CS 188 Fall 2018 | Introduction to Artificial Intelligence | **Final Exam**

- You have 180 minutes. The time will be projected at the front of the room. You may not leave during the last 10 minutes of the exam.
- Do NOT open exams until told to. Write your SIDs in the top right corner of every page.
- If you need to go to the bathroom, bring us your exam, phone, and SID. We will record the time.
- In the interest of fairness, we want everyone to have access to the same information. To that end, we will not be answering questions about the content. If a clarification is needed, it will be projected at the front of the room. **Make sure to periodically check the clarifications**.
- The exam is closed book, closed laptop, and closed notes except your three-page double-sided cheat sheet. You are allowed a non-programmable calculator for this exam. Turn off and put away all other electronics.
- The last two sheets in your exam are scratch paper. Please detach them from your exam. Mark your answers ON THE EXAM IN THE DESIGNATED ANSWER AREAS. We will not grade anything on scratch paper.
- For multiple choice questions:
  - ☐ means mark ALL options that apply
  - ◯ means mark ONE choice
  - When selecting an answer, please fill in the bubble or square COMPLETELY (● and ■)

First name	
Last name	
SID	
Student to the right (SID and Name)	
Student to the left (SID and Name)	

Q1.	Agent Testing Today!	/1
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Q9.	Decision Trees	/8
	Total	/100

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Q1. [1 pt] Agent Testing Today!

It's testing time! Not only for you, but for our CS188 robots as well! Circle your favorite robot below.

Q2. [14 pts] Short Questions

(a) [2 pts] Which of the following properties must a set of preferences satisfy if they are rational:

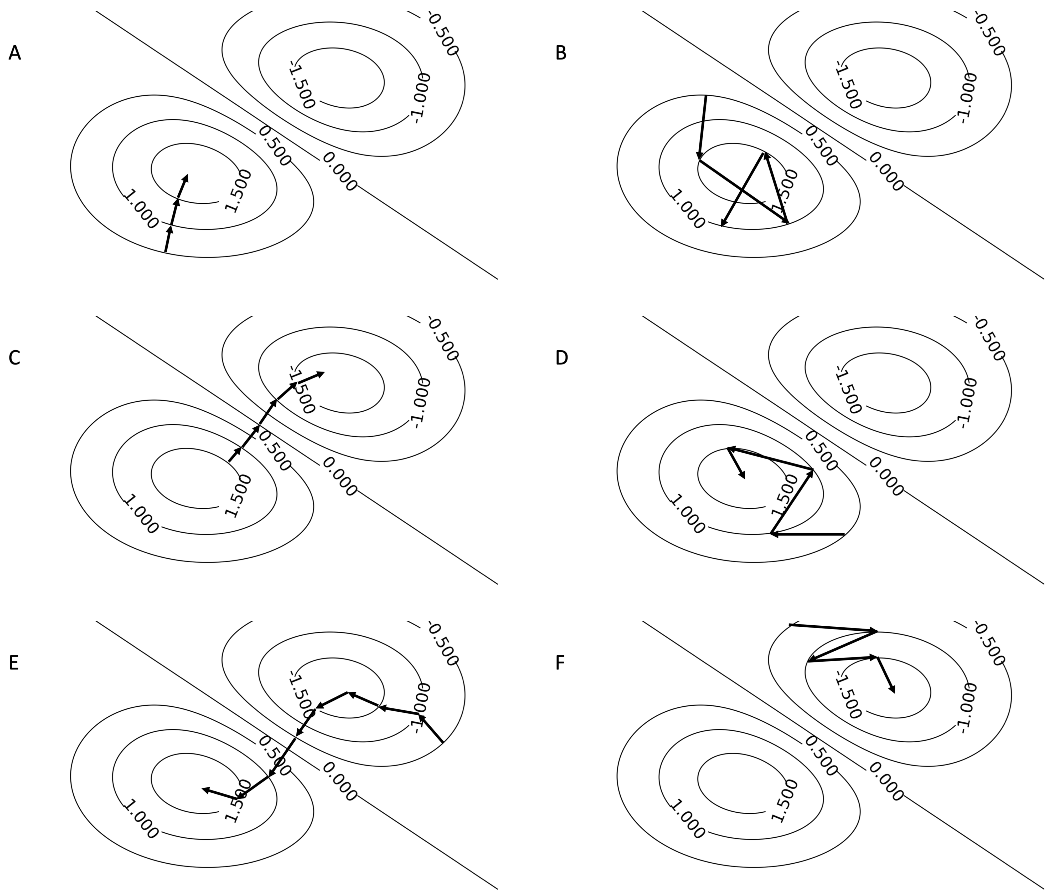
- ☐ $(A \succ B)$ OR $(B \succ A)$ OR $(B \sim A)$
- ☐ $(B \succ A)$ AND $(C \succ A) \Rightarrow (C \sim B)$
- ☐ $(A \sim B)$ AND $(B \sim C) \Rightarrow [p, A; 1-p, B] \sim [q, B; 1-q, C]$
- ☐ $A \succ B \succ C \succ D \Rightarrow [p, A; 1-p, C] \succ [p, B; 1-p, D]$
- ☐ $(B \succ A)$ AND $(C \succ B) \Rightarrow (C \succ A)$

(b) [2 pts] Which of the following are true?

