University of California, Berkeley *Spring 2020*
EE 105 *Prof. A. M. Niknejad*

Midterm Exam: Open book and open notes, no Internet

Name (Last, First)

SOLUTIONS

SID

HONOR CODE Copy the following statements into the box below and sign your name: I will respect my classmates and the integrity of this exam by following this honor code. I affirm:

1. All of the work submitted here is my original work.
2. I did not reference any sources on the internet.
3. I did not collaborate with any other human being on this exam.

Notes:
The resistance of a material is related to the physical dimensions and resistivity by

$$R = \frac{\rho L}{tW} = \frac{\rho}{t}\frac{L}{W} = R_{sq}\frac{L}{W}$$

The capacitance of a parallel plate structure is given by (per unit area)

$$C' = C/A = \frac{\epsilon}{d}$$

The conductivity of a material depends on charge density (n and p), mobility $\mu_{n,p}$, and charge of carriers q_e:

$$\sigma = q_e(\mu_n n + \mu_p p)$$

where $q_e = 1.60217662 \times 10^{-19}$C for an electron. The mobility is given by

$$\mu = \frac{q\tau}{m}$$

where q is the charge of the particle, τ is the average time between collisions, and m is the effective mass. Drift current $J = \sigma E$ flows due to fields. Diffusion currents flow due to concentration gradients. For a concentration gradient of positive charges, we have

$$J_{diff} = -qD_p \frac{dp}{dx}$$

where D_p is the diffusion coefficient, which is related to the mobility by $kT/q = D/\mu$. For silicon at room temperature, $kT/q = 26\text{mV}$, and assume $n_i = 10^{10}\text{cm}^{-3}$. For n-type materials, the potential ϕ_n defined with respect to intrinsic silicon is defined by

$$n = n_i e^{q\phi_n/kT}$$

Likewise, for p-type materials:

$$p = n_i e^{-q\phi_p/kT}$$

For a pn-junction, the equilibrium depletion region widths are given by

$$x_n(V_D) = \sqrt{\frac{2\varepsilon_s(\varphi_{bi} - V_D)}{qN_d}\left(\frac{N_a}{N_a + N_d}\right)} = x_{n0}\sqrt{1 - \frac{V_D}{\varphi_{bi}}}$$

$$x_p(V_D) = \sqrt{\frac{2\varepsilon_s(\varphi_{bi} - V_D)}{qN_a}\left(\frac{N_d}{N_a + N_d}\right)} = x_{p0}\sqrt{1 - \frac{V_D}{\varphi_{bi}}}$$

In the presence of excess minority carriers in a semiconductor, the continuity equation dictates that (for holes, similar for electrons)

$$\frac{dJ_p}{dx} = q\frac{\Delta p}{\tau_p}$$

where τ_p is the minority carrier lifetime. The diffusion length (for holes) is defined by $L_p^2 = \tau_p D_p$ and similarly for electrons. Boltzmann's Law:

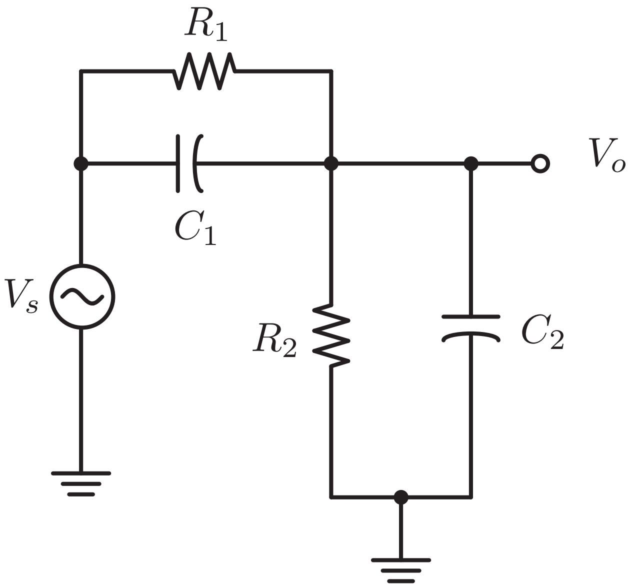
$$\frac{p_2}{p_1} = e^{-(Barrier\ Energy)/kT}$$

