Problem 1

We start by drawing a force diagram on the person. Since normal forces are always vertical, we have to assume that the normal forces on the left and right feet N_2 and N_1 are torques about the center of mass. Additionally, there are two forces of friction f_2 and f_1 on the feet, they also supply torques.

Since there is no vertical acceleration, the vertical equation of motion is

$$N_1 + N_2 - Mg = 0$$

Since there is an angular acceleration to the center of the circle of $\frac{v^2}{R}$, the radial equation of motion is

$$f_1 + f_2 = \frac{Mv^2}{R}$$

Finally, we also need an equation of motion for the torque. Let us take the pivot as the center of mass. We use

$$\tau = \mathbf{r} \times \mathbf{F} \qquad \Longrightarrow \qquad |\tau| = rF\sin\varphi$$

Where φ is the angle between the force and the lever-arm. the magnitude of the lever arm, r, is the same for all forces N_1, N_2, f_1, and f_2. Let θ be the acute angle between the vertical and the leg, so the angle between the normal forces and the lever arm is θ and the angle between the friction forces and the lever arm is $90° + \theta$. So, the four torques in the problem are

$$|\tau(f_1)| = rf_1\sin(90+\theta) = rf_1\cos\theta, \quad |\tau(f_2)| = rf_2\cos\theta, \quad |\tau(N_1)| = rN_1\sin\theta, \quad \text{and} \quad |\tau(N_2)| = rN_2\sin\theta$$

We will determine the directions in a bit. To find $\cos\theta$ and $\sin\theta$, we draw the right triangle with angle θ, the hypotenuse is the lever arm r, and the legs are of length $\frac{d}{2}$ and L. It is clear that

$$\cos\theta = \frac{L}{r} \quad \text{and} \quad \sin\theta = \frac{d}{2r}$$

. Now we can write an equation of motion for the torques. All the torques point in the same direction except for the torque from N_1, which we will make negative. There is no rotational acceleration.

$$-\frac{N_1 d}{2} + \frac{N_2 d}{2} + f_1 L + f_2 L = 0$$

We can combine this with the first two equations to solve for N_1 and N_2, the weights on the outside and inside feet.

$$0 = -\frac{N_1 d}{2} + \frac{N_2 d}{2} + L(f_1 + f_2) = -\frac{N_1 d}{2} + \frac{d}{2}(Mg - N_1) + L\left(\frac{Mv^2}{R}\right) = -N_1 d + \frac{Mgd}{2} + \frac{LMv^2}{R} = 0$$

$$\Longrightarrow \quad N_1 = \frac{Mg}{2} + \frac{Mv^2 L}{Rd} \quad \text{and} \quad N_2 = Mg - N_1 = \frac{Mg}{2} - \frac{Mv^2 L}{Rd}$$

Problem 2

(a) This is a simple application of conservation of energy.

$$\frac{1}{2}kb^2 = \frac{1}{2}mv^2 + \frac{1}{2}k(0)^2 \implies v = \sqrt{\frac{k}{m}}b \tag{1}$$

(b) This is subtle because there is a collision hidden in the problem. While is true that wheel is rolling without slipping (which implies that mechanical energy is conserved), this is only true once the wheel has started moving along the track. More precisely, when the wheel moves from the frictionless surface to the geared track, it suffers a collision on the first tooth (which is protruding). Energy is in general not conserved in a collision, and as we will see below is not conserved in this collision.

If we consider the system to be the wheel, then it is clear that momentum is not conserved. The first tooth of the track exerts an external force on the wheel that changes its velocity. Further, along the track the spring provides an external force.

Angular momentum around the point of contact is again subtle. We know that the around the point of contact, the torque due to static friction must be zero. We still have the force of the spring, which provides a torque. However, at the point of collision, the spring force is zero ($x = L$), which implies there is no external torque. At this point of collision, we can use conservation of angular momentum to calculate the necessary velocity.

(c) As noted above, we use the conservation of angular momentum.

$$L_i = mvR \tag{2}$$

$$L_f = mv'R + I_{cm}\omega = mv'R + (mR^2)\frac{v'}{R}, \tag{3}$$

where we used the rolling without slipping condition in the last equality. Equating L_i and L_f, and solving for v'

$$\boxed{v' = \frac{v}{2}.} \tag{4}$$

Note, energy is not conserved.

