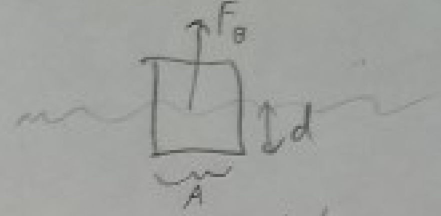
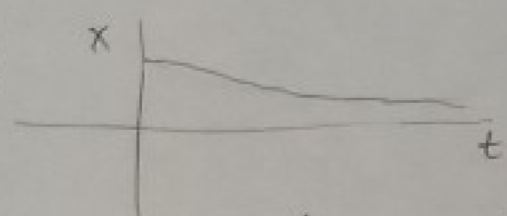
1) a) $p \propto d$

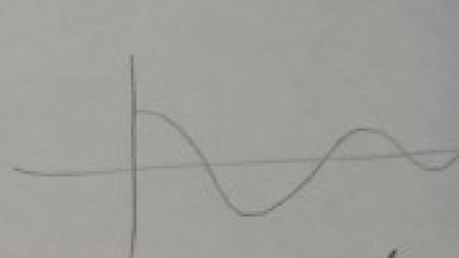


so $F_B = pA \propto d A$ = Volume displaced
We can fill in the proportionality factor by dimensional analysis or using $p = \rho g d$.
$\Rightarrow F_B = \rho g V$

b)



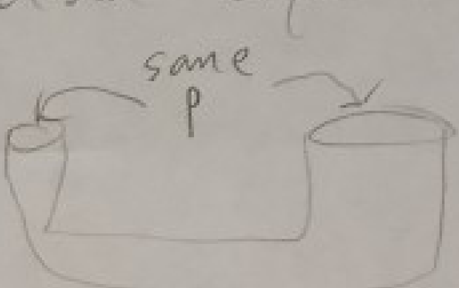
overdamped: exponential decay before any oscillations



underdamped: oscillations with decaying amplitude

critical: border between these

c) Pascal: pressure is transmitted undiminished in an enclosed liquid. $F = PA \Rightarrow$ greater area means greater force
(at cost of less distance raised)

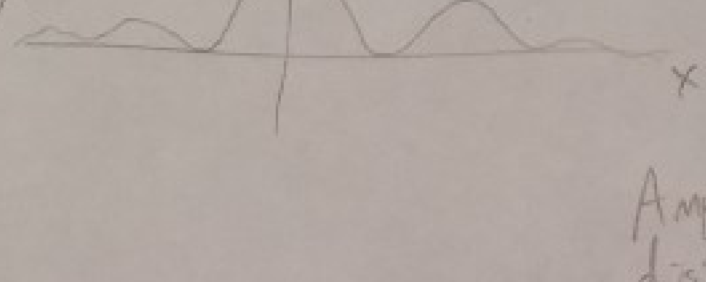


d)

$\xrightarrow{\vec{L}}$ $\vec{r}$, $\vec{F}$ so $\vec{\tau} = \vec{r} \times \vec{F}$ points out of page

So $\vec{L}$ rotates towards you, the reader, by $\frac{d\vec{L}}{dt} = \vec{\tau}$.

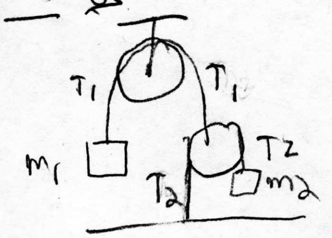
e)

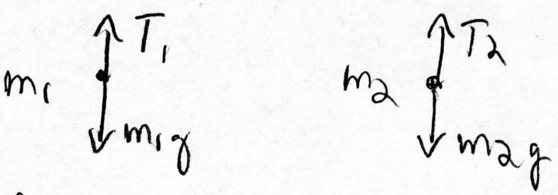


Constructive interference at center and other bumps, destructive interference between,
Amplitude decays at larger x due to distance from wave source.

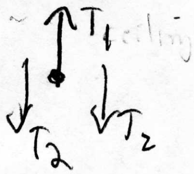
2 a

FBDs for two masses



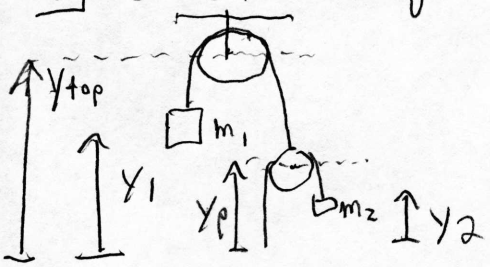


FBDs for lower pulley



Note tension in string is constant or pulleys would experience a net torque (but $I_{pulley} = 0$).

b) Constraint equations come from length of strings:



Note y_{top} is fixed. The length of the top string is

$$\boxed{l_{top,string} = (y_{top} - y_1) + (y_{top} - y_p) + \pi R_{top}}$$

R_{top} is radius of top pulley.

