University of California, Berkeley Physics

7B Spring 2020, Lecture 1, Midterm Exam II

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Please read the following honor code and follow the instructions below:

Honor Code (adapted from the Stanford University Honor Code)

This Honor Code is an undertaking of the student:

- *that they will not give, receive, or seek to obtain unpermitted aid or resources during this examination*
- *that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of this Honor Code.*

The instructors on their part manifest their confidence in the honor of their students by refraining from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above.

To indicate your agreement to abide by the honor code above, please write "I agree to abide by the Honor Code printed on the examination" at the beginning of your solutions, followed by a signature and date.

A submission without an agreement to abide by the Honor Code may not be graded.

PHYSICS 7B – Spring 2020 - Midterm 2
Lecture 1 regular exam and Lecture 2 alternate exam, C. Bordel & F. Wang
Monday, April 6th, 7-9pm PST

Make sure you show all your work and justify your answers in order to get full credit!

Problem 1 – DC circuit (20 pts)

A DC circuit is made of two batteries and seven identical resistors, as shown in Fig.1. The two batteries source an emf $\mathcal{E}_1$ and $\mathcal{E}_2$, and all the resistors have resistance R.

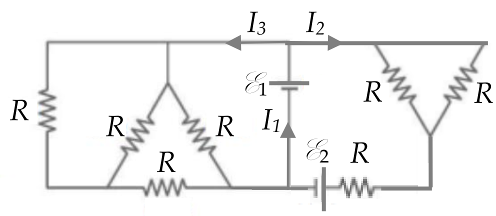

Figure 1

a) Determine the equivalent resistance on each side of the battery of emf $\mathcal{E}_1$ and redraw the circuit as a simple double-loop circuit with only two batteries and two resistors.

b) Determine the electric current passing through each branch of the simplified circuit.

c) Explain why one of the currents can never be equal to zero and determine under which conditions the other ones can be equal to zero.

Problem 2 – Charged tube (20 pts)

An uncapped tube of radius R and length L (Fig.2) carries uniform surface charge density σ.

a) Explain why you cannot use Gauss' law to determine the electric field created by the cylinder at any point on the x-axis.

b) Determine the electric field created by a very thin slice of the uniformly charged tube at any point on the x-axis. Take the origin at the center of the thin ring.

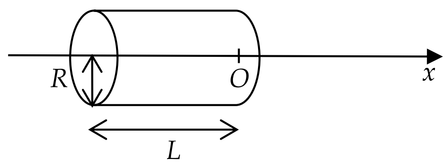

Figure 2

c) Determine the electric field created by the charged tube at any point on the x-axis (Fig.2). Take the origin at the right end of the cylinder.

Problem 3 – Charged solid sphere (20 pts)

A non-conducting solid sphere is carved out at the center so that the volumetric charge distribution $\rho_d(r)$ extends from inner radius R_1 to outer radius R_2, as shown in Fig.3.1. $\rho_d(r) = \alpha \frac{\exp(-r/R_1)}{r^2}$ where '*exp*' is the exponential function and α a positive constant.

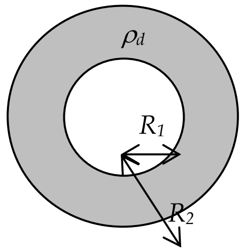

Figure 3.1

a) Determine the electric field created at any radial distance *r* from the center of the charge distribution.

Now you replace the non-conducting material by a conductor (still isolated) carrying the same amount of electric charge.

b) Calculate the surface charge densities σ_1 and σ_2 -on the inner and outer surfaces respectively- and the new volume charge density ρ_c in the bulk of the conductor.

The conducting material, which is assumed to have a uniform electrical resistivity ρ, is now connected to a battery that delivers a voltage V, as shown in Fig. 3.2. *You may assume that the wire leaving the negative terminal of the battery is inserted into a narrow insulated hole to connect to the inner radius of the conductor.*

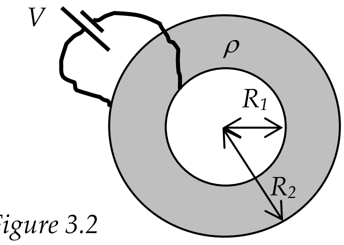

Figure 3.2

c) Determine the direction and absolute value of the electric current *I* passing through the resistor.

Problem 4 – CO_2 molecule (20 pts)

The CO_2 molecule can be represented by a linear triplet of point charges:
{*A(-q), O(+2q), B(-q)*} such that *AO=OB=a*, as shown in Fig.4.

