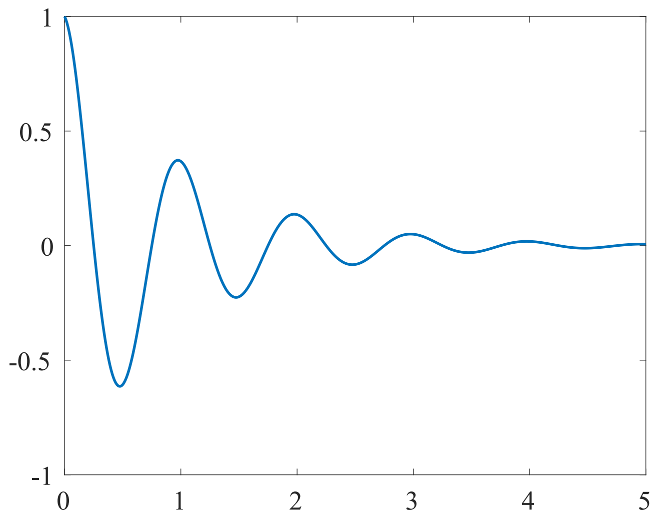
What are the equilibrium points?

- ○ $x = -1$ and $x = 0$
- ○ $x = 0$ is the only equilibrium point.
- ○ $x = 0$ and $x = 2$
- ◉ $x = 0$ and $x = 0.5$
- ○ $x = 0$ and $x = 1$

Question 24

1 / 1 pts

The transient response of a second-order linear continuous-time system is shown below. Which of the following could be one of the eigenvalues?

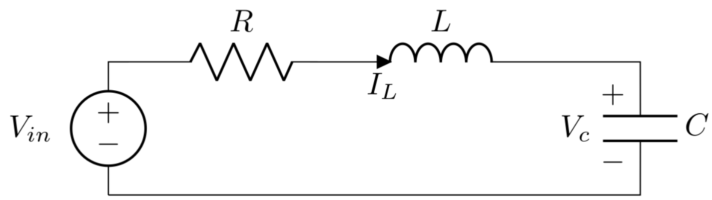


- $-1 + 2\pi j$
- $+1$
- $1 + j$
- $-10 + j$
- -10

Question 25

1 / 1 pts

You are given the following series RLC circuit:



Let the circuit's dynamics be modeled by the state space equation $\frac{d}{dt}\vec{x}(t) = \mathbf{A}\vec{x}(t) + \vec{b}u(t)$. What is a necessary condition for the eigenvectors of $\mathbf{A}$ to have non-zero imaginary components?

- (selected) The RLC circuit is underdamped.
- The RLC circuit is overdamped.
- The eigenvectors of an RLC circuit are always real-valued.
- Need more information.
- The RLC circuit is critically damped.

Quiz Score: **25** out of 25