For the bottom string (with R_{bottom} the radius of the bottom pulley),

$$\boxed{l_{bottom,string} = y_p + (y_p - y_2) + \pi R_{bottom}}$$

Taking two time derivatives, we get

$$\boxed{0 = -a_1 - a_p \quad \text{and} \quad 0 = a_p + a_p - a_2}$$

with a_p the acceleration of the bottom pulley, and a_i the acceleration of m_i. Either of these could be the constraint equations.

c) We write N2L from the FBD and use the constraint eq's to solve for a_1:

$N2L^{m_1}: T_1 - m_1 g = m_1 a_1$ $\quad N2L^{m_2}: T_2 - m_2 g = m_2 a_2$

$N2L^{pulley}: T_i - 2T_2 = 0 \Rightarrow m_1 a_1 = 2T_2 - m_1 g = 2(m_2 g + m_2 a_2) - m_1 g$

The constraint equation gives $a_p = \frac{a_2}{2}$ and $a_1 = -a_p = -\frac{a_2}{2}$

$$\Rightarrow m_1 a_1 = -m_1 g + 2 m_2 g + 2 m_2(-2a_1) \Rightarrow a_1(m_1 + 4m_2) = -(m_1+2m_2) g$$

$$\Rightarrow \boxed{a_1 = \frac{-(m_1 + 2m_2)}{(m_1 + 4m_2)} g}$$

Physics 7A - Lecture 2 Final Exam

Problem 3

Collision is elastic, we should apply conservation of energy (1) and conservation of momentum in the x and y direction (2) and (3):

$$\frac{1}{2}mV_0^2 + \frac{1}{2}MV^2 = \frac{1}{2}m\frac{V_0^2}{4} + \frac{1}{2}MV'^2 \tag{1}$$

$$mV_0 - MV = MV'cos(45) \tag{2}$$

$$0 = MV'sin(45) - m\frac{V_0}{2} \tag{3}$$

Where V and V' denote the initial and final velocity of mass M. We see we have 3 equations and 3 unknowns (M/m, V, and V'). Let's solve for V' in (3), then solve for V in (2), and finally substitute in (1) as follows:

$$\begin{aligned} V' &= \frac{\sqrt{2}m}{2M}V_0 \\ \implies V &= \frac{m}{M}V_0(1-\frac{1}{2}) = \frac{m}{2M}V_0 \\ \implies \frac{1}{2}mV_0^2 + \frac{m^2}{8M}V_0^2 &= \frac{1}{8}mV_0^2 + \frac{m^2}{4M}V_0^2 \\ \implies \frac{m}{M} &= 3 \\ \implies \frac{M}{m} &= \frac{1}{3} \end{aligned}$$

(4) (a)

The only force in the x-direction is from the spring, so we have

$F_x = -kx = m a_x$

$$\boxed{\frac{d^2 x}{dt^2} = -\frac{k}{m} x}$$

(b) we need a function $x(t)$ which looks kind of like itself after taking two derivatives. Let's try cosine:

$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

(c) This has three parameters, so we need to eliminate one.

$$\frac{d^2}{dt^2}\left(A\cos(\omega t + \phi)\right) = -\omega^2 A\cos(\omega t + \phi) = -\frac{k}{m} A\cos(\omega t + \phi$$

so $\boxed{\omega = \sqrt{\frac{k}{m}}}$

This gives the ~~[illegible]~~ solution with two free parameters:

$$\boxed{x(t) = A\cos\left(\sqrt{\frac{k}{m}}\, t + \phi\right)}$$

(d) we're given $x(0) = 0$ and $v(0) = v_0$

$$v(t) = -A\sqrt{\frac{k}{m}}\sin\left(\sqrt{\frac{k}{m}}\, t + \phi\right)$$

so

$x(0) = A\cos\phi = 0$

Either $A = 0$ (no oscillation) or $\cos\phi = 0$ $\boxed{(\phi = \frac{\pi}{2})}$

$$v(0) = -A\sqrt{\frac{k}{m}}\sin\left(\frac{\pi}{2}\right) = -A\sqrt{\frac{k}{m}} = v_0$$

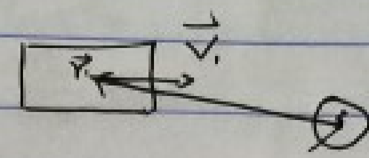
so

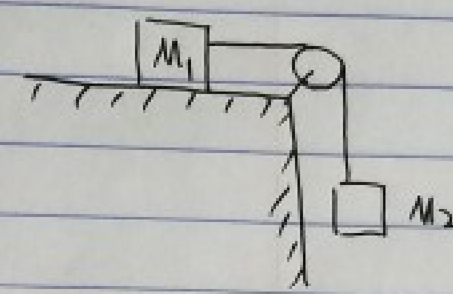
$$\boxed{A = -v_0\sqrt{\frac{m}{k}}}$$

(e) If we plug these into (b), we have

$$\boxed{x(t) = -v_0\sqrt{\frac{m}{k}}\cos\left(\sqrt{\frac{k}{m}}\, t + \frac{\pi}{2}\right)} = \boxed{v_0\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}\, t\right)}$$