The diode I-V relation is given by

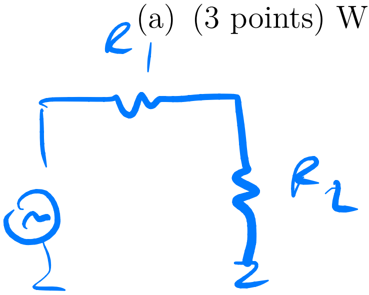
$$I_D + i_D = I_S\left(e^{\frac{q(V_d + v_d)}{kT}} - 1\right)$$

Electron charge $q = -1.6 \times 10^-19\text{C}$, silicon dielectric permittivity is given by $11.7\epsilon_0$ and silicon dioxide is $3.9\epsilon_0$. $\epsilon_0 = 8.854e - 12\text{F/m}$.

1. (25 points) Consider a circuit with input voltageV_s and output voltage V_o:



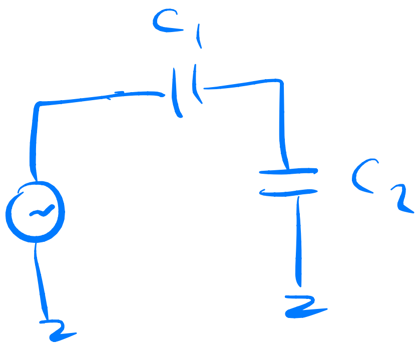
(a) (3 points) Without doing any "math" (lots of equations), what's the DC gain?



$$G_0 = \frac{R_2}{R_1 + R_2}$$

Resistive Divides

(b) (3 points) Again, without doing any "math" (lots of equations), what's the gain at very high frequencies $f \to \infty$?



$$G_\infty = \frac{C_1}{C_1 + C_2}$$

Capacitive Divider

(c) (4 points) Find the transfer function V_o/V_i. Make sure that your "inspection analysis" from the previous two parts matches your calculations.

$$Z_1 = \frac{R_1}{1+sR_1C_1} \qquad Z_2 = \frac{R_2}{1+sR_2C_2}$$

$$H(s) = \frac{Z_2}{Z_1+Z_2} = \frac{R_2(1+sR_1C_1)}{R_1(1+sR_2C_2)+R_2(1+sR_1C_1)}$$