- ☐ Given a set of preferences there exists a unique utility function.
- ☐ $U(x) = x^4$ is a risk prone utility
- ☐ $U(x) = 2x$ is a risk prone utility
- ☐ For any specific utility function, any lottery can be replaced by an appropriate deterministic utility value
- ☐ For the lotteries $A = [0.8, \$4000; 0.2, \$0]$, $B = [1.0, \$3000; 0.0, \$0]$ we have $A \succ B$

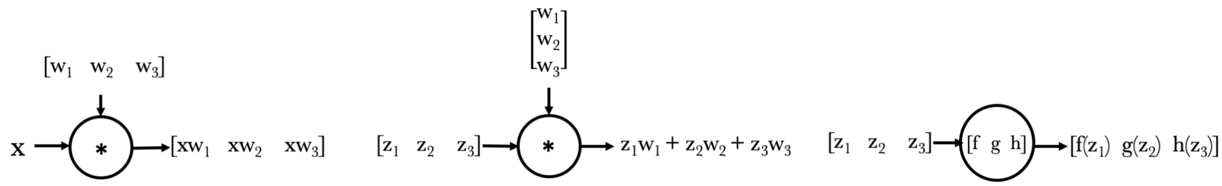
.

(c) [2 pts] Which of the following paths is a feasible trajectory for the gradient ascent algorithm?

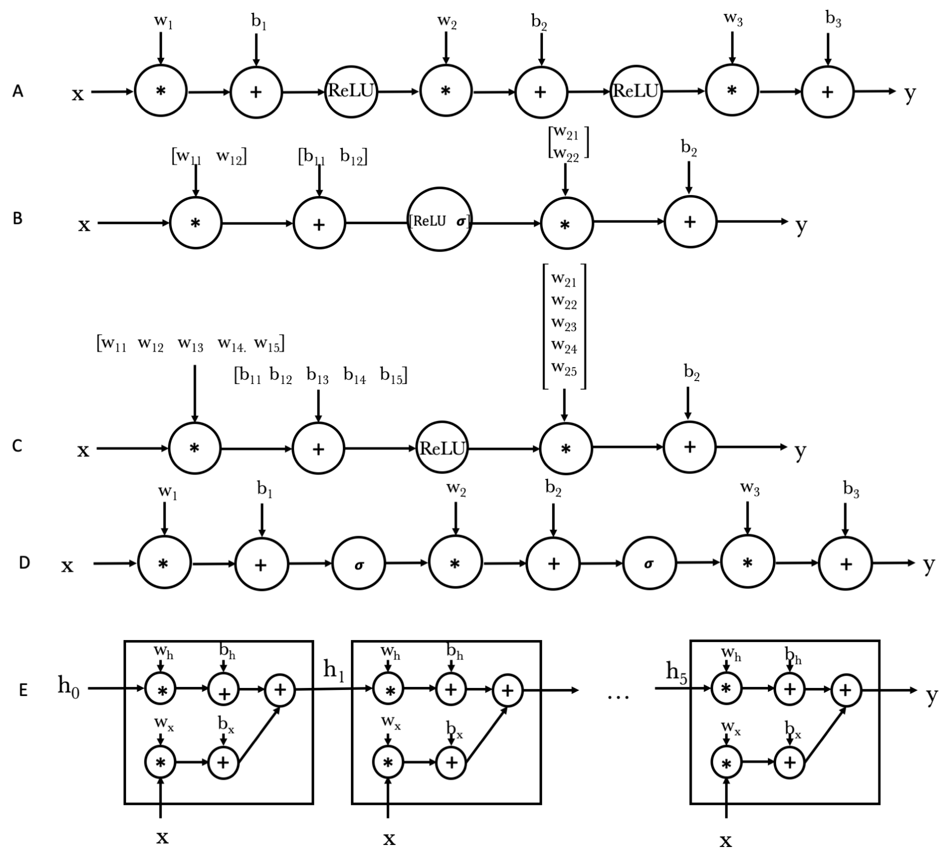


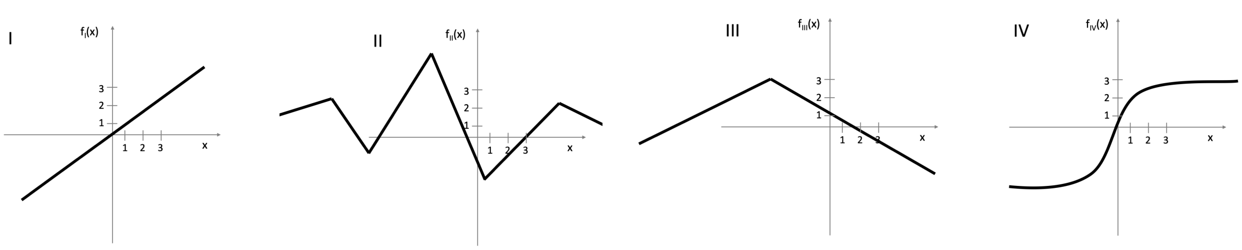
☐ A ☐ B ☐ C ☐ D ☐ E ☐ F

(d) We are given the following 5 neural networks (NN) architectures. The operation $*$ represents the matrix multiplication operation, $[w_{i1}...w_{ik}]$ and $[b_{i1}...b_{ik}]$ represents the weights and the biases of the NN, the orientation (vertical and horizontal) its just for consistency in the operations. The term [ReLU σ] in **B** means applying a ReLU activation to the first element of the vector and a sigmoid (σ) activation to the second element. These operations are depicted in the following figures:



Which of the following neural networks can represent each function?