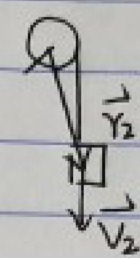
5. a) Angular momentum of M_1:





$$\vec{L_1} = \vec{r_1} \times \vec{p_1}$$
$$= M_1 v \cdot \vec{r_{1\perp}}$$
$$= M_1 v_0 R_0$$

Angular momentum of M_2:



$$\vec{L_2} = \vec{r_2} \times \vec{p_2}$$
$$= M_2 v \cdot \vec{r_{2\perp}}$$
$$= M_2 v R_0$$

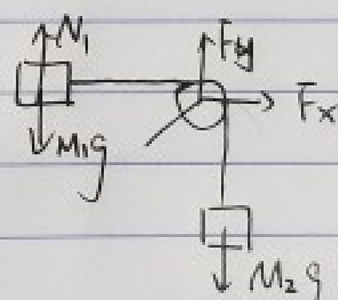
Angular momentum of pulley:
Since the string moves without slipping,
angular velocity $\omega = \frac{v}{R_0}$

$$\vec{L_3} = I\omega = \frac{Iv}{R_0}$$

Using Right hand principle, L_1, L_2, L_3 all pointing inwards
Thus angular momentum of the system is

$$\vec{L} = \vec{L_1} + \vec{L_2} + \vec{L_3} = (M_1 + M_2) v R_0 + \frac{Iv}{R_0}$$

b) Free body ~~dy~~ diagram:



F_x, F_y don't contribute torque on the axis, Torque from N_1 and $M_1 g$ cancel out.

$$\tau = M_2 g R_0 = \frac{dL}{dt} = \left[(M_1 + M_2) R_0 + \frac{I}{R_0}\right] a$$

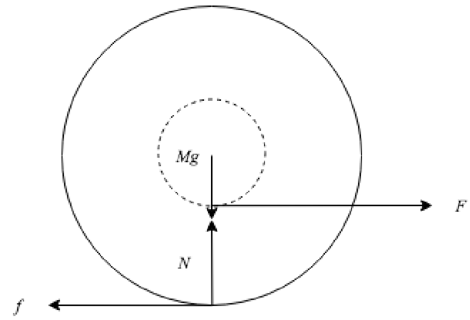
$$a = \frac{M_2 R_0^2}{M_1 R_0^2 + M_2 R_0^2 + I} g$$

Or: Moment of inertia of the system is:

$$\Sigma I = M_1 R_0^2 + M_2 R_0^2 + I$$
$$\tau = M_2 g R_0 = \Sigma I \alpha$$
$$\alpha = \frac{M_2 R_0}{M_1 R_0^2 + M_2 R_0^2 + I}$$

$$a = \alpha \cdot R_0 = \frac{M_2 R_0^2}{M_1 R_0^2 + M_2 R_0^2 + I}$$

6.



The FBD should include F acting at a radius b from the centre of the yo-yo, the weight Mg acting downwards at the centre, the normal force acting upwards at a radius R, and the friction f pointing to the left, acting at a radius R.

We first use the linear Newton's Second Law. The net vertical force must be zero because the yo-yo has no vertical motion:

$$\sum F_y = N - Mg = 0 \implies N = Mg.$$

The net horizontal force is

$$\sum F_x = F - f.$$

We would like to find the threshold value for F between slipping and not slipping; the value of static friction at that threshold is its maximum achievable amount

$$f = f_{\text{max}} = \mu N = \mu Mg.$$

Hence,

$$\sum F_x = F - \mu Mg.$$

By N2L, we find the acceleration of the centre of mass of the yo-yo:

$$F - \mu Mg = Ma_{\text{CM}} \implies a_{\text{CM}} = \frac{F}{M} - \mu g.$$

Next, we use the rotational Newton's Second Law. Setting clockwise to be the positive direction (anticipating the yo-yo to roll clockwise when pulled), the net torque on the yo-yo around its centre of mass is

$$\sum \tau = -bF + R\mu Mg.$$

Note that the torque due to Mg and N is zero because they are parallel to the radius. The angular acceleration can be found:

$$-bF + R\mu Mg = I\alpha \implies \alpha = \frac{1}{I}(R\mu Mg - bF).$$

Rolling without slipping implies

$$a_{\text{CM}} = R\alpha.$$

(This can be obtained by taking a time-derivative of the more familiar $v_{\text{CM}} = R\omega$.) Plugging in the two expressions as well as $I = \frac{1}{2}MR^2$, we have

$$\frac{F}{M} - \mu g = \frac{2}{MR}(R\mu Mg - bF) \implies F = \frac{3\mu Mg}{1 + \frac{2b}{R}}.$$