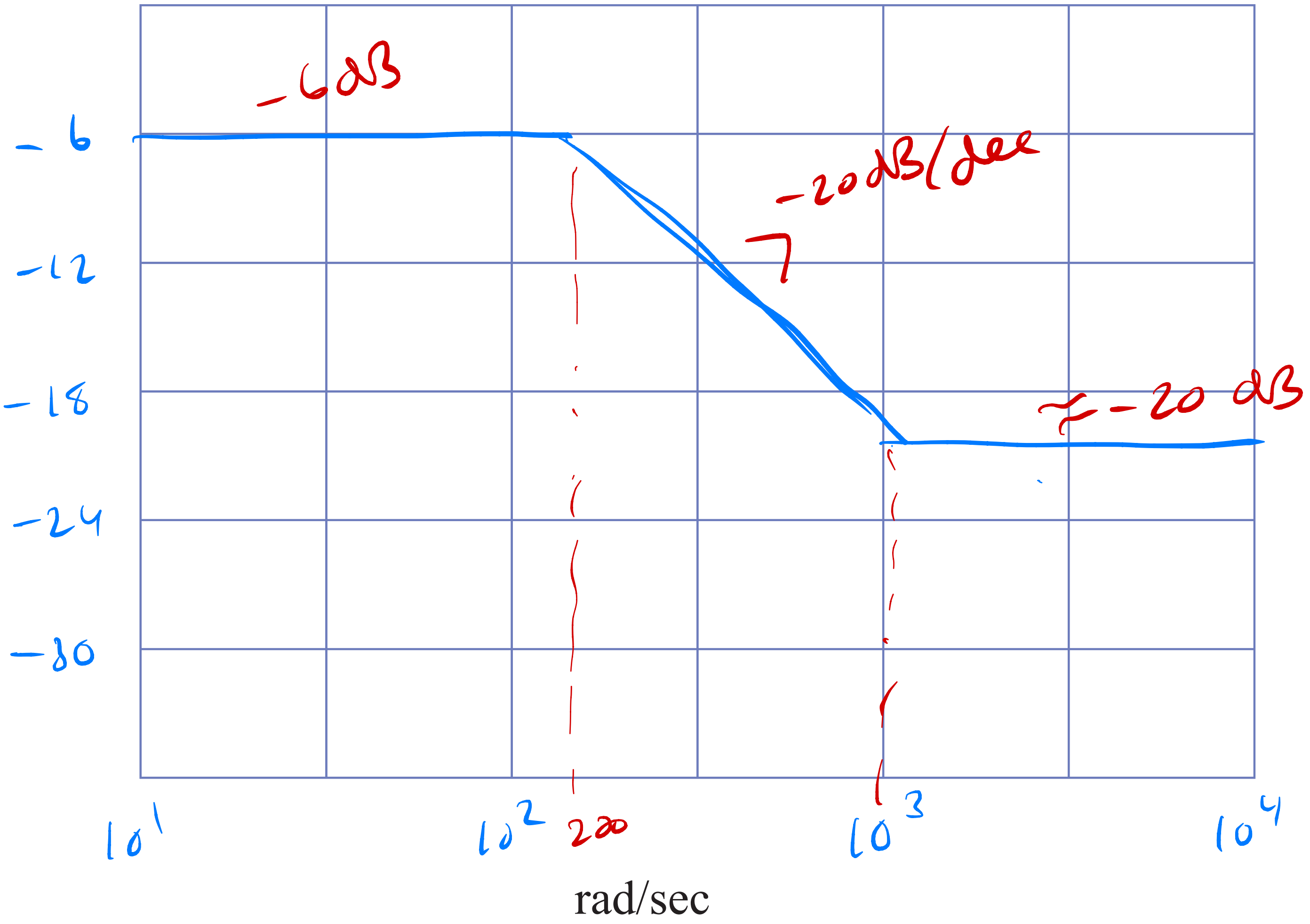
$$= \frac{R_2}{R_1+R_2} \cdot \frac{(1+sR_1C_1)}{1+s\frac{R_1R_2C_2}{R_1+R_2}+s\frac{R_1R_2C_1}{R_1+R_2}}$$

$$= \frac{G_0\,(1+sR_1C_1)}{1+s(R_1\|R_2)(C_1+C_2)}$$

(d) (2 points) Identify the the poles and zeros?

$$Z = -\frac{1}{R_1C_1} \qquad P = -\frac{1}{R_1\|R_2(C_1+C_2)}$$

(e) (4 points) Draw the magnitude Bode plot using the template. Clearly label the graph with the location of the poles and zeros, including the x-axis intercept point and any breakpoints. You may approximate the plot by using straight lines.



Assume $R_1 = R_2 = 1\text{k}\Omega$ and $C_1 = 10^{-6}$F and $C_2 = 10^{-5}$F.