(d) After the collision, energy is conserved. We can use this to calculate how close it gets to the wall. First, we calculate the amount of energy that makes it through the collision:

$$E = \frac{1}{2}mv'^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}m\frac{v^2}{4} + \frac{1}{2}mR^2\frac{v^2}{R^2}\frac{1}{4} = \frac{1}{4}kb^2. \tag{5}$$

Then,

$$\frac{1}{2}k(x-l)^2 = \frac{1}{4}kb^2 \implies (x-l) = \pm\frac{b}{\sqrt{2}} \tag{6}$$

The only physically acceptable solution is $\boxed{x = l - b/\sqrt{2}}$.

(e) Note that there is no collision when the wheel comes off the track. However, also note that the velocity (and thus angular velocity because of no-slip) are reversed at this point. The imporant nuance here is that once the wheel moves on the frictionless surface, it *will* slip. Furthermore, since there is no friction, there is no force that changes the angular momentum of the wheel around the CM. Thus, the farthest distance is when $v = 0$ (but throughout $\omega = \text{const} \neq 0$).

$$\frac{1}{4}kb^2 = \frac{1}{2}m(0)^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(x-l)^2 \tag{7}$$

$$\frac{1}{4}kb^2 = \frac{1}{8}kb^2 + \frac{1}{2}k(x-l)^2. \tag{8}$$

Therefore,

$$\boxed{x = l + b/2.} \tag{9}$$

(f) Again, we calculate the angular momentum around the (stationary) point of collision. However, the angular momentum consists of two terms: one due to the translational motion of the CM, and the other due to the rotation about the CM. Then, calling the clockwise direction positive, note that

$$L_i = m(-v')R + m\omega^2 R = -mv'R + mv'R = 0. \tag{10}$$

Therefore,

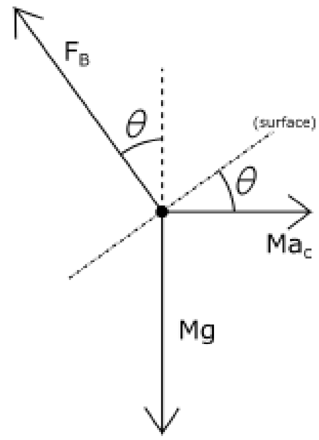
$$L_f = mv''R = L_i = 0. \tag{11}$$

In other words, the wheel will instantaneously stop.

Problem 3

We start by drawing a force diagram on a small volume element of water (of mass M) on the surface of the water. I will draw it in a rotating frame which has acceleration $a_c = \omega^2 R$ to the left, towards the center of the circle. This makes for a fictitious force of Ma_c pointing to the right.

Since the small volume element of water is in equilibrium, there must be a buoyancy force up and to the left at an angle θ with respect to the vertical so the water does not accelerate. Note: this buoyancy force will be proportional to g_{eff} since we are in an accelerating reference frame, not g. However, we are only interested in the direction of this buoyancy force, or θ, since θ is related to the shape of the surface.



Notice that the slope of the surface of the water is equal to $\tan\theta$ (rise over run). If we find the slope of the surface of the water as a function of R,

$$\frac{dh}{dR} = f(R)$$

then we can integrate to find the height of the surface h as a function of R:

$$\frac{dh}{dR} = f(R) \quad \Longrightarrow \quad dh = f(R)\,dR \quad \Longrightarrow \quad h(R) = \int_0^R f(R)\,dR$$

However, we claimed that the slope was equal to $\tan\theta$:

