EECS 120 **Fall 2004**

Sample Midterm 1

- The exam is for one hour and 50 minutes.

- The maximum score is 100 points. The maximum score for each part of each problem is indicated.

- The exam is closed-book and closed-notes. Calculators, computing, and communication devices are NOT permitted.

- Two double-sided sheets of notes are allowed. These should be legible to normal eyesight, i.e. the lettering should not be excessively small.

- No form of collaboration between students is allowed.

1. *(20 points)* State whether the following are true or false. In each case, give a brief explanation. A correct answer without a correct explanation gets 1 point. A correct answer with a correct explanation gets 4 points.

   (a) The periodic signal $g(t)$ with period T, where $\omega_0 = \frac{2\pi}{T}$,

   $$g(t) = \cos(\omega_0 t) + \sin^3(\omega_0 t) ,$$

   has only finitely many nonzero Fourier series coefficients.

   (b) A linear causal continuous time system is always time invariant.

   (c) The continuous time system whose output $y(t)$ for input $x(t)$ is given by

   $$y(t) = (1 + x^2(t))^{\cos(t)} ,$$

   is BIBO stable.

   (d) The signal

   $$s(t) = \sin(\frac{t}{1000})$$

   is an energy type signal.

   (e) The discrete time signal $x[n] = \cos(n)$ is a periodic signal.

2. *(10 points)*

   Consider a continuous time system whose output $y(t)$ for input

   $$x(t) = \cos(t) + \cos(2t)$$

   is

   $$y(t) = \frac{1}{2}(1 + \cos(t) + \cos(2t) + \cos(3t)) .$$

   Explain why this system cannot be a linear time-invariant (LTI) system.

3. *(10 points)*

   Consider the two signals

$$x_1(t) = \begin{cases} 1 & \text{if } -1 \le t \le 0 \\ 1-t & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   and

$$x_2(t) = \delta(t-1) - \delta(t-2) + \frac{1}{2}\delta(t-4) \ .$$

   Let

$$y(t) = x_1(t) * x_2(t)$$

   denote the convolution of these two signals. Sketch a plot of $y(t)$. Label the axes carefully so that there is no ambiguity in your answer. If there is any ambiguity, points will be taken off !

4. *(10 points)*

   A continuous time signal $x(t)$ has Fourier transform

$$X(j\omega) = \begin{cases} |\,\omega\,|\, e^{-j\omega} & \text{if } |\,\omega\,| < 1 \\ 0 & \text{otherwise} \end{cases} .$$

   Determine the signal $x(t)$.

5. *(5 + 5 + 5 + 5 + 5 points)*

   Consider the causal discrete time system defined by the equation

$$y[n] = x[n] + \alpha x[n-1] + \beta y[n-1] \ .$$

   (a) Draw a system diagram for this system using only delay elements, multiplications, and additions.

   (b) Suppose we start from the condition of initial rest at time 0, i.e. $y[-1] = 0$, and we apply the unit step input $x[n] = u[n]$. Find the corresponding output of the system, i.e. its step response.

   (c) Find the impulse response of the system. *Hint :* $\delta[n] = u[n] - u[n-1]$.

   (d) Find the frequency response $H(e^{j\omega})$ of the system, i.e. the discrete time Fourier transform of its impulse response.

   (e) For what values of α and β is the system BIBO stable ? Justify your answer.

6. *(10 points)*

   Consider the continuous time signal $x(t)$ with Fourier transform

$$X(j\omega) = \begin{cases} 2 & \text{if } \frac{9\pi}{4} < |\,\omega\,| \le 3\pi \\ 1 & \text{if } \frac{3\pi}{2} < |\,\omega\,| \le \frac{9\pi}{4} \\ \frac{3\pi}{2} - |\,\omega\,| & \text{if } |\,\omega\,| \le \frac{3\pi}{2} \\ 0 & \text{otherwise} \end{cases} .$$

This signal is passed through a linear time-invariant system with impulse response

$$h(t) = 2\text{sinc}(2t) \ ,$$

where, as usual,

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t} \ .$$

Let $y(t)$ denote the output of the system corresponding to the input $x(t)$. Determine

$$\int_{-\infty}^{\infty} \mid x(t) - y(t) \mid^2 dt \ .$$

7. *(10 + 5 points)*

A linear time-invariant system with input $x(t)$ and output $y(t)$ satisfies

$$\frac{d^2}{dt^2}y(t) + 2a\frac{d}{dt}y(t) + a^2 y(t) = x(t) \ .$$

(a) Find the frequency response $H(j\omega)$ of this system.

(b) Sketch a plot of the magnitude of the frequency response when $a = \frac{1}{2}$. Is the system high-pass or low-pass for this value of a ?