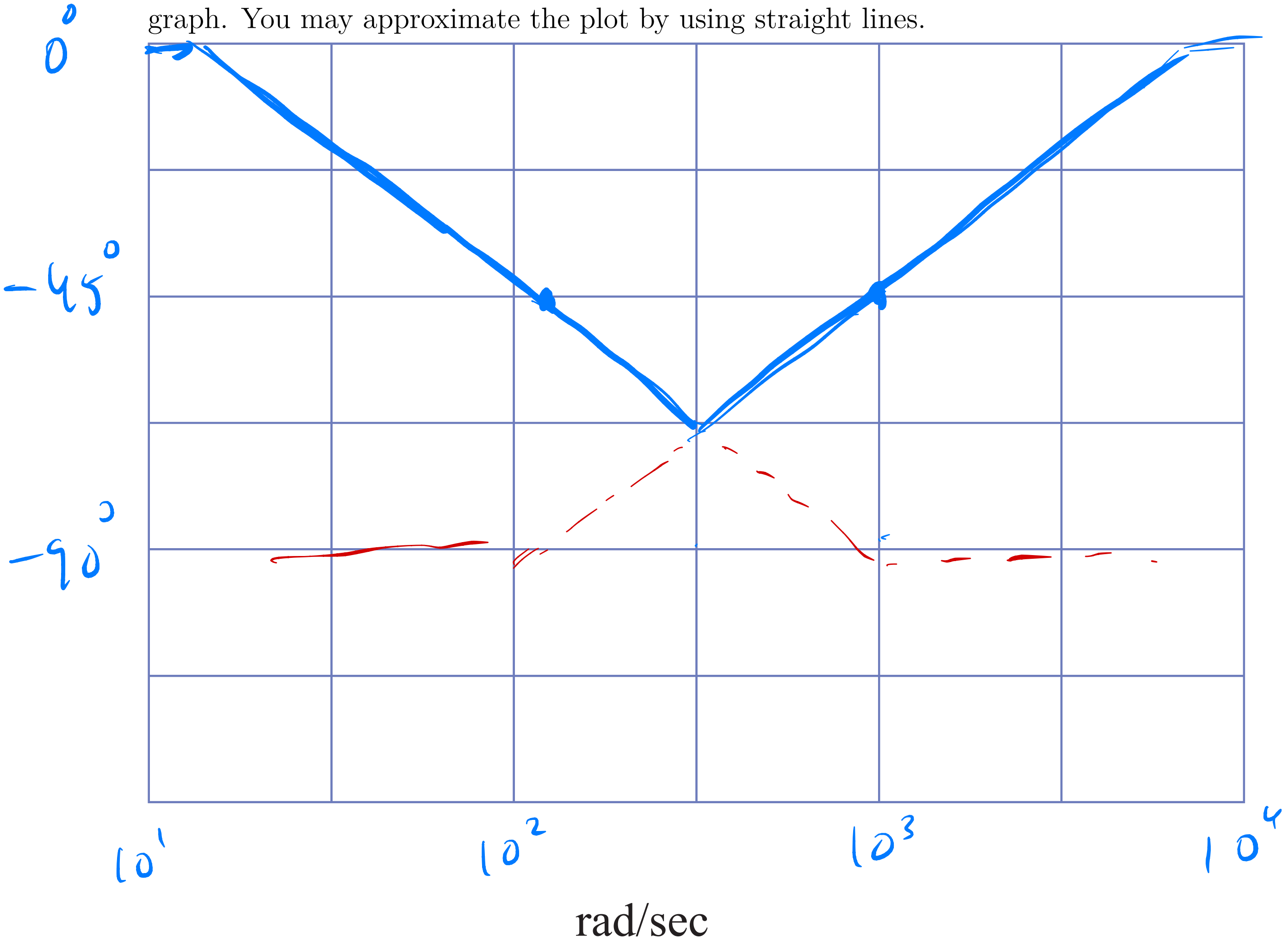
$\tau_z = R_1 C_1 = 10^{3} \cdot 10^{-6}\ s = 10^{-3}\ s$

$\tau_p = R_1 \| R_2 \cdot (C_1 + C_2) \simeq 500 \cdot 10^{-5}\ s$

$z = \frac{1}{\tau_z} = 10^{3}$ rad/sec | $G_0 = \frac{1}{2}$

$p = 200$ rad/sec | $G_\infty \simeq \frac{10^{-6}}{10^{-5}} = 10^{-1}$

(f) (4 points) Draw the phase Bode plot using the template. Clearly label the graph. You may approximate the plot by using straight lines.



Assume $R_1 = R_2 = 1\text{k}\Omega$ and $C_1 = 10^{-6}\text{F}$ and $C_2 = 10^{-5}\text{F}$.