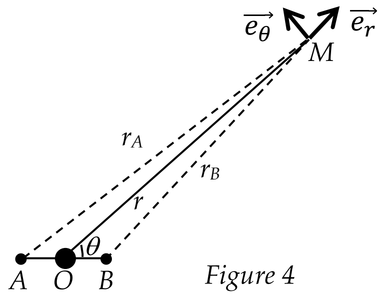

Figure 4

a) Determine the electric potential $V(r,\theta)$ at any point *M* such that $r >> a$.

Hints: Distance r_A can be written as $\left((\overrightarrow{AO} + \overrightarrow{OM})^2\right)^{1/2} = (\overrightarrow{AO}^2 + 2\,\overrightarrow{AO}.\overrightarrow{OM} + \overrightarrow{OM}^2)^{1/2}$.
Additionally, the function $(1+\varepsilon)^{-1/2}$ *should be expanded to the second order in* ε*, where* $\varepsilon \ll 1$.

$$(1+\varepsilon)^{-1/2} \approx 1 - \frac{\varepsilon}{2} + \frac{\varepsilon^2}{8}$$

b) Explain how you would calculate the components (E_r, E_θ) of the electric field created by this charge distribution. *You do not need to calculate them.*

Hint: The polar basis $(\vec{e_r}, \vec{e_\theta})$ *-which is a truncated cylindrical basis- represented at point M might be useful.*

Problem 5 – **Capacitor (20 pts)**

Two metallic coaxial cylindrical shells are placed vertically in a plastic bucket containing some oil of density ρ and dielectric constant κ. The length L of the shells is assumed to be very large compared to the inner radius a and outer radius b. A constant voltage V is applied across the two conducting shells, and the oil is observed to go up to a height h between the plates, as shown in Fig.5.

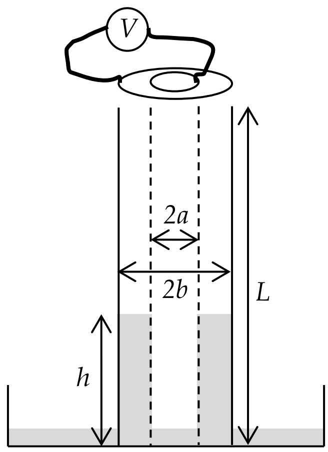

Figure 5

a) Determine the capacitance of the cylindrical capacitor when there is no oil between the shells.

b) Determine the capacitance of the cylindrical capacitor when the oil reaches height h between the shells.

c) Calculate ΔU_{batt} and ΔU_{capa}, the changes in potential energy of the battery and of the capacitor, resulting from the change in the height (from 0 to h) reached by the oil between the shells.
Hint: ΔU_{batt} is the negative of the work done by the battery to push some charge ΔQ on the capacitor's plates.

d) Determine the sign of the change in total potential energy of the battery and capacitor. If $\Delta U_{tot} > 0$, explain where the added energy comes from, and if $\Delta U_{tot} < 0$, explain where the released energy is stored.

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r} = \frac{kQ_1Q_2}{r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r} = \frac{k\,dQ}{r^2}\hat{r}$$

$$\rho = \frac{dQ}{dV}, \quad \sigma = \frac{dQ}{dA}, \quad \lambda = \frac{dQ}{dl}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}, \quad V = -\vec{p}\cdot\vec{E}$$

$$\Phi_E = \int \vec{E}\cdot d\vec{A}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon}$$

$$U = QV$$

$$V = -\int \vec{E}\cdot d\vec{l} = \int \frac{dQ}{4\pi\epsilon_0 r} = \int \frac{k}{r} dQ$$

$$E = -\nabla V$$

$$Q = CV$$

$$C_{\text{eq}} = C_1 + C_2 \text{ (parallel)}$$

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (series)}$$

$$\epsilon = \kappa\epsilon_0$$

$$C = \kappa C_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \frac{\epsilon}{2}\int |\vec{E}|^2\, dV,$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$P = IV$$

$$R = \frac{\rho l}{A}$$

$$R_{\text{eq}} = R_1 + R_2 \text{ (series)}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ (parallel)}$$

$$\sum_{\text{junction}} I = 0$$

$$\sum_{\text{loop}} V = 0$$

$$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi \text{ (spherical)}$$

$$dV = \rho\, d\rho\, d\phi\, dz \text{ (cylindrical)}$$

$$dA = r\, dr\, d\theta \text{ (polar)}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$