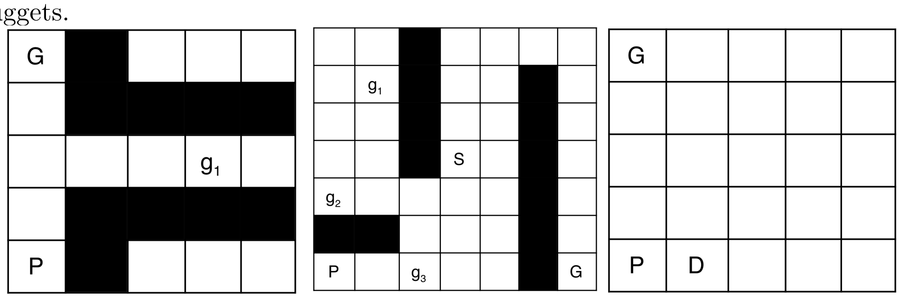




(i) [2 pts] $f_I(x)$:	☐ A	☐ B	☐ C	☐ D	☐ E
(ii) [2 pts] $f_{II}(x)$:	☐ A	☐ B	☐ C	☐ D	☐ E
(iii) [2 pts] $f_{III}(x)$:	☐ A	☐ B	☐ C	☐ D	☐ E
(iv) [2 pts] $f_{IV}(x)$:	☐ A	☐ B	☐ C	☐ D	☐ E

Q3. [12 pts] Treasure Hunting MDPs

In each below Gridworld, Pacman's starting position is denoted as P. G denotes the goal. At the goal, Pacman can only "Exit", which will cause Pacman to exit the Gridworld and gain reward +100. The Gridworld also has K gold nuggets $g_1, ..., g_k$ which will have the properties listed below in each part. Pacman automatically picks up a gold nugget if he enters a square containing one. Finally, define P_0 as Pacman's initial state, where he is in position P and has no gold nuggets.



(a) (b) (c)

(a) Pacman now ventures into (a). Each gold nugget Pacman holds when he exits will add +100 to his reward, and he will receive +100 when he exits from the goal G.

(i) [2 pts] When conducting value iteration, what is the first iteration at which $V(P_0)$ is nonzero?

Answer:

(ii) [2 pts] Assume Pacman will act optimally. What nonzero discount factor γ ensures that the policy of picking up g_1 before going to goal G and the policy of going straight to G yield the same reward?

Answer:

(b) Pacman is now at (b), which contains a Gold Store (S). He will receive +5 per nugget. When at the Store, Pacman can either "Sell" to sell all his gold for +5 per nugget or "Exit" to exit the Gridworld. Exiting from the Store yields 0 reward. Exiting from goal G will give $+100 + 5k$, where Pacman has k nuggets.

Note that Pacman can also only carry one gold nugget at a time.

(i) [2 pts] When conducting value iteration, what is the first iteration at which $V(P_0)$ is nonzero?

Answer:

(ii) [2 pts] Now Pacman is in a world with a Store that is not necessarily the Gridworld (b). Assume Pacman is acting optimally, and he begins at the Store. It takes Pacman time T_1, T_2, T_3 to go from the Store, pick up the nuggets g_1, g_2, g_3 respectively, return to the store, and sell each nugget. It takes time T_G to go from the Store and exit from the goal G. Assume $T_1 < T_2 < T_3 < T_G$.

What must be true such that the better policy for Pacman would be to gather and sell all nuggets and exit from the store rather than to gather all nuggets and exit from goal G?

- ○ $5(\gamma^{T_1}+\gamma^{T_1+T_2}+\gamma^{T_1+T_2+T_3}) > 115\gamma^{T_1+T_2+T_3+T_G}$
- ○ $5(\gamma^{T_1}+\gamma^{T_1+T_2}+\gamma^{T_1+T_2+T_3}) > 100\gamma^{T_G}$
- ○ $5(\gamma^{T_1}+\gamma^{T_2}+\gamma^{T_3}) > 100\gamma^{T_G}$
- ○ $15\gamma^{T_1+T_2+T_3} > 115\gamma^{T_G}$
- ○ $5(\gamma^{T_1}+\gamma^{T_1+T_2}+\gamma^{T_1+T_2+T_3}) > 115\gamma^{T_G}$
- ○ None of the above

(c) Finally, Pacman finds himself in Gridworld (c). There is no store. However, Pacman finds that there is now a living reward! He gets the living reward for every action he takes except the Exit action. Pacman receives +0 exiting from the Door, and +100 exiting from the Goal. Once in the Door, Pacman can only Exit.

(i) [2 pts] Suppose $\gamma = 0.5$. For what living reward will Pacman receive the same reward whether he exits via the Door or exits via the goal?

Answer:

(ii) [2 pts] Suppose $\gamma = 0.5$. What is the living reward such that Pacman receives the same reward if he traverses the Gridworld forever or if he goes straight to and exits from the goal? Hint: $\sum_{t=0}^{\infty} r\gamma^t = \frac{r}{1-\gamma}$

Answer:

Q4. [8 pts] Approximate Q-learning

(a) [2 pts] Pacman is trying to collect all food pellets, and each treasure chest contains 10 pellets but must be unlocked with a key. Pacman will automatically pick up a pellet or key when he enters a square containing them, and he will automatically unlock a chest when he enters a square with a chest and has at least one key. A key can only unlock on chest; after being used, it vanishes.

To finish, Pacman must exit through either goal G_1 or G_2.

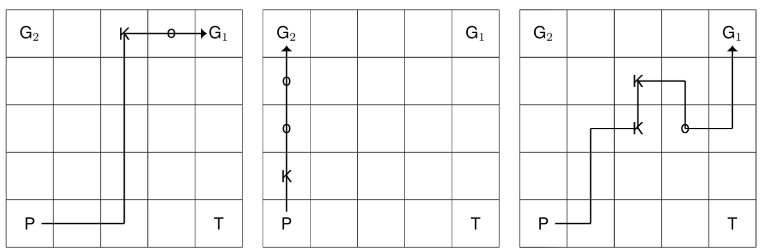
The keys are shown on the map as a K, treasure chests as T, and food pellets as circles. Pacman's starting position is shown as P. The goals are G_1, G_2.