$\angle H(0) = 0^\circ$

$\angle H(\infty) = 0^\circ$

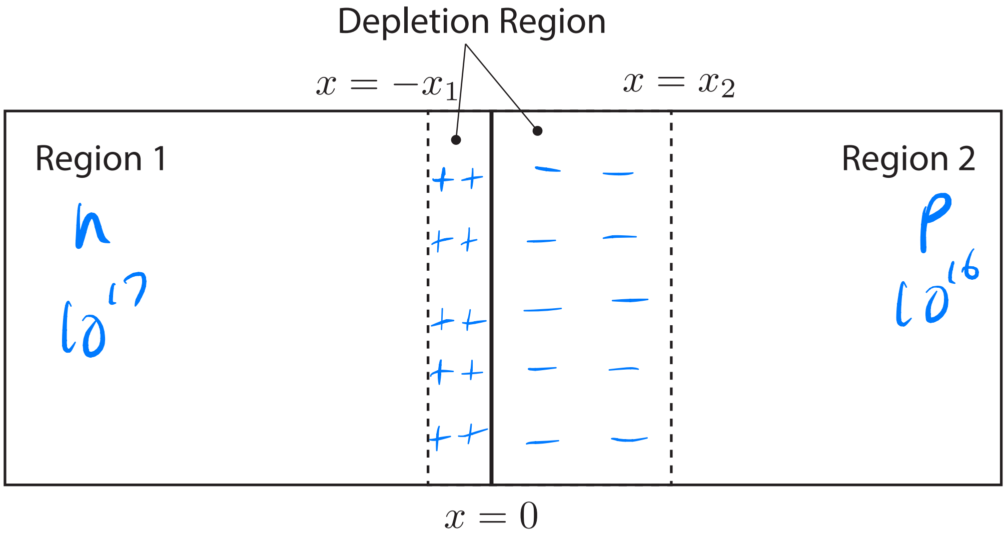
(g) (5 points) How much average power is dissipated in the circuit as a function of frequency normalized to a source $V_{in} = 1\text{V}$. Make sure you setup the problem correctly to get partial credit.

$$Z_{in} = Z_1 + Z_2 = \frac{R_1(1+j\omega R_2C_2) + R_2(1+j\omega R_1C_1)}{(1+j\omega R_1C_1)(1+j\omega R_2C_2)}$$

$$= (R_1+R_2)\cdot\frac{1 + j\omega R_1\|R_2(C_1+C_2)}{(1+j\omega R_1C_1)(1+j\omega R_2C_2)}$$

$$P = \frac{1}{2}\mathrm{Re}(V\cdot I^*) = \frac{1}{2}\mathrm{Re}\left(V_{in}\cdot\frac{V_{in}^*}{Z_{in}^*}\right)$$

$$= \frac{1}{2}|V_{in}|^2\,\mathrm{Re}\left(\frac{Z_{in}}{|Z_{in}|^2}\right) = \frac{1}{2}\frac{|V_{in}|^2}{|Z_{in}|^2}\mathrm{Re}(Z_{in})$$



2. (25 points) Consider the pn-junction shown above at equilibrium (zero bias). The p-region is doped with 10^{16}cm^{-3} dopants and the n-region is doped with a concentration of 10^{17}cm^{-3} dopants.

   (a) (7 points) Label where the p and n regions (region 1 or 2). The dashed lines indicate the border of the depletion region under the depletion approximation. Briefly explain your logic.

   Region 1: n
   Region 2: p
   Reason:

