Physics 7A, Spring 2018, Section 2 Instructor: Professor Adrian Lee
Final Examination, Thursday, May 10, 2018

Please do work in your blue/greenbooks. Show your reasoning carefully so that we can be sure that you derived the answer rather than guessing it or relying on memory; in addition, this enables us to give partial credit. You may use three double-sided 3.5 in. x 5 in. index cards of notes. Test duration is three hours. Simple calculators are allowed.

1 Qualitative Questions [25 pts. total].

Answers should be a short paragraph only, perhaps with a drawing, and very little, if any, math.

a) Please explain Archimedes Principle starting with the fact that the pressure in a body of water is proportional to depth. A few equations may be needed for the answer here. [5 pts.]

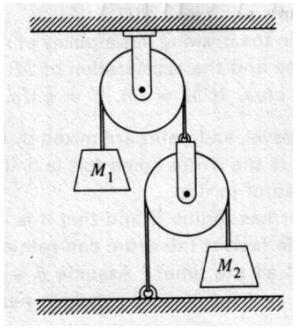
b) For a damped oscillator, please explain how the oscillator behaves in the overdamped, critically damped, and underdamped regimes. Make small plots of the oscillator movements as a function of time. [5 pts.]

c) Please explain how Pascal's Principle allows mechanical leverage in a hydraulic lift. [5 pts.]

d) In class, I made a gyroscope by spinning a bike wheel on its axis and then lifting it using a string tied to the axle a distance 10cm away from the center of the axle. With a small vector diagram explain why the gyroscope precesses and why it rotates in the direction that it does. [5 pts.]

e) In your large bathtub at home, you make waves in the same phase with both of your hands a few feet apart in the x axis. What will the pattern of interference be on the far side of the bathtub? Make a plot of the wave amplitude vs. the x variable but far away in the y direction which is perpendicular to the x axis. What causes this pattern? You likely do not need equations for your answer. [5 pts.]

2 Weights and Pulleys [25 pts. total]



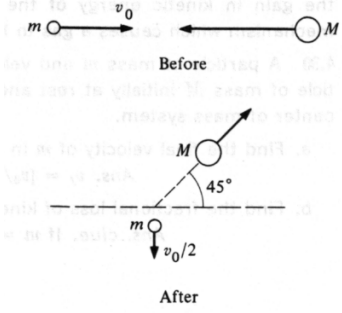
Masses M_1 and M_2 are connected to a system of strings and pulleys as shown in the figure. The ceiling and floor are fixed in height. The strings are massless and inextensible (can't stretch), and the pulleys are massless and frictionless.

a) Draw free-body diagrams as appropriate for this system. [5 pts.]

b) Write constraint equations as appropriate for this system. [10 pts.]

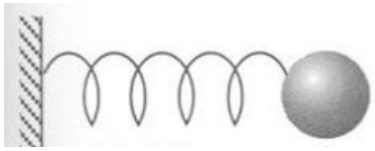
c) Find the acceleration of M_1. [10 pts.]

3 A Collision [25 pts. total]



A particle of mass m and initial velocity v_0 collides elastically with a particle of unknown mass M coming from the opposite direction as shown in the figure. After the collision m has velocity $V_0/2$ at right angles to the incident direction, and M moves off in the direction shown in the figure. Find the ratio M/m.

4 Harmonic Oscillator [25 pts. total]



Consider a mass m connected to a horizontal spring with spring constant k and the other end of the spring is connected to a fixed wall. The mass slides on the ground without friction.

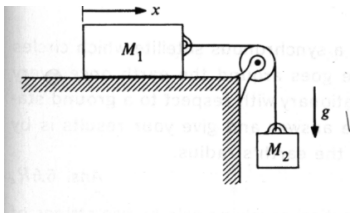
a) Write down a differential equation for the position of the mass x as a function of time t. [5 pts.]

b) Guess a solution with two free parameters. [5 pts.]

c) Plug your guess solution into the differential equation and find the frequency of oscillation. [10 pts.]

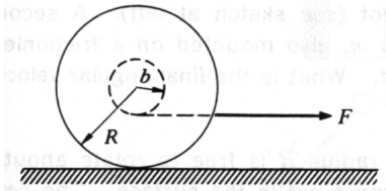
d) Assume that at $t = 0$, the velocity of the mass is v_0 and the spring is at its natural, relaxed length. Find the free parameters of the solution you found in part *b*. [5 pts.]

5 A pulley with mass [25 pts. total]



The figure to the right shows two masses connected by a cord passing over a pulley of radius R_0 and moment of inertia I. Mass M_1 slides on a frictionless surface, and M_2 hangs freely. The string moves without slipping along the pulley.
a) Determine a formula for the angular momentum of the system about the pulley axis, in terms of the speed v of mass M_1 or M_2. [20 pts]
b) Determine the acceleration of the masses. [5 pts]

6 Yo-Yo [25 pts. total]



A Yo-yo of mass M has an axle of of radius b and a spool of radius R. Its moment of inertia around its center can be taken as to be $MR^2/2$. The Yo-yo is placed upright on a table and the string is pulled with a horizontal force F as shown. The coefficient of friction between the yo-yo and the table is μ. What is the maximum value of F for which the yo-yo will roll without slipping? [25 pts]

Physics 7A, Spring 2018, Section 3 Instructor: Professor Adrian Lee
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1 Qualitative Questions [25 pts. total].