Note that if you defined anticlockwise to be the positive direction instead, then you might have gotten

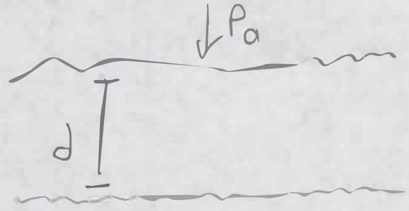
$$F = \frac{\mu Mg}{\frac{2b}{R} - 1}.$$

This is incorrect, because the rolling without slipping condition has to have the correct relative sign. If $a_{\mathrm{CM}} > 0$, that means the wheel rolls clockwise, and so

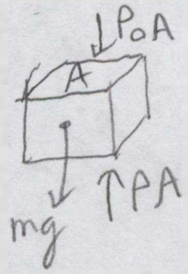
$$a_{\mathrm{CM}} = -R\alpha.$$

Then you would recover the correct result.

1 a If P_0 is pressure at the top of a body of water, then the pressure at a depth d is $P = P_0 + \rho g d$ where $\rho = 1\, g/cm^3$ is the density of water



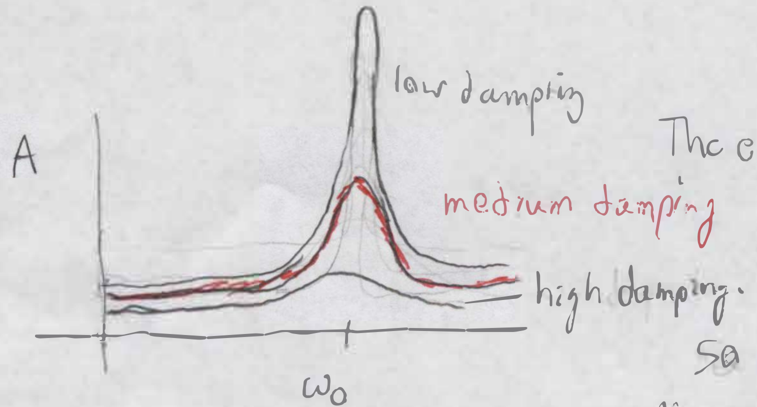
This happens because if we imagine that the water has surface area A, then on the volume of water $A \cdot d = V$ there are three forces: P_0A, PA, and $F_{grav.}$



to not accelerate $P_0A + mg - PA = 0 \Rightarrow P = P_0 + \frac{mg}{A}$

but the mass of water is $m = \rho A d$. $\Rightarrow P = P_0 + \rho d g$.

1 b

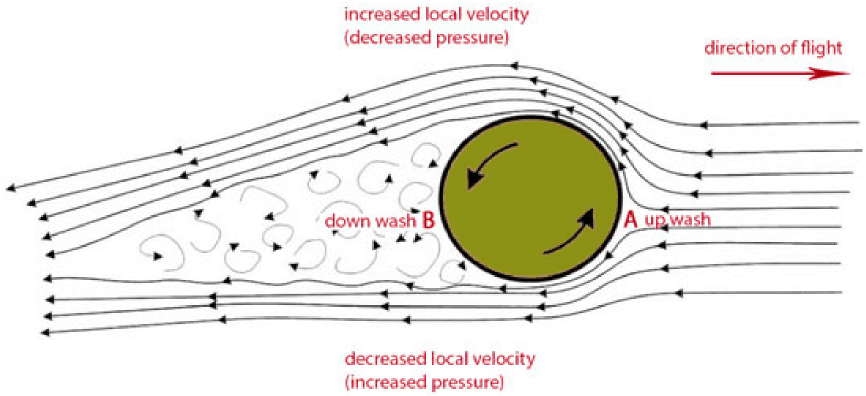


The equation is (and isn't necessary)

$$A = \frac{F_0/m}{\sqrt{(\omega_0-\omega)^2 + 4\gamma^2\omega^2}}$$

So as γ decreases (damping decreases) the peak of A goes up around ω_0 but not much else changes. The width of peak at half max decreases.