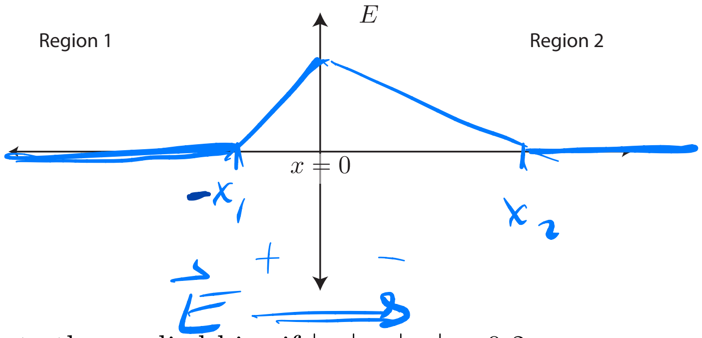
Higher Doping ⇒ smaller depletion region because the charges on each side need to cancel out aka "charge neutrality"

this means $Q_+ = Q_-$ $qAx_{po}N_a = qAx_{no}N_d$

(b) (6 points) Indicate the charge in each depletion region by using circles "plus" and "minus" signs to indicate ionized (immobile) dopants. Re-use the figure above.

(c) (6 points) Suppose the peak electric field in the structure is measured at -2MV/m. Draw the electric field strength on the provided graph and indicate the location and direction of the peak electric field by labeling the graph with $x = x_1$ and $x = x_2$. Note that you must label the x-axis to receive credit.

Typo!



(d) (6 points) Calculate the applied bias if $|x_1| + |x_2| = 0.2\mu\text{m}$.

$$V_{bi} = 60\,\text{mV} \log \frac{10^{17} \cdot 10^{16}}{10^{20}}$$

$$= 60\,\text{mV} \times 13 = 780\,\text{mV}$$

$$x_{dep} = \sqrt{\frac{2\varepsilon_s (V_{bi} - V_D)}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

Only unknown is V_D … solve

$$V_D = +0.497\,V$$

The diode is forward biased!

Alternative answer:

$$V_{bi} - V_D = \int_{-x_1}^{x_2} \mathcal{E}(x)\,dx = \frac{1}{2}\mathcal{E}_0 W$$

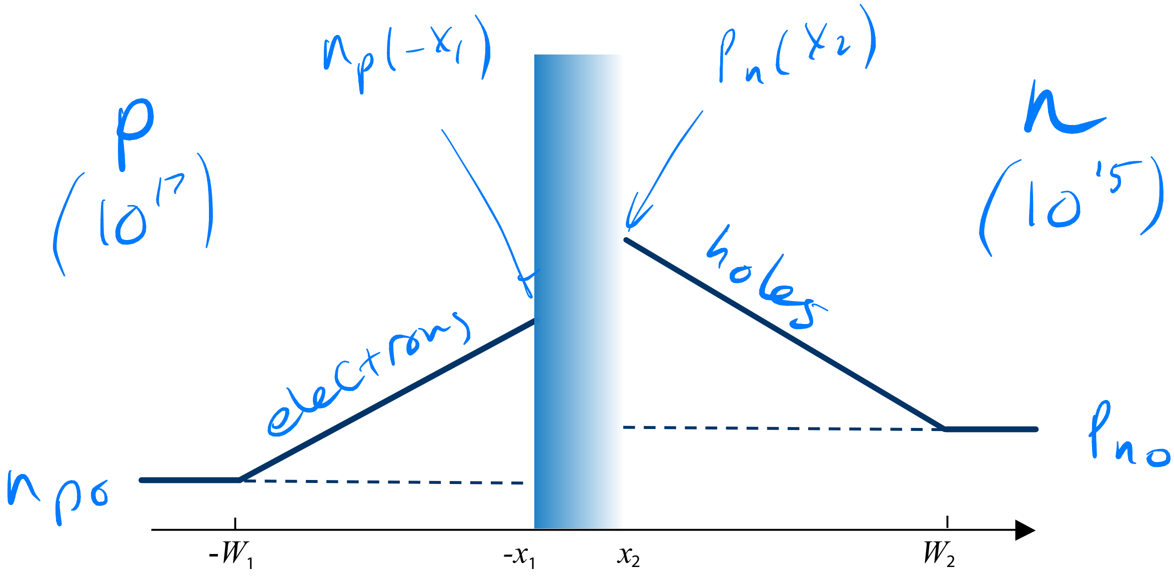
$$= \frac{1}{2}(2E6\,V/m)(0.2\,E{-}6\,m)$$