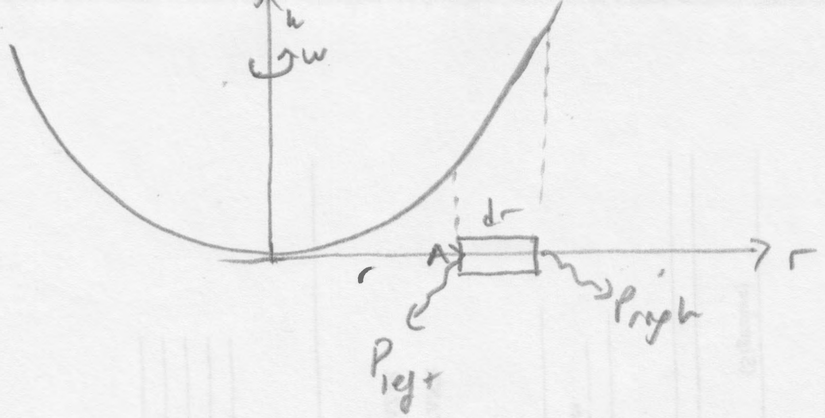
$$\frac{dh}{dR} = \tan\theta$$

So, the goal now is clearly to find $\tan\theta$ as a function of R. To do so we go back to our force diagram and assume that acceleration is 0.

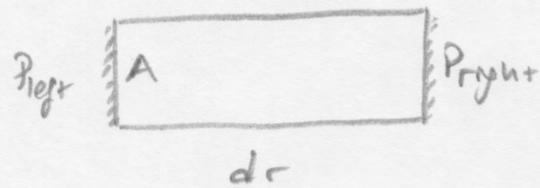
$$F_B \sin\theta = Ma_c \quad \text{and} \quad F_b\cos\theta = Mg \quad \Longrightarrow \quad \tan\theta = \frac{F_b \sin\theta}{F_B\cos\theta} = \frac{Ma_c}{Mg} = \frac{a_c}{g} = \frac{\omega^2}{g}R$$

$$\Longrightarrow \quad \frac{dh}{dR} = \frac{\omega^2}{g}R \quad \Longrightarrow \quad h(R) = \frac{\omega^2}{g}\int dR\,R = \frac{\omega^2}{2g}R^2$$

$$\Longrightarrow \quad A = \frac{\omega^2}{2g}$$



Zoom in:



$$P_{left} = P_0 + \rho g h(r) \qquad P_{right} = P_0 + \rho g h(r+dr)$$

$$\Delta P = P_{right} - P_{left} = \rho g [h(r+dr) - h(r)]$$

because water is doing uniform circular motion

$$\Sigma F_r = -m w^2 r$$

$$-\Delta P \cdot A = -m w^2 r \qquad m = \rho \cdot dV = \rho A dr$$

$$\rho g [h(r+dr) - h(r)] \cdot A = \rho A dr w^2 r$$

$$\frac{h(r+dr) - h(r)}{dr} = \frac{w^2 r}{g}$$

$$\frac{dh}{dr} = \frac{w^2 r}{g}$$

$$\int_0^h dh = \int_0^r \frac{w^2 r}{g} dr$$

$$h = \frac{w^2}{2g} r^2 \qquad A = \frac{w^2}{2g}$$

4. Bullet Train

The bullet train "pulls" air with it
the air around the train has the same velocity
as the train.
The pressure around the train is lower than
the pressure further away.

The pressure difference will ~~cause~~ result in
a force towards the train

Yildiz Final Problem 5

December 2019

I'll use energy to solve this. Letting the gravitational potential energy be zero when the ball is at the bottom of the dish, we obtain that, when the marble is displaced by an angle *theta* relative to the vertical,

$$U = mg(R - b)(1 - \text{Cos}\theta) \tag{1}$$

we can expand the cosine:

$$\text{Cos}\theta = 1 + \frac{\theta^2}{2} + \text{higher order terms} \tag{2}$$

and we neglect the higher order terms to obtain

$$U = \frac{1}{2}mgR\theta^2 \tag{3}$$

Cool. The marble is rolling so it has both types of kinetic energy. The linear kinetic energy is

$$E_{tr} = \frac{1}{2}mv^2 = \frac{1}{2}mR^2\theta'^2 \tag{4}$$

and rotational energy is

$$E_{rot} = \frac{1}{2}I\phi'^2 \tag{5}$$

we want to relate ϕ to θ and subsequentially θ'. Note that rolling without slipping demands $b\phi' = R\theta'$ and

$$E_{rot} = \frac{1}{2}I\frac{R^2}{b^2}\theta'^2 \tag{6}$$

then the total energy is

$$E_{tot} = \frac{1}{2}mgR\theta^2 + \frac{1}{2}mR^2(1 + \frac{2}{5})\theta'^2 = \frac{1}{2}mgR\theta^2 + \frac{1}{2}mR^2\frac{7}{5}\theta'^2 \tag{7}$$