When calculating features for a Q-function $Q(s, a)$, the state the features are calculated with is the state s' Pacman is in after taking action a from state s. The possible features the Q-learning can use are:

- N_{keys}: Number of keys Pacman holds.
- $D_m(K)$: Manhattan distance to closest key.
- N_{chests}: Number of chests Pacman has unlocked.
- $D_m(T)$: Manhattan distance to closest chest.
- N_{food}: Number of food pellets Pacman has eaten.
- $D_m(F)$: Manhattan distance to closest food pellet.
- $D_m(G_1)$: Manhattan distance to G_1.
- $D_m(G_2)$: Manhattan distance to G_2.

Note that the approximate Q-learning here can be any function over the features, not necessarily a weighted linear sum.

Suppose we finished training an agent using approximate Q-learning and we then run the learned policy to observe the following.



Assume we observe all the above paths. What is the minimal set of features that could have been used to learn this policy?

- ☐ N_{keys}
- ☐ $D_m(T)$
- ☐ $D_m(G_1)$
- ☐ $D_m(K)$
- ☐ N_{food}
- ☐ $D_m(G_2)$
- ☐ N_{chests}
- ☐ $D_m(F)$
- ☐ No features

(b) Suppose Pacman is now in an empty grid of size $M \times M$. For a Q-value $Q(s, a)$, the features are the x- and y-position of the state Pacman is in after taking action a.

Select "Possible with weighted sum" if the policy can be expressed by using a weighted linear sum to represent the Q-function. Select "Possible with large neural net" if the policy can be expressed by using a large neural net to represent the Q-function. Select "Not Possible" if expressing the policy with given features is impossible no matter the function.

(i) [2 pts] Pacman's optimal policy is always to go upwards.

☐ Possible with large neural net ☐ Possible with weighted linear sum ☐ Not Possible

(ii) [2 pts] We draw a vertical line and divide the Gridworld into two halves. On the left half, Pacman's optimal policy is to go upwards, and on the right half, Pacman's optimal policy is to go downwards.

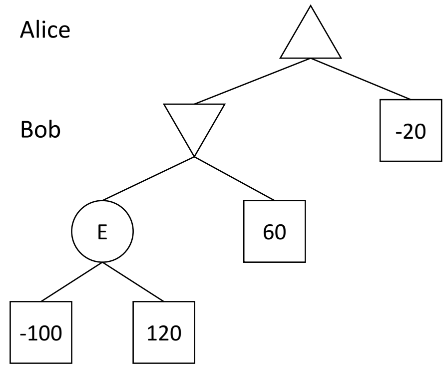
☐ Possible with large neural net ☐ Possible with weighted linear sum ☐ Not Possible

(iii) [2 pts] We draw a vertical line and divide the Gridworld into two equal halves. On the left half, Pacman's optimal policy is to go upwards, and on the right half, Pacman's optimal policy is to go right.

☐ Possible with large neural net ☐ Possible with weighted linear sum ☐ Not Possible

Q5. [16 pts] Value of Asymmetric Information

Alice and Bob are playing an adversarial game as shown in the game tree below. Alice (the MAX player) and Bob (the MIN player) are both rational and they both know that their opponent is also a rational player. The game tree has one chance node E whose outcome can be either $E = -100$ or $E = +120$ with equal 0.5 probability.



Each player's utility is equal to the amount of money he or she has. The value x of each leaf node in the game tree means that Bob will pay Alice x dollars after the game, so that Alice and Bob's utilities will be x and $-x$ respectively.

(a) [2 pts] Suppose neither Alice nor Bob knows the outcome of E before playing. What is Alice's expected utility?

Answer: ☐

(b) Carol, a good friend of Alice's, has access to E and can *secretly* tell Alice the outcome of E before the game starts (giving Alice the true outcome of E without lying). However, Bob is not aware of any communication between Alice and Carol, so he still assumes that Alice has no access to E.

(i) [1 pt] Suppose Carol secretly tells Alice that $E = -100$. What is Alice's expected utility in this case?

Answer: ☐

(ii) [1 pt] Suppose Carol secretly tells Alice that $E = +120$. What is Alice's expected utility in this case?

Answer: ☐

(iii) [1 pt] What is Alice's expected utility if Carol secretly tells Alice the outcome of E before playing?

Answer: ☐

We define the *value of private information* $V_{\text{A}}^{\text{pri}}(X)$ of a random variable X to a player A as the difference in player A's expected utility after the outcome of X becomes a private information to player A, such that A has access to the outcome of X, while other players have no access to X and are not aware of A's access to X.

(iv) [2 pts] In general, the value of private information $V_{\text{A}}^{\text{pri}}(X)$ of a variable X to a player A

- ☐ always satisfies $V_{\text{A}}^{\text{pri}}(X) > 0$ in all cases.
- ☐ always satisfies $V_{\text{A}}^{\text{pri}}(X) \geq 0$ in all cases.
- ☐ always satisfies $V_{\text{A}}^{\text{pri}}(X) = 0$ in all cases.
- ☐ can possibly satisfy $V_{\text{A}}^{\text{pri}}(X) < 0$ in certain cases.

(v) [1 pt] What is $V_{\text{Alice}}^{\text{pri}}(E)$, the value of private information of E to Alice in the specific game tree above?

Answer: ☐

(c) David also has access to E, and can make a *public* announcement of E (announcing the true outcome of E without lying), so that both Alice and Bob will know the outcome of E and are both aware that their opponent also knows the outcome of E. Also, Alice cannot obtain any information from Carol now.

(i) [1 pt] Suppose David publicly announces that $E = -100$. What is Alice's expected utility in this case?

Answer: ☐

(ii) [1 pt] Suppose David publicly announces that $E = +120$. What is Alice's expected utility in this case?

Answer: ☐

(iii) [1 pt] What is Alice's expected utility if David makes a public announcement of E before the game starts?