$V_{bi} - V_D = 0.2\,V$

$V_D = \quad V_b$

$V_D = V_{bi} - 0.2V = 0.580V$ or $0.980V$ if you used $-2E6\,V/m$

3. (30 points) A pn-junction is formed with one region doped with $N_A = 10^{17}\text{cm}^{-3}$ acceptor dopants and the other side doped with $N_D = 10^{15}\text{cm}^{-3}$ donor dopants. A voltage of magnitude $V_D = 0.7\text{V}$ is applied to the junction and the resulting minority carrier distribution is shown in the figure above. $W_1 = .25\mu\text{m}$ and $W_2 = 0.8\mu\text{m}$. For this problem assume electrons have a mobility of $\mu_n = 1000\text{cm}^2/\text{V}\cdot\text{s}$ and holes have a mobility of $\mu_p = 500\text{cm}^2/\text{V}\cdot\text{s}$. Assume the device has a cross-sectional area of $10\mu\text{m}^2$ (into the page).



(a) (2 point) What region of operation is the diode in ? In other words, what is the sign of V_D? Explain.

$V_D > 0$ Forward biased

since minority carriers are injected.

(b) (8 points) Label the graph above: (i) You must label p and n regions on the graph, (ii) Indicate which curve is the electron profile and which curve is the hole profile. (iii) Calculate and label n_{p0} and p_{n0} .

$$N_A > N_D \Rightarrow n_{p0} < p_{n0}$$

$$n_{p0} = \frac{n_i^2}{N_A} = \frac{10^{20}}{10^{17}} = 10^3 \text{ cm}^{-3}$$

$$p_{n0} = \frac{10^{20}}{10^{15}} = 10^5 \text{ cm}^{-3}$$

(c) (5 points) Calculate the concentration of minority carriers n_p and p_n at the depletion edge. Also label the graph.

$V_d = 0.7V$ $\qquad \frac{kT}{q} = 26mV$

$n_p(-x_1) = n_{p0} e^{qV_D/kT}$

$= 5 \times 10^{14} \text{ cm}^{-3}$

$p_n(x_2) = p_{n0} e^{qV_D/kT}$

$= 5 \times 10^{16} \text{ cm}^{-3}$

(d) (5 points) Calculate the current flow due to diffusion. Use $kT/q = D/\mu$.

$q = 1.6 \times 10^{-19}$ C

$\mu_p = 500 \ \mathrm{cm^2/V \cdot s}$ $\mu_n = 1000$

$D_p = \mu_p \frac{kT}{q} = 13 \ \frac{\mathrm{cm^2}}{\mathrm{s}}$ $D_n = 26$

$$J_p = -q D_p \frac{dp}{dW_p} = 1.6 \times 10^{-19} \cdot 13 \cdot \frac{5 \times 10^{16}}{0.8 \times 10^{-4}}$$

$$J_n = q D_n \frac{dn}{dW_n} = 1.6 \times 10^{-19} \cdot 26 \cdot \frac{5 \times 10^{14}}{0.25 \times 10^{-4}}$$

$J_p = 1281 \ \mathrm{A/cm^2}$ $J_n = 82 \ \mathrm{A/cm^2}$

$A = 10 \ \mu\mathrm{m}^2 = 10 \times 10^{-8} \ \mathrm{cm^2}$

$I = (J_p + J_n) \cdot A = 0.136 \ \mathrm{mA}$

(e) (5 points) Qualitatively, discuss the origin of the diode saturation current I_S. First explain why a current flows when the diode is reverse biased. Next discuss why the current is nearly independent of the reverse bias. Finally, discuss the temperature dependence of th saturation current.

