UNIVERSITY OF CALIFORNIA AT BERKELEY

Physics 7A – Lecture 1 (Stahler)

Fall 2019

FINAL EXAM

Please do all your work in this exam, in the blank space provided. If you have your solutions on scratch paper, **staple that paper to the last page of this exam.** Do not just insert the paper into the exam. **Only the front side of each page (including scratch work) will be scanned.**

You must attempt all six problems. If you become stuck on one, go on to another and return to the first one later. Be sure to show all your reasonimg, since partial credit will be allotted. No credit will be given for unjustified answers. **Remember to circle your final answer.**

Please complete the following. On each subsequent page, please write your SID in the upper right corner, where indicated.

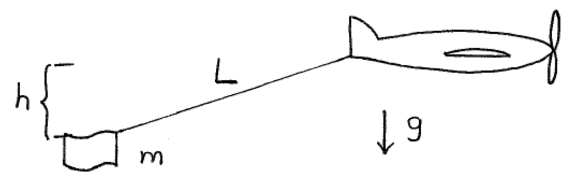
Full name:

SID:

Signature:

Problem 1 (15 points)

An airplane, flying horizontally, tows a small flag of mass m. The flag is tied to the plane by a massless cord of length L. The speed of both the plane and the flag are V_0, and both are subject to air drag, where the drag force is proportional to the velocity. The proportionality constants (drag coefficients) for the plane and flag are b_p and b_f, respectively.



(a) Find an expression for h, the vertical distance of the flag below the plane. (Consider drawing a free-body diagram.)

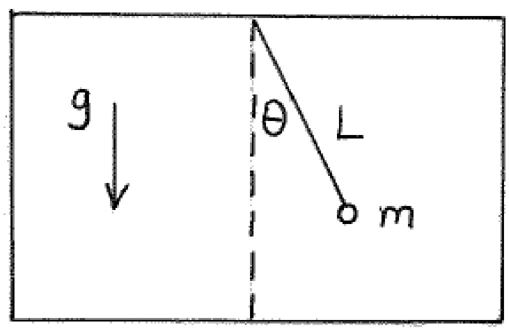
(b) What is P, the power exerted by the airplane's motor?

(c) At $t = 0$, the cord is cut and the flag descends. Find $V_x(t)$, the flag's horizontal speed as a function of time.

Problem 2 (20 points)

A pendulum of mass m and length L undergoes frictionless oscillations in a room that can accelerate. It may be assumed that $\theta \ll 1$ throughout the motion, where θ is the angle of the bob with respect to the vertical.

Suppose that the pendulum obeys $\theta(t) = \theta_0 \sin(\omega_0 t)$ for $t < 0$. At $t = 0$, the room begins to accelerate upward at rate $\alpha\, g$, where $0 < \alpha < 1$.



(a) Let ω_1 be the new angular frequency of oscillation. Find ω_1/ω_0. (*Hint*: Draw a free-body diagram in a frame moving with the room. Include both real and fictitious forces.)

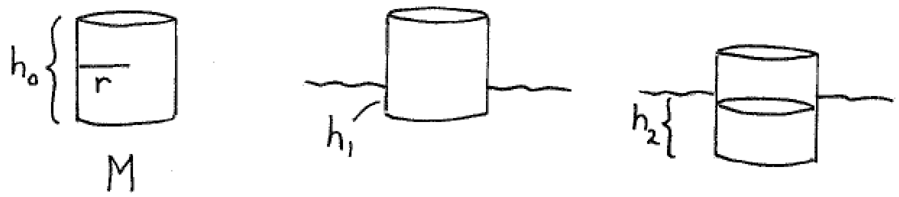
Now suppose the pendulum is at rest for $t < 0$ (i.e. $\theta = 0$). At $t = 0$, the room begins to accelerate to the right, at rate βg, where $0 < \beta < 1$. The pendulum now begins to swing.

(b) Let ω_2 be the new angular frequency of oscillation. Find ω_2/ω_0.

(c) What is θ_2, the amplitude of the motion for $t > 0$? (*Hint*: The pendulum is initially stationary, even in the accelerating room.)

Problem 3 (10 points)

A plastic cup, of mass M, is in the shape of a cylinder. The cylinder has radius r and height h_0. The cup is placed empty in a large vat of water, and floats. Let ρ be the mass density of water.