Answers should be a short paragraph only, perhaps with a drawing, and very little, if any, math.

a) How does the pressure in a body of water vary with depth? Assume that there is gravity described by "g" and that the water is incompressible. Please explain why this dependence occurs using simple reasoning and a few equations. [5 pts.]

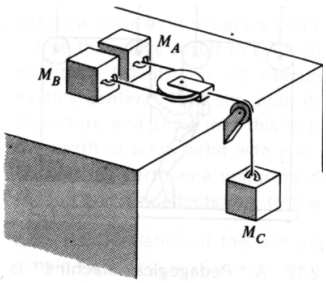
b) For a forced harmonic oscillator, sketch its response in amplitude as a function of frequency for the underdamped case for several values of the damping. No or very few equations are needed. [5 pts.]

c) Explain why a spinning tennis ball curves while flying using Bernouli's Principle. The "curve" is compared to the trajectory with no spin, which is usually a parabola under gravity. No or very few equations are needed. Please make a sketch. [5 pts.]

d) Consider a bicycle wheel rolling without slipping on a road. If the center of the wheel is moving with v, what is the relative velocity of the top and bottom of the wheel to the ground. This question requires a few simple equations and a drawing. [5 pts.]

e) If a traveling wave is described by $y = Acos(kx - \omega t)$, what is the velocity of the wave in terms of k and ω? There are at least two ways to explain the velocity, but please give a short derivation rather than just quoting the answer. [5 pts.]

2 Weights and Pulleys [25 pts. total]



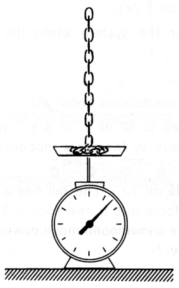
Two masses, M_A and M_B, lie on a frictionless table as shown in the figure. They are attached to either end of a light rope of length l which passes around a pulley of negligible mass. The pulley is attached to a rope connected to a hanging mass, M_C.

a) Draw free-body diagrams as appropriate for this system. [5 pts.]

b) Write constraint equations as appropriate for this system. [10 pts.]

c) Find the acceleration of each mass. [10 pts.]

3 Chain Reaction [25 pts. total]

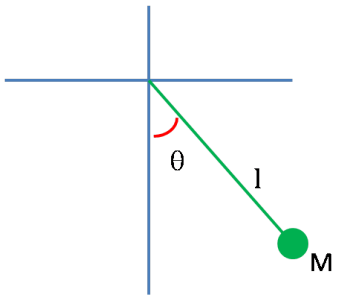


A chain of mass M and length l is suspended vertically with its lowest end touching a scale. The links in the chain are small, so you can consider it uniform in density. The chain is released and falls onto the scale.

a) What is the reading of the scale when a length of chain, x has fallen onto the scale? [20 pts]

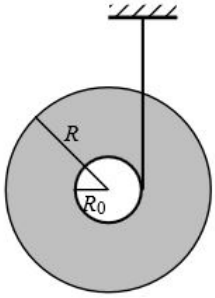
b) What is the maximum reading and for what value of x does it occur? [5 pts]

4 Pendulations



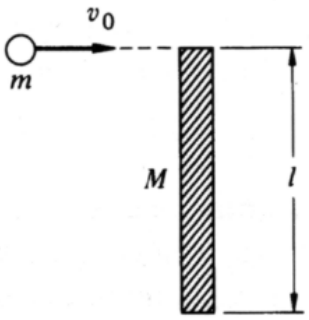
Consider a simple pendulum consisting of a mass m hanging on a massless string of length l.
a) In the limit of very small amplitude oscillations, find the frequency and period of oscillation. Please do not quote a known result, but derive the result from elementary equations. [20 pts]
b) Consider the case where the mass of the string is substantial so the pendulum is better described by a moment of inertia I rotating around the pivot point. Again in the limit of small oscillations, find the period of oscillation. Assume that the center of mass is at a distance h from the pivot point. And again, please calculate the result beginning from elementary equations. [5 pts]

5 Falling Yo-Yo [25 pts. total]



A thin string is wrapped around a cylindrical yo-yo of radius R and mass M with an axle with radius R_0. One end of the string is fixed, and the yo-yo is allowed to fall vertically, starting from rest, as the string unwinds. The moment of inertia of the yo-yo about its center is $1/2\ MR^2$.
a) Determine the angular momentum of the yo-yo about its center-of-mass as a function of time. [15 pts.]
b) What is the tension in the string as a function of time? [10 pts.]

6 Ball Hits Plank [25 pts. total]



A plank of length l and mass M lies on a frictionless plane. A ball of mass m and speed v_0 strikes its end in an elastic collision as shown.
a) Find the final velocity of the ball, v_f, if the velocity is along the original line of motion (that is, purely forward or backward). [15 pts.]
b) Find v_f assuming that the stick is pivoted at the lower end [10 pts.]