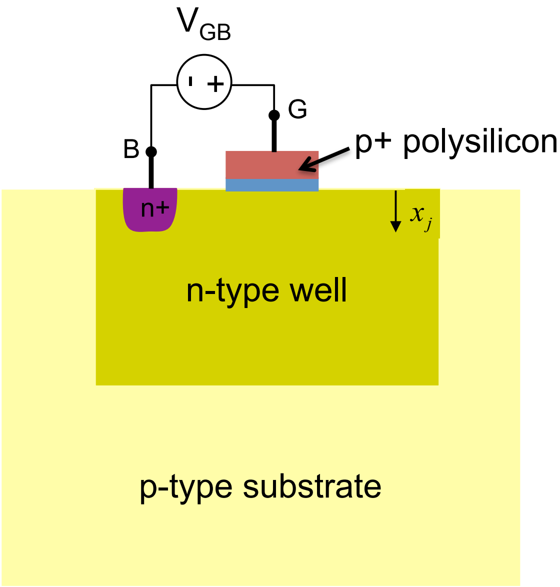
I_s is due to minority carriers generated within a diffusion length of the junction. Since these minority carriers are swept across the junction, the current only depends on the available number of minority carriers, and not the potential.
E field is very strong so they travel at v_{sat}.

Minority carrier density is a strong function of Temp. Current increases with T exponetially.

(f) (5 points) If a light source with a photon flux of $G_L = 10^{15}$photons · cm^{-3} uniformly illuminates the sample, how does this change the saturation current? You may answer this question qualitatively.

The saturation goes up. Every photon that impinges near the depletion region has a chance of creating an electron/hole pair, leading to more minority carriers near the junction.

4. (20 points) The following is a cross-section of a MOS Capacitor in a silicon wafer. For this problem, use Φ_n = 300mV, Φ_{p+} = -550mV, V_{DD} = 0.8V, V_T = -0.45V, t_{ox} = 15nm. Pay careful attention to voltage polarity!

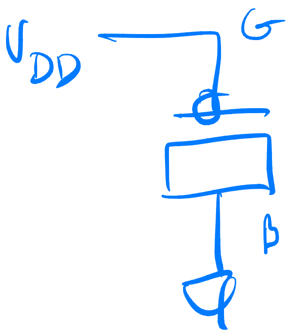


(a) (4 points) What bias voltage V_{GB} do you need to apply in order to achieve the flat-band condition (no charge in the capacitor)?

$$\phi_{bi} = \phi_{p+} - \phi_n = -550\,mV - 300\,mV$$

$$= -850\,mV \qquad V_{FB} = 850\,mV$$

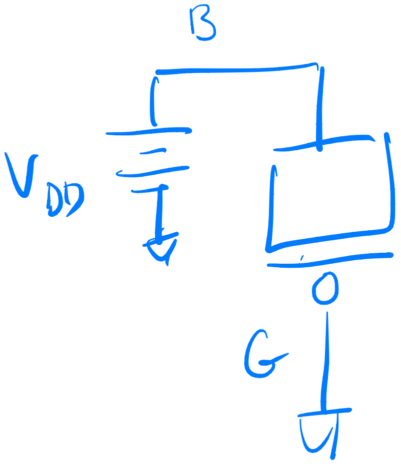
(b) (5 points) You decide to use this capacitor to filter noise on the supply V_{DD} (also known as decoupling). To do this you hook up the gate (G) to V_{DD} and the body (B) to ground. What region of operation is the MOS capacitor in? Justify your answer (a guess without justification will receive no credit).



$$V_{GB} = V_{DD} < V_{FB}$$

DEPLETION!

(c) (5 points) You compare this with the opposite configuration of hooking up the gate (G) to ground and the body (B) to V_{DD}. What region of operation is the MOS capacitor in now? Justify your answer (a guess without justification will receive no credit).



$$V_{GB} = -V_{DD} < V_T$$

$\Rightarrow$ INVERSION

(d) (6 points) Which configuration gives the highest capacitance per unit area? Calculate the capacitance per unit area in this preferred configuration.

INVERSION CAP HIGHER THAN DEPLETION CAP.

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.854 \times 10^{-12}\ F/m}{15 \times 10^{-9}\ m}$$

$$= 2.3 \times 10^{-3}\ F/m^2$$

$$= 2.3 \times 10^{-3}\ \frac{F}{m^2} \cdot \frac{1m^2}{(10^6\ \mu m)^2} = 2.3\ fF/\mu m^2$$