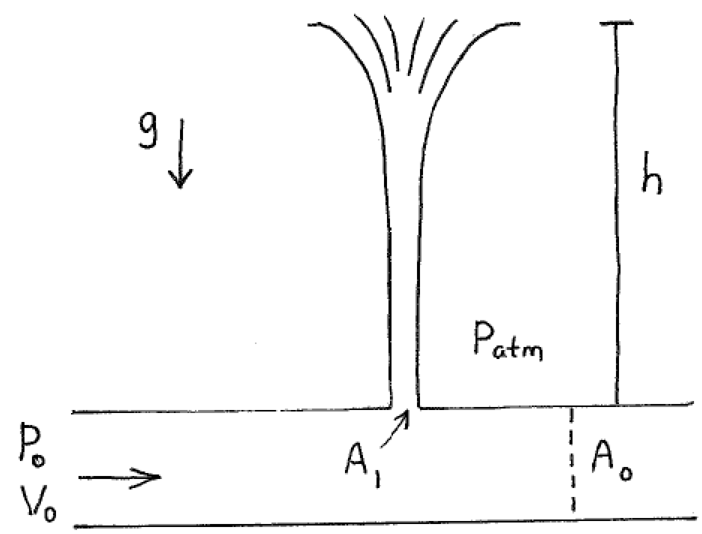


(a) Find h_1 the depth of the bottom of the cup below the surface of the water.

(b) You now pour water into the cup. If the added water fills exactly half the cup, what is h_2, the new depth of the cup's bottom below the surface of the water? Express your answer in terms of h_0 and h_1.

Problem 4 (20 points)

Water enters a horizontal pipe of cross sectional area A. The water, of mass density ρ, initially has pressure P_0 and speed V_0. Because of a small hole at the top of the pipe, a narrow jet spurts upward. Let A_1 be the area of this hole. Assume that the pressure everywhere inside the jet is the atmospheric value P_{atm}, where $P_{\text{atm}} < P_0$.



(a) What is $\dot{M}_0$, the mass per unit time of water entering the pipe from the left?

(b) What is h, the maximum height above the pipe attained by the upward jet of water?

(c) What is $\dot{M}_1$, the mass per unit time of water spurting up through the hole? Write your answer in terms of ρ, A_1, h, and g.

(d) Let A_2 be the cross sectional area of the jet at height $h/2$. Find A_2/A_1.

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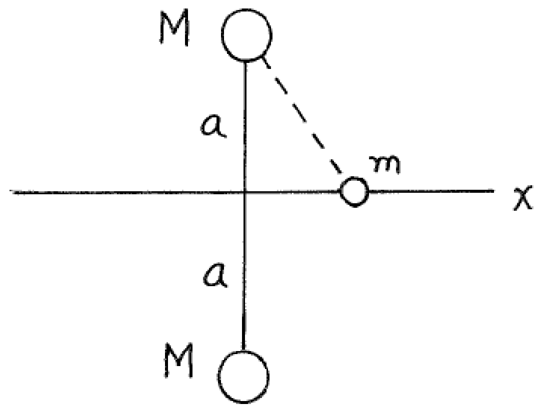
Problem 5 (15 points)

A small bead of mass m slides along a frictionless, massless rod lying in the horizontal ($x-$)direction. The bead is attracted gravitationally to two large, stationary masses M, separated by distance $2\,a$. Starting from rest, the bead oscillates about $x = 0$, the point midway between the two large masses.



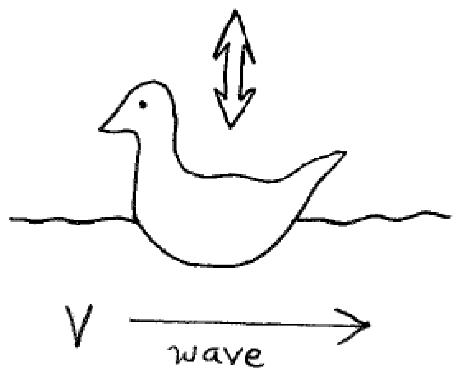
(a) Find $U(x)$, the gravitational potential energy of the bead as a function of its horizontal position x.

(b) Assuming that $x \ll a$, find an *approximate* expression for $U(x)$, valid through order $(x/a)^2$.

(c) Find ω, the angular frequency of the oscillation.

Problem 6 (20 points)

A duck floating in a pond bobs up and down because of a surface wave traveling to the right. At $t = 0$, the duck is at its lowest point. At $t = t_1$, it reaches its maximum height, which is Δh above the low point.



(a) What is ω, the angular frequency of the traveling wave?

(b) What is A, the amplitude of the wave?

Another duck on the pond is a distance D to the right. This second duck reaches its maximum height at $t = t_2 > t_1$.

(c) What is λ_{max}, the *longest* wavelength the traveling wave could have?

(d) Assuming the wavelength actually is λ_{max}, what is V, the wave speed?