Answer: ☐

We define the *value of public information* $V_{\text{A}}^{\text{pub}}(X)$ of a random variable X to a player A as the difference in player A's expected utility after the outcome of X becomes a public information, such that everyone has access to the outcome of X and is aware that all other players also have access to X.

(iv) [2 pts] In general, the value of public information $V_{\text{A}}^{\text{pub}}(X)$ of a variable X to a player A

- ☐ always satisfies $V_{\text{A}}^{\text{pub}}(X) > 0$ in all cases.
- ☐ always satisfies $V_{\text{A}}^{\text{pub}}(X) \geq 0$ in all cases.
- ☐ always satisfies $V_{\text{A}}^{\text{pub}}(X) = 0$ in all cases.
- ☐ can possibly satisfy $V_{\text{A}}^{\text{pub}}(X) < 0$ in certain cases.

(v) [1 pt] What is $V_{\text{Alice}}^{\text{pub}}(E)$, the value of public information of E to Alice in the specific game tree above?

Answer: ☐

(vi) [2 pts] Let $a = V_{\text{Alice}}^{\text{pub}}(E)$ be the value of public information of E to Alice. Suppose David will publicly announce the outcome of E if anyone (either Alice or Bob) pays him b dollars ($b > 0$), and will make no announcement otherwise. Which of the following statements are True?

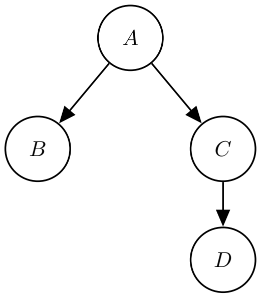
- ☐ The value of public information of E to Bob is $V_{\text{Bob}}^{\text{pub}}(E) = -a$.
- ☐ If $b < a$, then Alice should pay David b dollars.
- ☐ If $b > a$, then Bob should pay David b dollars.
- ☐ If $b < -a$, then Bob should pay David b dollars.
- ☐ If $b > -a$, then Alice should pay David b dollars.
- ☐ There exists some value $b > 0$ such that both Alice and Bob should pay David b dollars.
- ☐ There exists some value $b > 0$ such that neither Alice nor Bob should pay David b dollars.

Q6. [12 pts] Bayes Net Modeling

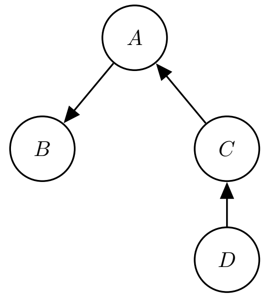
(a) Modeling Joint Distributions For each of the Bayes Net (BN) models of the true data distribution, indicate if the new Bayes Net model is guaranteed to be able to represent the true **joint** distribution. If it is not able to, draw the **minimal** number of edges such that the resulting Bayes Net can capture the **joint** distribution, or indicate if it is not possible.

(i) [2 pts]

BN Model of True Data Distribution



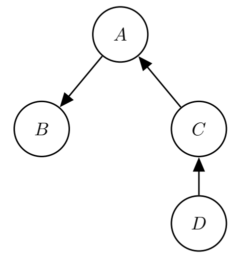
New Bayes Net Model



Can new BN represent joint distribution of True Data Distribution?

○ Yes ○ No

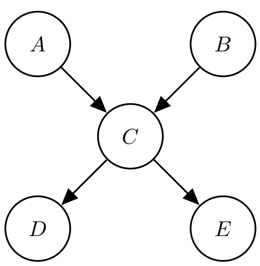
If no, draw arrows needed



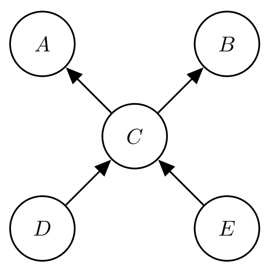
○ Not Possible

(ii) [2 pts]

BN Model of True Data Distribution



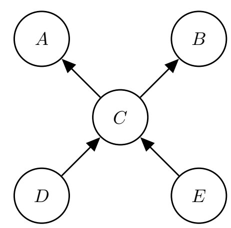
New Bayes Net Model



Can new BN represent joint distribution of True Data Distribution?

○ Yes ○ No

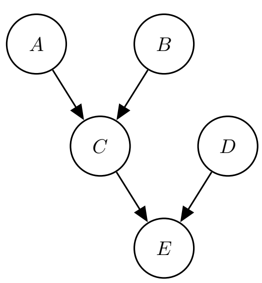
If no, draw arrows needed



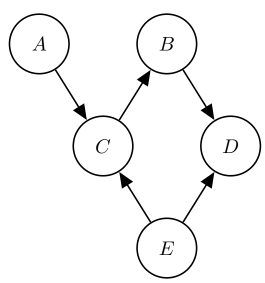
○ Not Possible

(iii) [2 pts]

BN Model of True Data Distribution



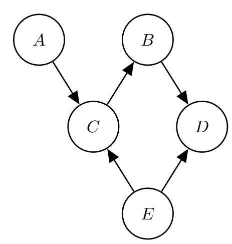
New Bayes Net Model



Can new BN represent joint distribution of True Data Distribution?

○ Yes ○ No

If no, draw arrows needed



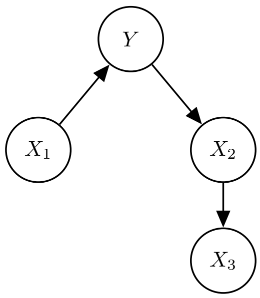
○ Not Possible

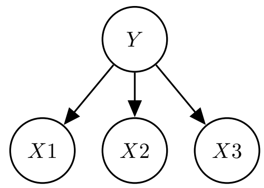
(b) Bayes Nets and Classification Recall from class that we can use Bayes Nets for classification by using the conditional distribution of $P(Y|X_1, X_2, \ldots, X_n)$, where Y is the class and each of the X_i are the observed features.