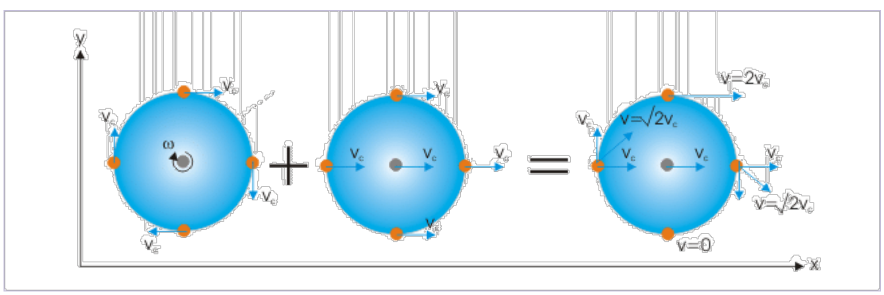
1.(c)



Bernoulli's principle states that an increase in speed in a flowing fluid corresponds to a decrease in pressure. Imagine that we are in the frame of the ball, so that the ball is "not moving" and the air around it is flowing past it. If the ball is not spinning, then the airflow is equal in speed on all sides of the ball. If the ball is spinning, then the side opposing the airflow causes the speed of the current to decrease, while the side promoting the airflow causes the speed of the current to increase. Correspondingly, the pressure on the side promoting the airflow is less than that on the side opposing the airflow. The pressure imbalance causes the ball to experience a "Magnus force" towards the side promoting the airflow, thus curving the trajectory of the flying ball.

The sketch should include the direction of spin, the direction of wind (or the ball's motion), and locations of higher and lower airflow speed or pressure.

1.(d)



Approach 1: To obtain the velocity of a point on the wheel relative to the ground, we add v [its velocity relative to the centre of mass of the wheel] to v_{cm} [the velocity of the centre of mass of the wheel relative to the ground]. For a wheel rotating at angular velocity ω, all points at the same radius from the centre have the same speed $v = \omega r$, while the velocity always points in the tangential direction. Rolling without slipping implies $v_{cm} = \omega r$.

For the top point, the velocity points "forwards" with magnitude $v = \omega r$. For the bottom point, the velocity points "backwards" with magnitude $v = \omega r$. The CM velocity points "forwards" with magnitude $v_{cm} = \omega r$. Thus the velocities of the top and bottom points relative to the ground are, respectively, $v_{cm} + v = 2\omega r = 2v_{cm}$ and $v_{cm} - v = 0$.

Approach 2: Rolling without slipping implies that the bottom of the wheel has zero velocity relative to the ground. Instantaneously, treating this point as the pivot, the centre of mass velocity is $v_{cm} = \omega r$, and the velocity of the top of the wheel is $2\omega r = 2v_{cm}$.

1.(e) We first write $k = 2\pi/\lambda$ and $\omega = 2\pi/T$ in terms of the more familiar wavelength λ and period T. The wavelength is the distance between one peak and the next at any given time, and the period is the time duration between one peak and the next on any given point on the wave. This means that the wave must travel one wavelength is exactly one period. The wave velocity is thus

$$v = \frac{\lambda}{T} = \frac{\omega}{k}.$$

This can be verified by a simple dimensional analysis: ω has units of s^{-1} and k has units of m^{-1}.

Alternatively, we may track the position x and time t where the peak ($y = A$) occurs. Since $y = A$ whenever $kx - \omega t = 0$, we have

$$x = \frac{\omega}{k} t.$$

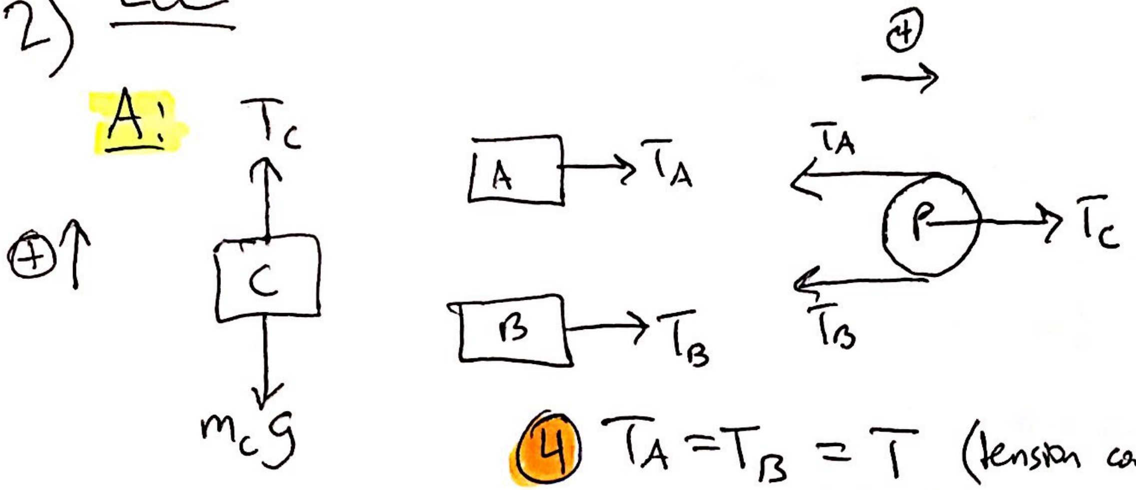
The speed of the wave is

$$v = \frac{dx}{dt} = \frac{\omega}{k}.$$

Note that it is incorrect to take the derivative dy/dt in an attempt to get the velocity. The time-dependent velocity obtained in this way is that of a single oscillating element along the wave, not the time-independent velocity of the wave pattern travelling along.