where we substituted in $I = \frac{2}{5}mb^2$
finally, we can arrange things to look just like a mass on a spring

$$E_{tot} = \frac{1}{2}mR(g\theta^2 + \frac{7}{5}R\theta'^2) \tag{8}$$

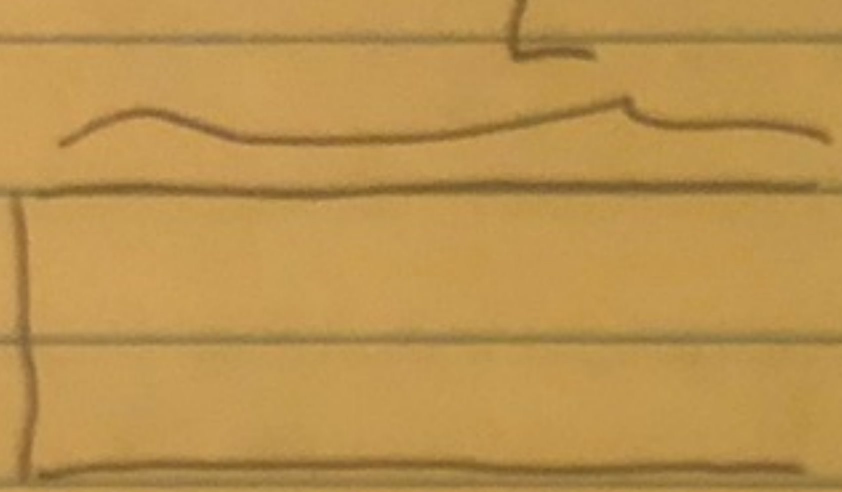
in analogy with a mass on a spring, whose total energy looks like

$$E_s = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \tag{9}$$

and has frequency $\sqrt{\frac{k}{m}}$, here we take g to be like k and $\frac{7}{5}R$ to be m to yield the answer

$$\omega = \sqrt{\frac{5g}{7R}} \tag{10}$$

6. Single open pipe



$L = \frac{n\lambda}{4}$ n - odd number

$n = 1, 3, 5, \ldots$

$L = 3cm$ $V = 330\, m/s$

$V = \lambda_n f_n$ $f_n = \frac{V}{\lambda_n}$

$f_n = n\frac{V}{4L}$ $\lambda_n = \frac{4L}{n}$

$f_n = n\frac{V}{4L} = n\frac{330\, m/s}{4(.03\, m)}$

$f_n = n(2750\ Hz)$

$f_1 = 2750\ Hz$ $f_3 = 8250\ Hz$

~~[illegible]~~

fundamental!

Yildiz Final Prob 7

December 16, 2019

Ror part a, first we note that $v_0 = 0$. Next, use trig to find θ_s. We find that

$$\tan\theta = \frac{30\text{m}}{40\text{m}} = \frac{3}{4} \tag{1}$$

we then plug into the provided equation:

$$f' = \frac{340}{340 - 25\text{Cos}(36.9)} 500 \text{ Hz} = 531 \text{ Hz} \tag{2}$$

Part B: The smallest angle we have would be $\theta = 0$, which is true when the train is infinitely far away on the left. The max angle we have is $\theta_s = \pi$, when the train is infinitely far away on the right (these are limiting cases). The first gives us the frequency

$$f' = \frac{340}{340 - 25} 500 \text{ Hz} = 539 \text{ Hz} \tag{3}$$

and at $\theta_s = \pi$ we get

$$f' = \frac{340}{340 + 25} 500 \text{ Hz} = 465 \text{ Hz} \tag{4}$$

and thus the full range is 469 to 539 Hz.

For the last part, we first find θ_o by realizing that since the velocity of the observer is perpendicular to that of the train, we have

$$\theta_o = 360 - (90 - \theta_s) = 306.87 \tag{5}$$

where we subtract by 360 since the angle is measured away from the line from the source to the observer. We can plug this in to obtain

$$f' = \frac{340 + 40\text{Cos}(306.87)}{340 - 25\text{Cos}(36.9)} 500 \text{ Hz} = 568 \text{ Hz} \tag{6}$$