Assume all we know about the true data distribution is that it can be represented with the "True Distribution Model" Bayes Net structure. Indicate if the new Bayes Net models are guaranteed to be able to represent the true **conditional** distribution, $P(Y|X_1, X_2, \ldots, X_n)$. Mark "Yes" if it can be represented and "No" otherwise.

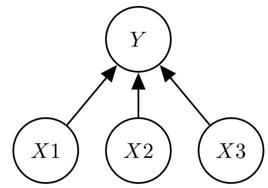
(i) [2 pts]

True Distribution Model





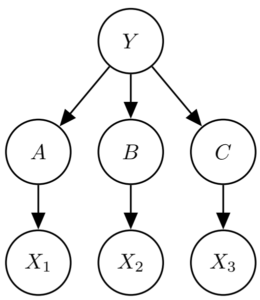
○ Yes ○ No

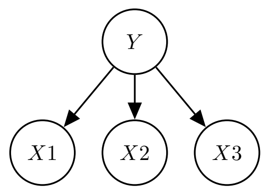


○ Yes ○ No

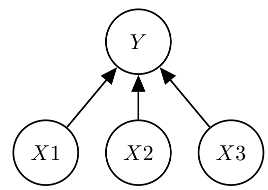
(ii) [2 pts]

True Distribution Model





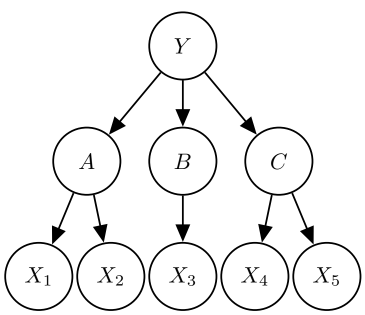
○ Yes ○ No

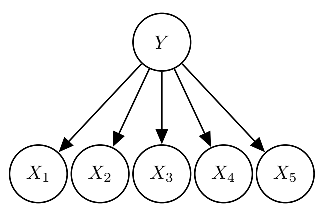


○ Yes ○ No

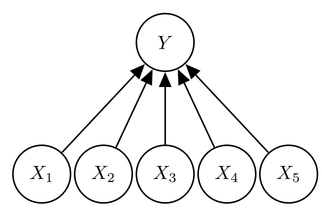
(iii) [2 pts]

True Distribution Model





○ Yes ○ No



○ Yes ○ No

Q7. [14 pts] Help the Farmer!

Chris is a farmer. He has a hen in his barn, and it will lay at most one egg per day. Chris collects data and discovers conditions that influence his hen to lay eggs on a certain day, which he describes below.

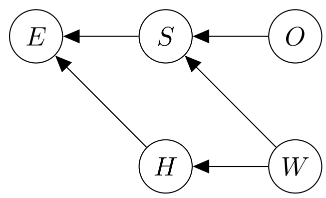
O	P(O)
$+o$	0.1
$-o$	0.9

W	P(W)
$+w$	0.7
$-w$	0.3

H	W	P(H|W)
$+h$	$+w$	0.9
$+h$	$-w$	0.5
$-h$	$+w$	0.1
$-h$	$-w$	0.5

S	W	O	P(S|W,O)
$+s$	$+w$	$+o$	0.6
$+s$	$+w$	$-o$	0.1
$+s$	$-w$	$+o$	0.8
$+s$	$-w$	$-o$	0.1
$-s$	$+w$	$+o$	0.4
$-s$	$+w$	$-o$	0.9
$-s$	$-w$	$+o$	0.2
$-s$	$-w$	$-o$	0.9

E	H	S	P(E|H,S)
$+e$	$+h$	$+s$	0.4
$+e$	$+h$	$-s$	0.8
$+e$	$-h$	$+s$	0.2
$+e$	$-h$	$-s$	0.6
$-e$	$+h$	$+s$	0.6
$-e$	$+h$	$-s$	0.2
$-e$	$-h$	$+s$	0.8
$-e$	$-h$	$-s$	0.4



For a single hen, variables O, W, S, H, and E denote the event of an outbreak (O), sunny weather (W), sickness (S), happiness (H), and egg being laid (E). If an event does occur, we denote it with a +, otherwise −, e.g., $+o$ denotes an outbreak having occurred and $-o$ denotes no outbreak occurred.

(a) Suppose Chris wants to estimate the probability that the hen lays an egg given it's good weather and the hen is not sick, e.g., $P(+e|+w,-s)$. Suppose we receive the samples:

$$(-o,+w,-s,-h,+e), \quad (-o,+w,-s,+h,-e), \quad (+o,+w,-s,-h,-e)$$

(i) [2 pts] Similar to the likelihood weighing method, Chris weighs each of his samples after fixing evidence. However, he weighs each sample only with $P(-s|+w,O)$, i.e. he omits weighing by $P(+w)$. Chris' method results in the correct answer for the query $P(+e|+w,-s)$.

○ True ○ False

(ii) [2 pts] Using likelihood weighting with the samples listed above, what is the probability the hen lays an egg given it's good weather and the hen is not sick, or $P(+e|+w,-s)$? Round your answer to the second decimal place or express it as a fraction simplified to the lowest terms.

(b) Chris uses Gibbs sampling to sample tuples of (O, W, S, H, E).

(i) [2 pts] As a step in our Gibbs sampling, suppose we currently have the assignment of $(-o,-w,+s,+h,+e)$. Then suppose we resample the "sickness" variable, i.e., S. What is the probability that the next assignment is the same, i.e., $(-o,-w,+s,+h,+e)$? Round your answer to the second decimal point, or express it as a fraction simplified to the lowest terms.