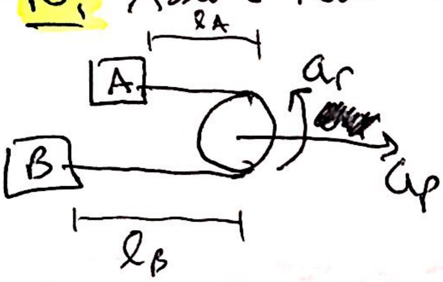
2) Lee

A:



④ $T_A = T_B = T$ (tension constant throughout rope)

B: Assume relative direction for ~~[illegible]~~ A, B accelerations



① $a_A = a_p - a_r$

② $a_B = a_p + a_r$

③ $|a_p| = |a_c|$ (since connected on same rope)

$\ell_{A_r} + \ell_{B_r} = \ell_{total} = \text{constant}$

$\frac{d^2}{dt^2} \rightarrow a_{A_r} + a_{B_r} = 0$

$\boxed{a_{Ar} = -a_{Br}}$

$\boxed{|a_{Ar}| = |a_{Br}| = a_r}$

C: ⑤ $\Sigma F_c = T_c - m_c g = m_c a_c$

⑥ $\Sigma F_B = T_B = m_B a_B$

⑦ $\Sigma F_A = T_A = m_A a_A$

⑧ $\Sigma F_p = T_c - T_A - T_B = 0$

8 unknowns: $a_A, a_B, a_c, T_A; T_B, T_c, a_R, a_p$

8 equations => good to go

→ continued on next page

C:

- Focusing on the left pulley side

(6) $T = m_B a_B$

(7) $T = m_A a_A$

$$m_B a_B = m_A a_A$$

Plug in (1) + (2)

$$m_B(a_p + a_r) = m_A(a_p - a_r)$$

$$m_B a_p + m_B a_r = m_A a_p - m_A a_r$$

$$(m_B - m_A) a_p = -(m_B + m_A) a_r$$

$$(m_A - m_B) a_p = (m_B + m_A) a_r$$

$$\boxed{a_r = \left(\frac{m_A - m_B}{m_B + m_A}\right) a_p}$$

Plugging a_r back into (1) + (2)

(1) $a_A = a_p - \left(\frac{m_A - m_B}{m_B + m_A}\right) a_p = \boxed{\left(\frac{2m_B}{m_A + m_B}\right) a_p}$

(2) $a_B = a_p + \left(\frac{m_A - m_B}{m_A + m_B}\right) a_p = \boxed{\left(\frac{2m_A}{m_A + m_B}\right) a_p}$

- Focusing on the pulley

(8) $T_c = T_A + T_B$

$* \boxed{T_c = 2T}$

- Focusing on the right pulley side

(5) $T_c - m_c g = -m_c a_c \rightarrow$ plug in (3)

$T_c - m_c g = -m_c a_p \rightarrow$ plug in (8) *

$2T - m_c g = -m_c a_p \rightarrow$ plug in (6) or (7) for T

$2(m_B a_B) - m_c g = -m_c a_p \rightarrow$ plug in (2)

$2 m_B\left(\frac{2m_A}{m_A + m_B}\right) a_p - m_c g = -m_c a_p$

$\frac{4 m_A m_B}{m_A + m_B} a_p - m_c g = -m_c a_p$

$$\left(\frac{4 m_A m_B}{m_A + m_B} + m_c\right) a_p = m_c g$$

$$\left(\frac{4 m_A m_B + m_c m_A + m_c m_B}{m_A + m_B}\right) a_p = m_c g$$

$$\boxed{a_p = \frac{m_c(m_A + m_B) g}{4 m_A m_B + m_c(m_A + m_B)}}$$

Plug back into (1), (2), (3)

$\rightarrow$

C:

$$a_A = \frac{2m_B}{m_A+m_B}\left(\frac{m_c(m_A+m_B)g}{4m_Am_B+m_c(m_A+m_B)}\right) \quad \text{right}$$

$$a_B = \frac{2m_A}{m_A+m_B}\left(\frac{m_c(m_A+m_B)g}{4m_Am_B+m_c(m_A+m_B)}\right) \quad \text{right}$$

$$a_c = \frac{m_c(m_A+m_B)g}{4m_Am_B+m_c(m_A+m_B)} \quad \text{down}$$

7A Lecture 3 Final exam Question 3 Solutions

7A GSI

May 11, 2018

1 Part A

As seen in the diagram 1, we have a chain of length L falling onto the scale. At some time instance, one can see that the chain has dropped x amount of distance and the speed of the chains is v going downwards. One should expect that there is a normal force by the chain onto the scale which creates the reading of the scale.

