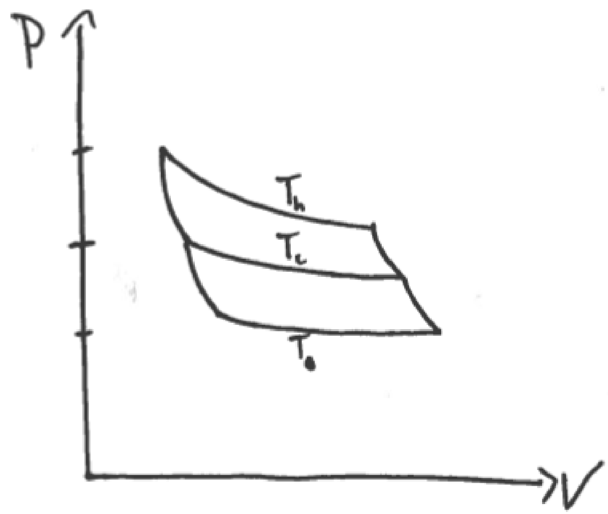
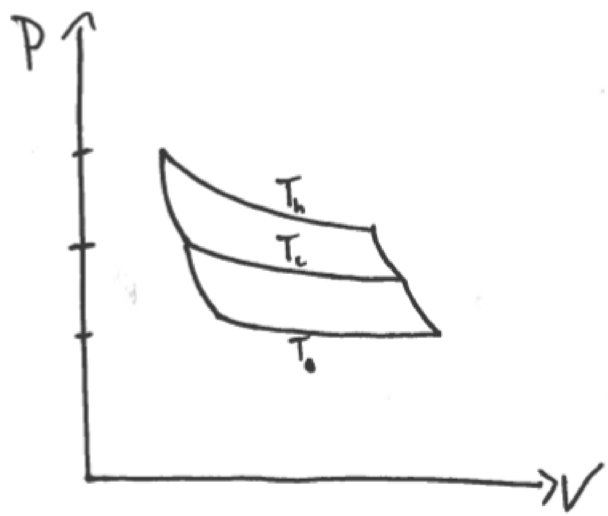
Problem 1

a) We stack the engines on top of one another in the P-V diagram:



b) We simply combine the engines:



c) From the diagram of the equivalent engine, one could simply write

$$\epsilon_{eff} = 1 - \frac{T_0}{T_h} \tag{1}$$

One can also show that

$$\epsilon_{eff} = \frac{W_1 + W_2}{Q_h} = \frac{\epsilon_1 Q_1 + \epsilon_2 (Q_1 - W_1)}{Q_1} = \epsilon_1 + \epsilon_2(1 - \epsilon_1) = 1 - \frac{T_0}{T_h} \tag{2}$$

Problem 2 The total heat is the sum of the heat requires to raise the temperature of the water and vaporize the wtaer:

$$Q = m_w c_w (T_f - T_i) + m_w L_w = m_w (c_w (T_f - T_i) + L_w) \tag{3}$$

$$= \frac{1}{2}(4 * 80 + 2200) \text{ kJ} = 1260 \text{ kJ} \tag{4}$$

Problem 3

a)

b) (i) As given in the problem statement, the current has reached a steady state, so the current through each surface is the same.

(ii) Using $J = \sigma E$ and the fact that $J = I/A$, we have $E = I/A\sigma$ so that through any of the surfaces, $\Phi_E = I/\sigma$. Since I and A are constant, the flux through all the surfaces is the same

(iii) From the expression $E = I/A\sigma$, we see that the electric field decreases as one goes from the larger end to the smaller end

Problem 4

a) This is effectively two capacitors in parallel. The effective capacitance is

$$C_{eff} = \frac{\kappa_1 \epsilon_0 A}{2d} + \frac{\kappa_2 \epsilon_0 A}{2d} = \frac{(\kappa_1 + \kappa_2)\epsilon_0 A}{2d} \tag{5}$$

b) This is two capacitors in series. Thus

$$\frac{1}{C_{eff}} = \frac{d_1}{\kappa_1 \epsilon_0 A} + \frac{d_2}{\kappa_2 \epsilon_0 A} \tag{6}$$

$$C_{eff} = \frac{\kappa_1 \kappa_2 \epsilon_0 A}{\kappa_1 d_2 + \kappa_2 d_1} \tag{7}$$

Problem 5 Since the two branches have different resistances, we have

$$I = I_1 + I_2 \tag{8}$$

$$I_1 R_1 = I_2 R_2 \tag{9}$$

so

$$I_1 = I - I_2 = I - I_1 \frac{R_1}{R_2} \rightarrow I_1 = \frac{IR_2}{R_1 + R_2} \tag{10}$$

$$I_2 = \frac{IR_1}{R_1 + R_2} \tag{11}$$

The force on a current carrying wire in a uniform magnetic field is given by $\vec{F} = I\vec{L} \times \vec{B}$. There is no force on the top and bottom parts of the loop. On the left side, there is a force of magnitude $F_1 = I_2 LB$ that points out of the page. On the right side, there is a magnetic force $F_2 = I_2 LB$, also out of the page. Thus the total torque is given by

$$\vec{\tau} = \frac{dI_2 LB}{2}\hat{y} - \frac{dI_1 LB}{2}\hat{y} = \frac{BLd}{2(R_1 + R_2)}(R_1 - R_2)\hat{y} \tag{12}$$

Problem 6

a). $R = \frac{\rho L}{A} = \frac{\rho}{A}(r\theta + 2r)$

$\int_0^r \int_0^\theta r' dr' d\theta$

b). $\Phi_B = \frac{B r^2 \theta}{2}$

$\frac{r^2}{2}\theta$

c). $I = \frac{\varepsilon}{R} = \frac{B\omega r^2}{2R} = \frac{BA\omega r}{2\rho(\theta+2)}$

$\theta = \alpha \frac{t^2}{2}$

$\omega = \alpha t$

$I = \frac{BA\alpha r t}{\rho(\alpha t^2 + 4)}$

$\frac{dI}{dt} = 0 = -\frac{ABr\alpha(t^2\alpha - 4)}{\rho(4 + t^2\alpha)^2}$

$t = \frac{2}{\sqrt{\alpha}}$

$\theta_m = \frac{\alpha}{2} \cdot \frac{4}{\alpha} = 2$

d). $\omega_m = \sqrt{2\alpha\theta_m}$

Problem 7

a) Since the emf drop across an inductor is $V = -L\frac{dI}{dt}$, we use the fact that two inductors in series carry the same current so that

$$V_{eff} = V_1 + V_2 = -(L_1 + L_2)\frac{dI}{dt} = -L_{eff}\frac{dI}{dt} \tag{17}$$

$$L_{eff} = L_1 + L_2 \tag{18}$$

b) The inductors are in parallel, so they have the same voltage drop

$$L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt} = L_{eff}\frac{dI}{dt} \tag{19}$$

where $I = I_1 + I_2$. Plugging this into the middle equation, we find

$$\frac{dI_1}{dt} = \frac{L_2}{L_1 + L_2}\frac{dI}{dt} \tag{20}$$

plugging this into the equation on the left and setting it equal to the voltage drop across the effective inductor, we have

$$L_{eff} = \frac{L_1 L_2}{L_1 + L_2} \tag{21}$$