(ii) [2 pts] What will be the most observed tuple of (O, W, S, H, E) if we keep running Gibbs sampling for a long time? Select one value from each column below to denote the assignment.

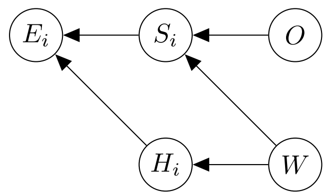
☐ $+o$	☐ $+w$	☐ $+s$	☐ $+h$	☐ $+e$
☐ $-o$	☐ $-w$	☐ $-s$	☐ $-h$	☐ $-e$

(c) [3 pts] Suppose we adopt a sampling procedure where at each evidence node with probability 0.5 we fix to the evidence, otherwise we sample the outcome and reject if it doesn't match. Upon seeing an evidence node, write an expression for the value we will **multiply into the weight** of the sample to make this procedure consistent. The weight is initialized at the start as `weight = 1`.

Your answer may use the variables s and p, where s is 1 if the coin flip told us to sample the evidence node, and p is the probability of the evidence given its parents.

`weight *=` [ ]

Now, suppose there are 1000 hens, each independently modeled by the Bayes Net model below. Denote the random variables for sickness, happiness, and laying an egg as S_i, H_i, E_i for hen i. The conditional probability tables are the same as above for each hen.

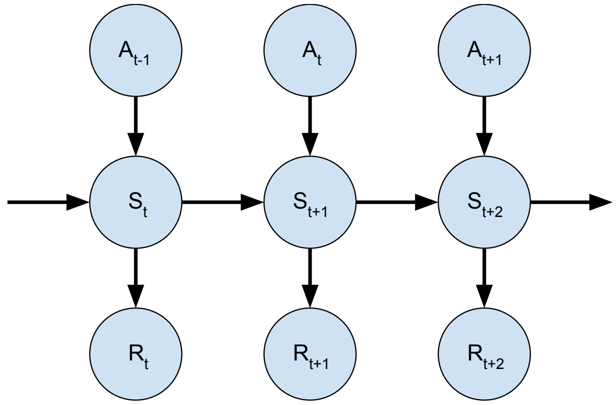


(d) [3 pts] One day, Chris observed that all the hens lay eggs and the weather is bad. What's the probability of an outbreak happening? Round your answer to the second decimal point. *Hint:* $P(O = +o, W = -w, E_i = +e_i) = 0.0114$ and $P(O = -o, W = -w, E_i = +e_i) = 0.1782$ for all i.

[ ]

Q8. [15 pts] Bayes Nets and RL

In this question, you will see that variable elimination can solve reinforcement learning problems. Consider the following Bayes net, where $S_t \in \mathcal{S}$, $A_t \in \mathcal{A}$, and $R_t \in \{0, 1\}$:



(a) [2 pts] From the list below, select all the (conditional) independencies that are guaranteed to be true in the Bayes net above:

- ☐ $S_{t+1} \perp\!\!\!\perp S_{t-1} | S_t, A_t$
- ☐ $R_{t+1} \perp\!\!\!\perp R_{t-1} | S_t, A_t$
- ☐ $R_{t+1} \perp\!\!\!\perp R_t$
- ☐ $A_{t+1} \perp\!\!\!\perp R_t | S_t$
- ☐ $A_{t+1} \perp\!\!\!\perp R_t | S_t, A_t$
- ☐ None of the above

Let $+r_{t:T}$ denote the event $R_t = R_{t+1} = \cdots = R_T = 1$, and assume that $P(a_t) = 1/|\mathcal{A}|$. Define the following functions:

$$\beta_t(s_t, a_t) = P(+r_{t:T} | s_t, a_t), \qquad \beta_t(s_t) = P(+r_{t:T} | s_t) = \frac{1}{|\mathcal{A}|} \sum_{a_t} \beta_t(s_t, a_t)$$

Perform variable elimination to compute $P(A_t | S_t, +r_{t:T})$.

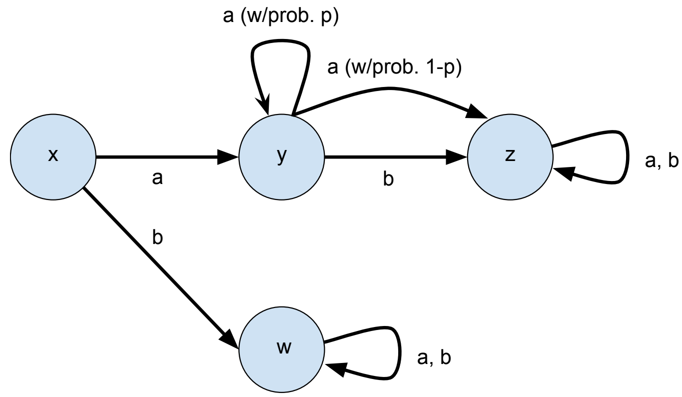
(b) [2 pts] Which of the following recursions does $\beta_t(s_t, a_t)$ satisfy?

- ○ $\beta_t(s_t, a_t) = P(+r_t | s_t) \sum_{s_{t+1}} \beta_{t+1}(s_{t+1})$
- ○ $\beta_t(s_t, a_t) = P(+r_t | s_t) \sum_{s_{t+1}} \beta_{t+1}(s_{t+1}, a_t)$
- ○ $\beta_t(s_t, a_t) = P(+r_{t+1} | s_t) \sum_{s_{t+1}} \beta_{t+1}(s_{t+1}, a_t)$
- ○ $\beta_t(s_t, a_t) = P(+r_t | s_t) \sum_{s_{t+1}} P(s_{t+1} | s_t, a_t) \beta_{t+1}(s_{t+1})$
- ○ $\beta_t(s_t, a_t) = \sum_{a_{t+1}} P(+r_{t+1} | s_{t+1}) \frac{1}{|\mathcal{A}|} \sum_{s_{t+1}} P(s_{t+1} | s_t, a_t) \beta_{t+1}(s_{t+1})$
- ○ None of the above

(c) [2 pts] Write $P(a_t | s_t, +r_{1:T})$ in terms of $\beta_t(s_t, a_t)$, $\beta_t(a_t)$, and relevant probabilities from the Bayes net.