Also, one should see that the normal force compose of 2 parts. 1. the weight of the chain which is already at rest on the scale. 2. the force needed to stop the chain for the dx portion.

Define a linear density $\lambda = \frac{M}{L}$, we have the following:

$$F_{\text{normal}} = F_{\text{stopping}} + F_{\text{gravity}}. \tag{1}$$

It is easy to see that

$$\begin{aligned} F_{\text{gravity}} &= mg \qquad (2) \\ &= \lambda x g. \qquad (3) \end{aligned}$$

While on the term, we need to apply the relationship between momentum and force

$$\begin{aligned} F_{\text{stopping}} &= \frac{dp}{dt} \qquad (4) \\ &= \frac{dmv}{dt} \qquad (5) \\ &= \frac{\lambda dxv}{dt} \qquad (6) \\ &= \lambda v^2 \qquad (7) \end{aligned}$$

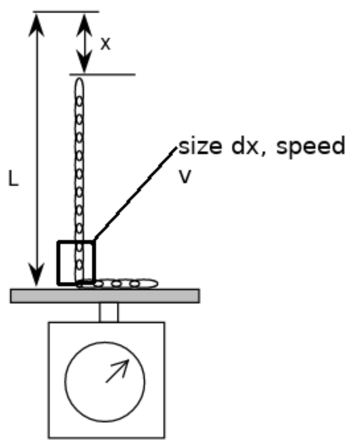


Figure 1: Diagram for the exam

But the speed v is changing, we need to express it in terms of x. We can apply kinematics

$$
\begin{aligned}
v^2 &= u^2 + 2a\Delta x && (8)\\
&= 2gx. && (9)\\
& && (10)
\end{aligned}
$$

Therefore, putting everything together

$$
\begin{aligned}
F_{\text{normal}} &= F_{\text{stopping}} + F_{\text{gravity}} && (11)\\
&= \lambda xg + 2\lambda xg && (12)\\
&= 3\lambda xg. && (13)\\
& && (14)
\end{aligned}
$$

The problem did not specify what scale it is, lets assume it gives the mass reading by measuring the normal force.

Then the final answer is $3\lambda x$.

2 Part B

The reading increases linearly with the length which has fallen, so the maximum reading occurs at $x = L$

3 Remarks

Many student only used only the weight of the chain which is at rest on the pan, that will *severely* reduce the points that one will get because one is not setting up the forces correctly, no calculation using the force-momentum relation and also losing points for getting incorrect answer in both parts.

Energy conservation does not work in here since the collision is inelastic. Any answers related to energy conservation may have points deducted.

Any reasonable mentioning of force-momentum relation will earn one's quite a few points.

Physics 7A - Lecture 3 Final Exam

Problem 4

(For full credit result needed to be derived not simply stated)

a) Consider the sum of the Torques about the pivot:

$$\sum \tau = I\alpha = \vec{r} \times \vec{F}_{grav} \tag{1}$$

$$\implies ml^2 \frac{d^2\theta}{dt^2} = -mglsin(\theta) \approx -mgl\theta \quad \text{for small } \theta \tag{2}$$

$$\implies \frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \tag{3}$$

This is the differential equation describing simple harmonic motion, we can read off the angular frequency $\omega = \sqrt{\frac{g}{l}}$, which implies the regular freuency $\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}}$ and the period is $\boxed{T = \frac{1}{f} = 2\pi\sqrt{\frac{l}{g}}}$

b) Follow a similar derivation only now we consider the moment of inertia, I and the force of gravity now acts at the center of mass rather than at a distance l:

$$\sum \tau = I\alpha = \vec{r} \times \vec{F}_{grav} \tag{4}$$

$$\implies I \frac{d^2\theta}{dt^2} = -mghsin(\theta) \approx -mgh\theta \quad \text{for small } \theta \tag{5}$$

$$\implies \frac{d^2\theta}{dt^2} = -\frac{mgh}{I}\theta \tag{6}$$

Again, we read off the angular frequency $\omega = \sqrt{\frac{mgh}{I}}$, which implies the regular freuency $\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{mgh}{I}}}$ and the period is $\boxed{T = \frac{1}{f} = 2\pi\sqrt{\frac{I}{mgh}}}$

Final Exam Physics 7A Lecture 3 Problem 5 Solution

GSI: James Reed Watson

May 10, 2018

(a) Set up free body diagrams and read of the equations of motions for translation and rotation:

$$Mg - T = Ma \tag{1}$$
$$TR_0 = I\alpha \tag{2}$$
$$a = \alpha R_0 \tag{3}$$

Plug (3) into (2) to find that $T = Ia/R_0^2$. Solve (1) for $T = M(g-a)$ and plug in the result from before to find $Ia/R_0^2 = M(g-a)$. This means that:

$$a = \frac{Mg}{M + I/R_0^2} = \frac{g}{1 + I/MR_0^2} = \frac{g}{1 + \frac{R^2}{2R_0^2}} \tag{4}$$

