

MIDTERM 1
MATH 54

Your name

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Student ID number

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Section number and GSI name

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1	2	3	4	5	total

Instructions: Be sure to write your name, your id number and your GSI name. Write your solutions in the space below the problem. If you run out of space use the back of the page. **Show your work and justify your answer. Indicate final answers by circling them.** Good luck!

1. Check if the following matrices are invertible and find the inverse when possible

(a) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix},$

(b) $\begin{bmatrix} -1 & -2 & -3 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}.$

2. Find bases in $\text{Nul } A$ and $\text{Col } A$ for the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 & 5 & 7 \\ -1 & 2 & 2 & 0 & 1 \end{bmatrix}.$$

3. Consider the linear transformation T from \mathbb{R}^2 to \mathbb{R}^3 defined by the formula

$$T \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} 2x_1 + x_2 \\ 0 \\ 0 \end{bmatrix}.$$

(a) Write the matrix of T .

(b) Describe the kernel and the range of T .

4. Let S be the circle in \mathbb{R}^2 with center at $(1, 1)$ and radius 3 and $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation with matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find the area of $T(S)$.

5. Let $\{v_1, v_2, v_3\}$ be a linearly independent set of vectors in a vector space V . Show that the set $\{v_1 + v_2, v_2 + v_3, v_1 + v_3\}$ is also linearly independent.