- ○ $P(a_t | s_t, +r_{1:T}) = \frac{\beta_t(s_t, a_t)}{\beta_t(s_t)} P(+r_t | s_t)$
- ○ $P(a_t | s_t, +r_{1:T}) = \frac{\beta_t(s_t, a_t)}{\sum_{a'_t} \beta_t(s_t, a'_t)}$
- ○ $P(a_t | s_t, +r_{1:T}) = \frac{\beta_t(s_t, a_t)}{\beta_t(s_t)}$
- ○ $P(a_t | s_t, +r_{1:T}) = \frac{\beta_t(s_t, a_t)}{P(+r_t | s_t) |\mathcal{A}|}$
- ○ $P(a_t | s_t, +r_{1:T}) = \frac{P(+r_t | s_t)}{\beta_t(s_t) \beta_t(s_t, a_t) |\mathcal{A}|}$
- ○ None of the above

So far, we have only discussed variable elimination in a certain Bayes net. Now, we will associate the Bayes net with an MDP with two parameters $p, q \in (0,1)$. The states are $\mathcal{S} = \{x, y, z, w\}$, and the actions are $\mathcal{A} = \{a, b\}$. The transitions are described in this diagram:



All transitions are deterministic except when taking action a starting in state y – this transition is determined by $P(S_{t+1} = y \,|\, S_t = y, A_t = a) = p$. The rewards are stochastic and depend only on state, taking on values in $\{0, 1\}$ with probabilities

$$P(+r_t|S_t = x) = 1, \quad P(+r_t|S_t = y) = 1, \quad P(+r_t|S_t = z) = 0, \quad P(+r_t|S_t = w) = q$$

Throughout the following questions, assume that $p, q \in (0,1)$.

(d) [3 pts] Consider running the uniform policy $\pi_{\text{uniform}}(A_t = a) = \frac{1}{2}$ for $T+2$ timesteps starting in state x. What is $P(A_1 = a|S_1 = x, +r_{1:T+2})$?

- ○ $p^{T+1}/(p^{T+1} + (2q)^{T+1})$
- ○ $(2q)^{T+1}/(p^{T+1} + (2q)^{T+1})$
- ○ $p^T/(p^T + 2^T q^{T+1})$
- ○ $2^T q^{T+1}/(p^T + 2^T q^{T+1})$
- ○ $p^T/(p^T + (2q)^T)$
- ○ $(2q)^T/(p^T + (2q)^T)$
- ○ $p^T/(p^T + q^T)$
- ○ $q^T/(p^T + q^T)$
- ○ $p^{T+1}/(p^{T+1} + q^{T+1})$
- ○ $q^{T+1}/(p^{T+1} + q^{T+1})$
- ○ None of the above

(e) [2 pts] Suppose $p > 2q^{(T+1)/T}$. When running π_{uniform}, what is $\arg\max_z P(A_1 = z|S_1 = x, +r_{1:T+2})$?

- ☐ $\arg\max_z P(A_1 = z|S_1 = x, +r_{1:T+2}) = a$
- ☐ $\arg\max_z P(A_1 = z|S_1 = x, +r_{1:T+2}) = b$
- ☐ Cannot be determined

(f) [2 pts] Suppose $q > 2^{-T/(T+1)}$. When running π_{uniform}, what is $\arg\max_z P(A_1 = z|S_1 = x, +r_{1:T+2})$?

- ☐ $\arg\max_z P(A_1 = z|S_1 = x, +r_{1:T+2}) = a$
- ☐ $\arg\max_z P(A_1 = z|S_1 = x, +r_{1:T+2}) = b$
- ☐ Cannot be determined

(g) [2 pts] Consider running the optimal policy π^* for $T+1$ timesteps starting in state x. When is π^* always guaranteed to choose b as its first action?

- ○ $T > \frac{1}{(1-p)q}$
- ○ $T > \frac{1}{(1-q)p}$
- ○ $T > \frac{1}{(1-p)(1-q)}$
- ○ $T > \frac{1}{pq}$
- ○ $T < \frac{1}{(1-p)q}$
- ○ $T < \frac{1}{(1-q)p}$
- ○ $T < \frac{1}{(1-p)(1-q)}$
- ○ $T < \frac{1}{pq}$
- ○ None of the above

Q9. [8 pts] Decision Trees

You are given a dataset for training a decision tree. The goal is to predict the label (+ or -) given the features A, B, and C.

A	B	C	label
0	0	0	+
0	0	1	+
0	1	0	+
0	1	1	-
1	0	0	-
1	0	1	-
1	1	0	+
1	1	1	-

First, consider building a decision tree by greedily splitting according to information gain.

(a) [2 pts] Which features could be at the root of the resulting tree? Select all possible answers.

- ☐ A
- ☐ B
- ☐ C

(b) [2 pts] How many edges are there in the longest path of the resulting tree? Select all possible answers.

- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4
- ☐ None of the above

Now, consider building a decision tree with the smallest possible height.

(c) [2 pts] Which features could be at the root of the resulting tree? Select all possible answers.

- ☐ A
- ☐ B
- ☐ C

(d) [2 pts] How many edges are there in the longest path of the resulting tree? Select all possible answers.

- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4
- ☐ None of the above

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