Everything is in-plane, so $L = I\omega$. The acceleration is uniform, so the angular acceleration is also uniform. This means that $L = I\alpha t = Iat/R_0$. Plugging in the above result:

$$L(t) = \frac{MR^2}{2R_0} \frac{gt}{1 + \frac{R^2}{2R_0^2}} = \frac{MR_0 gt}{1 + 2R_0^2/R^2} \tag{5}$$

Which has the correct units of $[kg][m^2][s^{-1}]$.

There is a second route to the answer, using energy conservation.

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \tag{6}$$
$$Mg\omega R_0 t/2 = \frac{1}{2}M\omega^2 R_0^2 + \frac{1}{2}I\omega^2 \tag{7}$$
$$\omega = \frac{gt}{R_0(1 + I/MR_0^2)} \tag{8}$$
$$L = I\omega = \frac{gt}{R_0(1/I + 1/MR_0^2)} = \frac{MR_0 gt}{1 + 2R_0^2/R^2} \tag{9}$$

(b) The tension in the string is constant in time. We can plug in the result for a into the equation we solved before to find:

$$T = \frac{MR^2}{2R_0^2} \frac{g}{1 + \frac{R^2}{2R_0^2}} = \frac{Mg}{1 + 2R_0^2/R^2} \tag{10}$$

Which has the correct units of $[kg][m][s^{-2}]$.

In either method, using forces or energy conservation, one can simply take $T = L/(R_0 t)$.

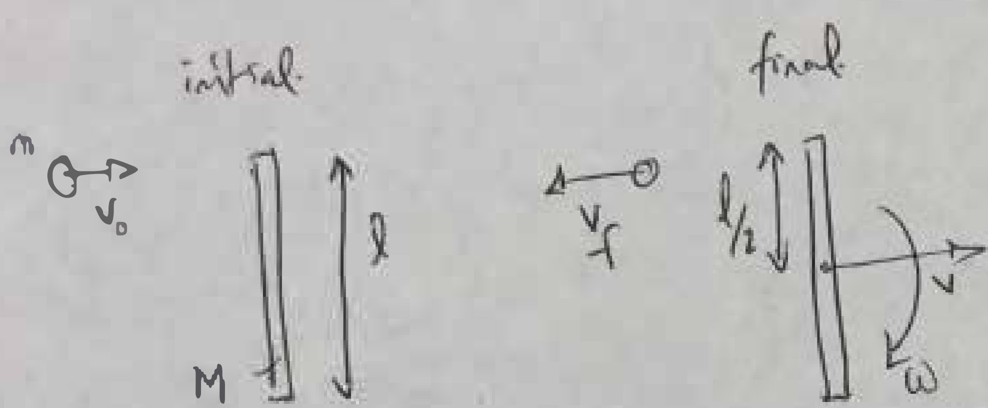
Problem #6

$$I_0 = \frac{1}{12} M l^2$$

a) Momentum & Energy Conservation

(1) $m v_0 = M v - m v_f$ (3)

(2) $\frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} M v^2 + \frac{1}{2} I_0 \omega^2$ (3)



Angular momentum about COM

(3) $m v_0 \frac{l}{2} = - m v_f \frac{l}{2} + I_0 \omega$ (3)

(3) from eq (1) $v = \frac{m}{M} (v_0 + v_f)$

from eq (3) $\omega = m (v_0 + v_f) \frac{l}{2 . I_0}$

substitute in eq (2) and solve for v_f

(2) $$v_f = \frac{(4 I_0 m - 4 I_0 M + l^2 m M) v_0}{4 I_0 m + 4 I_0 M + l^2 m M}$$

b) Energy conservation

$$① \quad \frac{1}{2} M v_0^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} I_{piv} \omega^2 \qquad ③$$

Angular momentum about pivot.

$$I_{piv} = \frac{1}{3} M \ell^2$$

$$② \quad M v_0 \ell = -m v_f \ell + I_{piv} \omega \qquad ③$$

from eq. ②

$$② \quad \omega = \frac{\cancel{2} m \ell (v_0 + v_f)}{I_{piv}}$$

substitute in eq ① and solve for v_f.

$$② \quad v_f = \frac{-\ell^2 m^2 v_0 - \sqrt{I_{piv}^2 m\, m v^2 + I_{piv} \ell^2 m^2\, m v^2 - I_{piv} \ell^2 m^3 v^2}}{I_{piv} m + \ell^2 m^2}$$

=(M - 3 m) v0 / (M + 3m)