Mathematics 54.1
Midterm 1, 4 October 2019
50 minutes, 50 points

NAME: _____   ID: ____ ___ _____ ___

GSI: _____

INSTRUCTIONS:

Justify your answers, except when told otherwise.

All the work for a question should be on the respective sheet.

This is a CLOSED BOOK examination, NO NOTES and NO CALCULATORS are allowed.

NO CELL PHONE or EARPHONE use is permitted.

Please turn in your finished examination to your GSI before leaving the room.
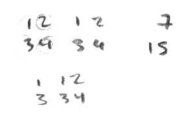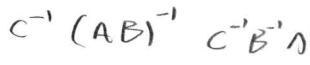
Question 1. (15 points)   ●○○○○

Mark each statement as T for true or F for False. Mark "T" only if the statement *must* be true without further assumptions. Justification is not needed, but **incorrect answers carry a 1-point penalty**, so random guessing will not help. You may leave any answer blank for 0 points. You will also not get a negative total score on any group of five questions.

(F)   The dimensions of the nullspace and the left nullspace of any $m \times n$ matrix A agree.

(T)   If a set S of vectors spans $\mathbb{R}^n$, then S contains at least n vectors.

(T)   The map $T : \mathbb{R}^3 \to \mathbb{R}^3$ which reflects points about the plane $x + y + z = 0$ is a linear transformation.

(F)   The first column of AB is the first column of the matrix A multiplied on the right by B.

( )   If one row in the echelon form of the augmented matrix of a system is $[0\,0\,0\,1\,1]$, then the system is inconsistent.

(T)   The solution set of a consistent $m \times n$ linear system $A\mathbf{x} = \mathbf{b}$ is a linear subspace of $\mathbb{R}^n$.

(F)   The rows of any 4×5 matrix are linearly dependent.

(F)   The determinant of a square matrix is the product of its pivots.

(F)   For any two $n \times n$ matrices A, B, we have $(AB)^T = A^T B^T$.

(T)   If the linear system $A\mathbf{x} = \mathbf{b}$ is inconsistent, then the coefficient matrix A does not have a pivot position in every row.

(F)   The kernel of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the left nullspace of A.

(T)   The determinant of a change-of-coordinates matrix cannot be zero.

(T)   For invertible $n \times n$ matrices A, B, C, we have $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

(T)   If a set of vectors in $\mathbb{R}^n$ is linearly independent, then it contains at most n vectors.

(T)   If the coefficient matrix A has a pivot position in every column, then the system $A\mathbf{x} = \mathbf{b}$ has at most one solution.

Question 2. (4+8+2 points)

All questions pertain to the matrix A below. In all cases, make sure that your methods are clear.

(a) Is the system $A\mathbf{x} = [1, 1, 4]^T$ consistent? If so, find a particular solution.

(b) Find bases of the nullspace, column space, row space and left nullspace.

(c) For what values of a, b does $[4, 7, a, b]$ lie in the row space?

Check your work! Small mistakes can cost you many points in this question.

$$A = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & 5 & 3 & 2 \\ 5 & 14 & 9 & 5 \end{bmatrix}$$

Question 3. (2+3+3 points)

The coordinate vector of $\mathbf{v} \in \mathbb{R}^2$ with respect to the basis $\mathcal{B} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ of $\mathbb{R}^2$ is: $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} a \\ 1 \end{bmatrix}$.

(a) What is the vector $\mathbf{v}$ in the standard basis?

(b) Find the coordinate vector $[\mathbf{v}]_{\mathcal{B}'}$ of $\mathbf{v}$ with respect to the basis $\mathcal{B}' = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. (Explain.)

(c) Represent graphically the bases $\mathcal{B}, \mathcal{B}'$ as they appear in standard coordinates, as well as the set of all vectors $\mathbf{v}$, as a ranges over all real values.

Question 4. (5+3+5 points)

(a) By using determinants, find the values of $k \in \mathbb{R}$ for which the matrix A is invertible.

(b) Find $\det(A^{-1})$ when A is invertible. (*Hint:* you don't need to compute A^{-1}.)

(c) Find the inverse matrix for $k = 1$ by your favorite method. Check your answer.

$$A = \begin{bmatrix} k & 1 & 2 \\ k^2 + 1 & 3 & 4 \\ 2k & 2 & 3 \end{bmatrix}$$

THIS PAGE IS FOR ROUGH WORK (not graded)